Heterotic and type II orientifold compactifications on SU(3) structure manifolds

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Abstract

We study the four-dimensional $N = 1$ effective theories of generic $SU(3)$ structure compactifications in the presence of background fluxes. For heterotic and type IIA/B orientifold theories, the $N = 1$ characteristic data are determined by a Kaluza-Klein reduction of the fermionic actions. The Kähler potentials, superpotentials and the D-terms are entirely encoded by geometrical data of the internal manifold. The background flux and the intrinsic torsion of the $SU(3)$ structure manifold, gives rise to contributions to the four-dimensional F-terms. The corresponding superpotentials generalize the Gukov-Vafa-Witten superpotential. For the heterotic compactification, the four-dimensional fermionic supersymmetry variations, as well as the conditions on supersymmetric vacua, are determined. The Yukawa couplings of the theory turn out to be similar to their Calabi-Yau counterparts.

Zusammenfassung

Wir untersuchen die vierdimensionalen effektiven $N = 1$ Theorien generischer $SU(3)$-Struktur-Kompaktifizierungen bei Anwesenheit von Flüssen. Die charakteristischen $N = 1$ Daten werden für heterotische und Typ IIA/B Orientifold Theorien durch Kaluza-Klein-Reduktion der fermionischen Wirkung bestimmt. Die Kähler-Potentiale, Superpotentiale und D-Terme sind gänzlich in den geometrischen Daten der internen Mannigfaltigkeit kodiert. Der Hintergrund-Fluss und die intrinsische Torsion der $SU(3)$-Struktur-Mannigfaltigkeit geben Anlass zu Beiträgen zu den vierdimensionalen F-Termen. Das korrespondierende Superpotential verallgemeinert das Gukov-Vafa-Witten-Superpotential. Für heterotische Kompaktifizierungen werden sowohl die vierdimensionalen Supersymmetrie-Variationen als auch die Bedingungen an supersymmetrische Vakua bestimmt. Die Yukawa-Kopplungen der Theorie stellen sich als ähnlich zu ihren Calabi-Yau-Gegenstücken heraus.

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Contents

Chapter 1

Introduction

Over the last centuries great progress has been made in understanding our universe and a wide range of physical phenomena at various length scales. One of the major scientific advancements was the discovery of the building blocks of matter. Quarks and leptons have been identified as the elementary constituents of any known form of matter. Their interactions, the strong, weak and the electromagnetic one are described by a gauge theory with gauge group $SU(3) \times SU(2) \times U(1)$ which is known as the Standard Model (SM) of particle physics (for a review see [1]). The Standard model describes particle physics up to the order of 100 GeV with very high precision. Up to the present date, all experimental tests have shown agreement with the Standard Model predictions. Despite of this impressive success the Standard model is not a 'complete theory' describing our universe. This is due to the fact that it contains a large number of free parameters such as particle masses. These are not predicted by the Standard Model but must be determined experimentally. A possible mechanism to generate masses for the particles is known as the Higgs effect. In this the so-called Higgs scalar gives masses to the particles as soon as it acquires a non-zero expectation value in the vacuum. However, the Standard Model with the Higgs mechanism as a mechanism for gauge symmetry breaking does not forbid large quantum corrections to the scalar masses, and one needs unnatural fine-tuning of the bare masses in order to arrange the desired scales. This is known as the naturalness problem. Furthermore the SM does not explain the hierarchy between the scales. It is still unclear why the scale of electroweak symmetry breaking is so tiny with respect to the Planck mass.

In addition to these drawbacks one has to remark that the Standard Model only describes particle physics phenomena for which gravity is negligible. This fact is related to another important defect of the Standard Model, which is its incompatibility with gravity. The latter, instead, is described by Einstein's General Theory of Relativity (GR). All these problems hint to the fact that the Standard Model is not a fundamental theory but rather an effective description of another underlying theory which includes general relativity.

One of the first attempts to reduce the number of free parameters in the Standard Model has been the construction of Grand Unification Theories (GUT) [2]. These theories have larger gauge groups, such as $SU(5)$ or $SO(10)$, which can accommodate the standard model gauge group $SU(3) \times SU(2) \times U(1)$. Via the mechanism of spontaneous symmetry breaking the unifying gauge group reduces to the standard model gauge group. However, not even the GUTs are complete in describing our world. One of the criticisms of these theories is related to their prediction of proton decay which has not been observed in nature.

Another attempt to address the problems of the Standard Model has been the implementation of a symmetry between fermions and bosons. This symmetry, called supersymmetry (SUSY), predicts a fermionic superpartner for each known boson and vice versa. The Supersymmetric Standard Model (SSM) [3] solves some other problems of the Standard Model as well. This is due to the fact that SUSY forbids large quantum corrections to the scalar masses. Hence there is a natural protection of the Higgs mass so that there is no more need to fine-tune this parameter. However, it is disappointing that the superpartners have not yet been seen in our world. Regarding this fact one can argue that if supersymmetry exists then it must appear in its broken phase. Luckily enough, even in its (softly) broken phase SUSY forbids large corrections to the scalar masses such that the Higgs mass remains of the order of the weak scale.

The Supersymmetric Standard Model predicts the unification of all three gauge couplings. However, General Relativity is not a part of this unification. This is due to the fact that GR is different in nature. Einstein's theory explains gravity not as being a force or an interaction but rather as a manifestation of curved spacetime due to its energy content. This theory has passed various kinds of tests for large scale phenomena with great success. But at the same time as GR is proving its unambiguous status it is also showing its incompatibility with quantum mechanics. Due to its ultra-violet divergences it is constrained to phenomena where the quantum effects are negligible. This incompatibility is expected to become a problem when considering regimes such as black hole physics and early time cosmology where quantum effects play a considerable role. Moreover, the presence of singularities in GR such as black holes is a disconcerting issue. As a result, there is also a reason to think of a theory beyond Einstein's theory. One can hope to find a more fundamental theory which furthermore intends to unify the Standard Model and General Relativity.

There were, and still are, many attempts to formulate this fundamental theory. One way to do so is the construction of a quantum theory of gravity which has GR as classical limit. One such construction is provided by Loop Quantum Gravity theory [4]. However, the efforts of this field are concentrated on a quantum reformulation of gravity not on the unification of the four existing interactions. On the latter there exists a significantly more promising solution known as String Theory.

String Theory [5, 6, 7] is a strong candidate for a fundamental unification of the SM and GR. Its building blocks are one-dimensional extended objects (strings) rather than zero-dimensional pointlike particles. Strings can be either closed or open. The fundamental string has a characteristic energy (tension) of the order of the Planck energy (10^{19} GeV) referred as $1/\sqrt{\alpha'}$ where α' is known as the Regge slope. The extended nature of the string appears only at the string scale. Particles in string theory arise as vibrations of the string. Included in these vibrations is a particle with zero mass and spin two which can be identified with the graviton, the proposed messenger of the gravitational force. Furthermore, String Theory predicts well-defined interactions for the graviton at any loop order. This is due to the extended nature of strings which avoids the problem associated with the presence of pointlike particles. Particle interactions occur at a single point of spacetime which is dramatic for the graviton because of the associated ultra-violet divergences. In String Theory, instead, strings are colliding over small but finite distances, and hence the ultra-violet divergences of the graviton scattering amplitudes in field theory are avoided by smearing out the location of the interactions. In this sense String Theory closes the gap between GR and Quantum theory which is one of its important triumphs.

In contrast to the other known theories, where the dimensions of spacetime are inserted by hand, String Theory predicts the number of those dimensions. In 10-dimensional universe String Theory is a consistent supersymmetric theory.

Higher dimensional theories were considered before String Theory. The first theory of this kind was proposed by Kaluza (1919) who added a fifth dimension to Einstein's theory in an attempt to unify gravity and electromagnetism. The reason for the unobservability of the fifth dimension is its compactness, as was suggested for the first time by Klein (1926). In the original Kaluza-Klein (KK) theory the five-dimensional fields are periodic in the internal direction parameterizing a circle (see appendix A). In this S^1 compactification the Fourier modes appear as fields in four dimensions. The mass of a mode ϕ_n depends on the size of the circle denoted by its radius r. It is of the order $\sim (n/r)$. Clearly the massless modes are the zero modes. The masses of the heavy modes get larger if r is small enough, and hence can be discarded if one limits the analysis to the low energy effective theory.

In String Theory, to make contact with the observed phenomena of the four-dimensional universe ($D = 4$), one adopts the same reasoning as in KK theory, i.e. one compactifies six out of the ten dimensions. In other words one specifies an ansatz for the ten-dimensional spacetime background $M_{10} = M_{(3,1)} \times \mathcal{M}_6$ where $M_{(3,1)}$ is our observable world and $\mathcal{M}_6$ is a compactification manifold. It is a six-dimensional manifold on which the extra dimensions curl. If $\mathcal{M}_6$ is small enough the extra six dimensions will not be seen and one ends up with an effective description of the four-dimensional observable world.

In such compactifications one usually insists on not breaking all of the supersymmetries of the ten-dimensional theory since String Theory is under better control in supersymmetric backgrounds. This in turn selects some properties which the internal manifold $\mathcal{M}_6$ should satisfy. It turns out that a Calabi-Yau (CY) manifold is a correct choice of a compactification manifold to result in a supersymmetric effective theory with a four-dimensional Minkowski background [8].

If String Theory can be trusted to be the unifying theory then it should reproduce the standard model after compactification to four-dimensions. However, the natural mass scale of String Theory $(1/\sqrt{\alpha'})$ is of the order of the Planck mass. This means that the modes of the string are way too heavy to be detected by current particle physics technology. Hence the massless sector is the only plausible sector where one can hope to find the standard model particles. Therefore, in attempts to find the standard model from String Theory one should, in a first step, restrict to the massless sector where α' effects are negligible. The dynamics is then encoded in the supergravity theories, which are regarded as low energy effective descriptions of the underlying String Theories.

As a second step, one needs to specify the ansatz of the compactification to reduce the dimensions to four. Here the same strategy as in the original Kaluza-Klein theory is adopted. The lower dimensional theory is obtained by expanding the fields in modes of the compactification manifolds. As in the S^1 compactification, the masses appear to be quantized in terms of $1/R$ where R is the 'radius' of the manifold. For small R one can truncate the spectrum to the massless fields which are in one to one correspondence with the zero modes of the Lapalace operator. In Calabi-Yau compactifications, for example, these modes are found upon expansion in the harmonics of the manifold. This procedure can be applied to all fields of the theory including the metric.

Among the scalars arising in the expansion in the internal modes one finds moduli fields which are the flat directions of the scalar potential. They parameterize the degeneracy of consistent compactification vacua. Generically there is a large number of moduli. However, in Standard-Model-like vacua, these moduli should get massive. Hence one needs some mechanism for their stabilization, or in other words one needs to generate a potential for these fields which fixes their values in the vacuum. There are various mechanisms attempting to do that such as background fluxes [9]–[23], instanton corrections [24, 25, 26] and gaugino condensates [27].

In this thesis we consider flux compactifications on $SU(3)$ structure manifolds to generate a scalar potential for the moduli fields. Including background fluxes amounts to allowing for non-trivial vacuum expectation values of certain fields which are extended in the internal directions. Unfortunately background fluxes cannot fix all the moduli in all compactifications. To fix the remaining ones one needs to include non-perturbative effects, such as instantons and gaugino condensates [28]–[32].

1.1 Dualities in String Theory

By the term 'String Theory' one actually refers to five kinds of consistent theories known as type I, types IIA and IIB, and the two heterotic string theories $SO(32)$ and $E_8 \times E_8$. Until the mid 90ies it was thought that only one of the five string theories could be successful in describing the four dimensional world, reducing the remaining four to beautiful, but non-realized, mathematical constructions. This naive picture was corrected after the discovery of transformations relating the five string theories among themselves. These transformations are called dualities. The so called S-duality, for instance, maps the states and vacua of a theory with coupling constant g to those of a theory with a coupling constant 1/g. This opens the door to the study of strongly coupled theories via the perturbation theory of the weakly coupled dual ones. An example is type IIB theory where the weak and strong coupling limits are related via S-duality. The same holds for type I and the heterotic $SO(32)$.

Another duality relating two theories is T-duality. This transformation acts on spaces in which at least one direction has the topology of a circle and changes the radius to its inverse. As an example, type IIA compactified on a circle of radius

R is dual to type IIB compactified on a circle of radius 1/R. The same holds for the two heterotic theories.

In a work by Strominger, Yau and Zaslow [33] it has been shown that T-duality is related to a surprising symmetry called mirror symmetry which relates two Calabi-Yau manifolds [34]. The compactifications of type IIA and IIB string theories on the respective dual manifolds can be proven to lead to identical physics in four dimensions. Mirror symmetry has led to the calculation of many quantities that seemed virtually incalculable before, by considering their 'mirror' descriptions, which may be much easier.

The fact that the five String Theories are connected to one another reveals that they are special cases or limits of some more fundamental theory which is referred to as M-theory. There is not much known about this theory apart from the fact that it incorporates the five string theories and its low energy limit is eleven-dimensional supergravity.

1.2 D-branes and O-planes

The study of String Theories has revealed, besides strings, further higher dimensional objects called p-branes (for p running from 0 to 9). The variable p refers to the spatial worldvolume dimensions of the brane. For instance, a 0-brane is a particle, a 1-brane is a string and a 2-brane is a membrane. Those branes, then, sweep-out a (p+1)-dimensional world-volume as they propagate through space-time. They are non-perturbative objects which is one of the reasons why it took some time to realize their existence.

In String Theory one calls D-branes (where the D stands for Johann Dirichlet) the p-branes on which the ends of open strings are localized. The D-branes are, then, typically classified by their dimensions. A D0-brane, for example is a single point, a D1-brane is a string often called a 'D-string' and a D2-brane is a plane. Different types of D-branes appear in different theories. In type IIA string theory one finds Dp-branes for p even $(0,\ldots,8)$ while in type IIB the Dp-branes appear for p odd $(1,\ldots,9)$. They are charged under the so called Ramond-Ramond (RR) fields. Each Dp-brane has an associated (p+1)-form RR potential.

D-branes are very important in string theory and have helped to understand the duality between open and closed strings. A loop of an open string attached to two D-branes can be viewed as a closed string leaving one D-brane to the other [35, 36]. Furthermore, D-branes can generate gauge theories. Open strings with both ends on the same D-brane correspond to a $U(1)$ gauge theory. The gauge group gets enhanced to $U(N)$ if one considers a stack of N D-branes on top of each other. The $SU(3)\times SU(2)\times U(1)$ of the Standard Model can, then, be realized on spacetime filling intersecting D-branes [37]. The matter fields arise from dynamical fluctuations of the brane around its background configuration which are charged under the corresponding gauge group. Another important application of D-branes has been to the study of black holes. They are important in understanding and counting the quantum states that lead to black hole entropy [38].

D-branes have a positive tension which should be balanced on the compactification manifold by a negative tension for the consistency of the theory [17]. The negative tension is provided by other objects called orientifolds which are a generalization of orbifolds [39]. These are quotients of manifolds under the action of the group of its isometries. Orientifold actions furthermore include an orientation reversal. They generate orientifold planes which are the locus where the orientifold action reduces to the change of the string orientation. Using the same analogy as for Dp-branes, orientifold planes are denoted as Op-planes. As an example, type I string theory is an orientifolding of type IIB string theory having a spacetime-filling O9-plane.

1.3 Outline of the thesis

After this general introduction let us now turn to the actual topic of this thesis and its organization. As mentioned earlier, compactifications on Calabi-Yau threefolds in the absence of fluxes lead to four-dimensional effective theories with no potential for the moduli fields and all vacua are Minkowskian preserving the full supersymmetry. This changes as soon as we include background fluxes and localized sources such as D-branes and orientifold planes. In these situations it is a non-trivial task to perform consistent compactifications such that the four-dimensional effective theory remains supersymmetric. In particular, this is due to the fact, that the inclusion of sources forces the geometry to back-react. For example, in the heterotic string the manifold has to allow for torsion to balance the effects of fluxes [40]–[43]. The internal manifold is then no longer directly related to a Calabi-Yau manifold and a more general class of compactification manifolds has to be taken into account.

In this thesis we discuss such general compactifications leading to (spontaneously broken) $N = 1$ four-dimensional effective theories which are of importance from a phenomenological point of view. More specifically, we determine, via a Kaluza-Klein reduction, the $N = 1$ four-dimensional low energy effective actions resulting from compactifications of the heterotic and type II theories on a $SU(3)$ structure manifolds. In the case of type II compactifications we include orientifold actions to ensure the reduction of supersymmetry form $N = 2$ to $N = 1$. We express the characteristic informations of the $N = 1$ four-dimensional theories in terms of geometric data of the internal manifold.

The thesis is organized as follows. In chapter 2 we briefly review the ten-dimensional heterotic and type II theories. We recall their spectra, effective actions and the supersymmetry transformations of the fermionic fields which are important in our discussion later on. We discuss the conditions leading to minimal supersymmetry in four dimensions. It turns out that compactifications on manifolds admitting one globally defined spinor satisfy these conditions [44]. These manifolds are known as $SU(3)$ structure manifolds and can be classified by their intrinsic torsion [45, 46]. We shortly recall the properties of these manifolds and then we review the $N = 1$ supergravity in four dimensions.

In chapter 3 we proceed by compactifying the heterotic $E_8 \times E_8$ supergravity on $SU(3)$ structure manifolds. This theory naturally includes chiral fermions. Moreover, its gauge group $E_8 \times E_8$ is big enough to accommodate the $SU(3) \times SU(2) \times U(1)$ of the standard model [47]–[51]. However the purpose of our study is to determine the resulting $N = 1$ four-dimensional effective theory before the breaking of the four-dimensional gauge group to $SU(3) \times SU(2) \times U(1)$. We use Kaluza-Klein reduction to derive the four-dimensional spectrum. Unlike Calabi-Yau compactifications there is no obvious relation between massless modes and the harmonic forms. Therefore we expand the ten-dimensional fields in forms on the $SU(3)$ structure manifolds which are not necessarily harmonic. Among these modes we do not keep any triplets of $SU(3)$ such that the $D = 4$ fields arrange in the standard $N = 1$ supermultiplets.

To specify the $N = 1$ effective action one needs to determine the Kähler potential and the superpotential. We compute these quantities from fermionic couplings where they appear linearly. This confirms the structure of these couplings derived previously from bosonic terms [53, 54, 55, 56]. Our aim in doing the computation at the level of the fermionic action is to provide a consistent reduction of the fermionic part which, to our surprise, is missing in the literature even for Calabi-Yau compactifications. The superpotential depends on the flux and the intrinsic torsion of the manifold. We compute the Yukawa couplings as well as F- and D-terms of the theory in sections 3.2.3 and 3.2.4. We evaluate the SUSY transformations of the fermionic fields of the theory in section 3.3. This sets the stage to the discussion of supersymmetry conditions for the background. Torsion and fluxes, then, can not be chosen arbitrarily but rather have to satisfy specific conditions which we derive by setting all fermionic SUSY transformations to zero. This recovers the results of Strominger in the heterotic string [40].

In chapter 4 we turn to compactifications of type II theories on $SU(3)$ structure manifolds [57, 58, 59]. These result in $N = 2$ four-dimensional effective theories. As we are interested in $N = 1,\ D = 4$ theories we include orientifold actions in the set-ups which reduce the supersymmetry by a half [60]. We review the properties of the orientifold projections of type IIA and type IIB in turn. We show in section 4.2 that the truncated spectra organize indeed in $N = 1$ supermultiplets which generalize the results found in Calabi-Yau orientifold compactifications [61, 62, 63]. We follow largely the method outlined in the heterotic compactification to determine the superpotentials from fermionic terms.

Finally we conclude in chapter 5 by a summary and a discussion. We collect our conventions and some technical details in five appendices. In appendix A we recall Kaluza-Klein reductions on a circle and on a six-dimensional manifold and we give Calabi-Yau manifolds as an example of six-dimensional compactification manifolds. Moreover, we summarize our spinor conventions in four and six dimensions in appendix B. In appendix C we present the geometry of the scalar manifold in $SU(3)$ compactifications. We summarize the results of [40] in appendix D and we review the Hitchin functional and stable forms in appendix E.

Chapter 2

$D = 10$ theories and $SU(3)$ structure manifolds

In this chapter we give a short review of the ten-dimensional theories we aim to compactify. These are the heterotic and type II supergravities. The compactification considered in this thesis is such that the resulting four-dimensional theories have $N = 1$ supersymmetry. This requirement puts some conditions on the compactification manifold which we will examine. For completeness we review as well the $N = 1$ four-dimensional supergravity.

2.1 Heterotic and type II theories in $D = 10$

2.1.1 Heterotic supergravity theory

The heterotic supergravity is an $N = 1$ supersymmetric theory in ten dimensions. It can be seen as the gravity multiplet coupled to a vector multiplet. Its spectrum, then, consists of the 10-dimensional metric $\hat{G}_{MN}$ where $M, N = 0, \ldots, 9$, an antisymmetric two-tensor $\hat{B}_{MN}$, the dilaton $\hat{\phi}$, a left-handed Majorana-Weyl gravitino $\hat{\psi}_M$ and a right handed Majorana-Weyl fermion, the dilatino $\hat{\lambda}$. These are the fields in the gravity multiplet. In the vector multiplet one finds a gauge boson $\hat{A}_M^{\mathcal{A}}$ and a gaugino $\hat{\chi}^{\mathcal{A}}$. The index $\mathcal{A}$ refers to the adjoint representation of either $E_8 \times E_8$ or $SO(32)$ gauge groups in which the fields of the vector multiplet transform. In the following we consider only the compactification of the heterotic $E_8 \times E_8$. The ten-dimensional fields are denoted by a 'hat' and are summarized in table 2.1.

	Bosons	Fermions
gravity multiplet	$\hat{G}_{MN}, \hat{B}_{MN}, \hat{\phi}$	$\hat{\psi}_M, \hat{\lambda}$
vector multiplet	$\hat{A}^{\mathcal{A}}_M$	$\hat{\chi}^{\mathcal{A}}$

Table 2.1: $N = 1$ *spectrum in* $D = 10$.

The low energy dynamics of the heterotic fields given in table 2.1 are encoded in the ten-dimensional $N = 1$ supergravity action [64, 65]. The action can be given as the sum of three distinct contributions $S^{(10)} = S_{\rm b} + S_{\rm f} + S_{\rm int}$ where $S_{\rm b}$ includes the purely bosonic terms and reads

$$S_{\rm b} = -\tfrac{1}{2}\int d^{10}x\sqrt{-\hat{G}_{10}}\,\Big[\hat{R} \;+\; \tfrac{1}{2}e^{-\hat{\phi}}\;\hat{H}_{MNP}\,.\,\hat{H}^{MNP} + \tfrac{1}{2}\,\partial_M\hat{\phi}\,\partial^M\hat{\phi} + \; e^{-\frac{\hat{\phi}}{2}}\;\hat{F}^{\mathcal{A}}_{MN}\,.\,\hat{F}^{\mathcal{A}\;MN}\Big]\;. \tag{2.1}$$

Here we gave $S_{\rm b}$ in the Einstein frame and $\hat{R}$ is the Ricci scalar in that frame. The contraction of n indices is defined with a factor of $\frac{1}{n!}$. More explicitly we define the product of n-forms as as $E_{M_1\ldots M_n}.E^{M_1\ldots M_n} = \frac{1}{n!}E_{M_1\ldots M_n}E^{M_1\ldots M_n}$. Finally $\hat{F}^{\mathcal{A}}_{MN}$ is the field strength for the gauge boson $\hat{A}^{\mathcal{A}}_M$, while $\hat{H}$ is the modified three-form field strength of $\hat{B}_2$ defined as

$$\hat{H} = d\hat{B}_2 - \,\omega_3^{\rm YM} + \omega_3^{\rm L}\;, \tag{2.2}$$

where $\omega_3^{\rm YM}$ is the Yang-Mills Chern-Simon 3-form while $\omega_3^{\rm L}$ is the Lorentz Chern-Simon 3-form. Their exterior derivatives are given as

$$d\omega_3^{\rm YM} = \mathbf{Tr}\big(\hat{F}\wedge\hat{F}\big)\;,\quad d\omega_3^{\rm L} = {\rm tr}\big(\hat{\rm R}_2\wedge\hat{\rm R}_2\big)\;. \tag{2.3}$$

Here $\mathbf{Tr}$ refers to 1/30 of the trace in the adjoint of $E_8\times E_8$ while tr denotes the trace in the vector representation of the Lorentz group $SO(9,1)$. $\hat{R}_2$ is the curvature two-form. If $\hat{H}$ admits a non trivial background value a three-form flux appears which is often referred as the NS-NS flux where NS refers to Neuveu-Schwarz and denoted by $H_3 =< \hat{H}_3 >$ where $< \ldots >$ indicates the vacuum expectation value.

The action $S_{\rm f}$ contains the kinetic terms for the fermions

$$S_{\rm f} = -\int d^{10}x\sqrt{-\hat{G}_{10}}\,\Big[\hat{\bar{\psi}}_M\Gamma^{MNP}D_N\hat{\psi}_P + \hat{\bar{\lambda}}\Gamma^M D_M\hat{\lambda} + \hat{\bar{\chi}}^{\mathcal{A}}\Gamma^M D_M\hat{\chi}^{\mathcal{A}}\Big]\;, \tag{2.4}$$

where the ten-dimensional fermions are Majorana-Weyl spinors and the conjugate spinor $\hat{\bar{\psi}}_M = \psi_M^\dagger \Gamma^0$ is obtained by hermitian conjugation and multiplication with the ten-dimensional gamma-matrix Γ^0. The action $S_{\rm int}$ contains the interactions

$$
\begin{aligned}
S_{\rm int} \quad = \quad & -\int d^{10}x\sqrt{-\hat{G}_{10}}\Big[\sqrt{\tfrac{1}{2}}\,\partial_N\hat{\phi}(\hat{\bar{\psi}}_M\Gamma^N\Gamma^M\hat{\lambda}) - \tfrac{1}{4}e^{-\frac{\hat{\phi}}{2}}\hat{H}_{MNP}\,.\,\hat{\bar{\chi}}^{\mathcal{A}}\Gamma^{MNP}\hat{\chi}^{\mathcal{A}} \\
& -\tfrac{1}{4}e^{-\frac{\hat{\phi}}{2}}H_{MNP}\,.\,(\hat{\bar{\psi}}_Q\Gamma^{QMNPR}\hat{\psi}_R + 6\hat{\bar{\psi}}^M\Gamma^N\hat{\psi}^P - \tfrac{1}{2}\hat{\bar{\psi}}_Q\Gamma^{MNP}\Gamma^Q\hat{\lambda}) \quad (2.5)\\
& +e^{-\hat{\phi}}\hat{F}^{\mathcal{A}}_{MN}\,.\,\big(\hat{\bar{\chi}}^{\mathcal{A}}\Gamma^Q\Gamma^{MN}(\hat{\psi}_Q + \sqrt{\tfrac{1}{72}}\,\Gamma_Q\hat{\lambda})\big) + \text{four Fermi terms}\Big]\ ,
\end{aligned}
$$

where $\Gamma^{M_1\ldots M_p} = \frac{1}{p!}\Gamma^{[M_1}\ldots\Gamma^{M_p]}$ denotes totally antisymmetrized products of p Γ-matrices.

The heterotic action $S^{(10)}$ is invariant under the supersymmetry variation of its fields. For the gravitino, gaugino and dilatino these are given as follows [40]

$$\delta\hat{\psi}_M \;=\; D_M\hat{\epsilon} + \frac{1}{16}e^{\frac{-\hat{\phi}}{2}}\hat{H}_{NPQ}\,.\,(\Gamma_M^{NPQ} - 9\delta_M^N\Gamma^{PQ})\ , \tag{2.6}$$

$$\delta\lambda \;=\; -\frac{1}{2}\sqrt{\frac{1}{2}}(D_M\hat{\phi})\Gamma^M\epsilon + \frac{1}{4}\sqrt{\frac{1}{2}}e^{-\frac{\hat{\phi}}{2}}H_{MNP}\,.\,\Gamma^{MNP}\epsilon\ , \tag{2.7}$$

$$\delta\chi^{\mathcal{A}} \;=\; e^{-\frac{\hat{\phi}}{2}}F^{\mathcal{A}}_{MN}\,.\,\Gamma^{MN}\hat{\epsilon}\ , \tag{2.8}$$

where we denote the ten-dimensional SUSY spinor by $\hat{\epsilon}$ and D_M denotes the ten-dimensional covariant derivative.

2.1.2 Type II supergravity theories

Type II theories are maximally supersymmetric theories in ten dimensions. Hence their field content is organized in the $N = 2$ supermultiplet. These fields result from four different sectors. These are NS-NS, R-R and finally R-NS and NS-R sectors. The bosonic NS-NS fields of both type IIA and type IIB supergravities are the scalar dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{G}_{MN}$ and the two-form $\hat{B}_2$. In the R-R sector type IIA consists of odd forms $\hat{C}_{2n-1}$, while type IIB consists of even forms $\hat{C}_{2n}$. The fermionic fields come from the R-NS and NS-R sectors[1] and are two gravitinos $\hat{\psi}^1_M$, $\hat{\psi}^2_M$ and two dilatinos $\hat{\lambda}^1$, $\hat{\lambda}^2$. The gravitinos have the same chiralities in type IIB theory while in type IIA they have different chiralities.

[1]The NS-R and R-NS sectors have the same field content.

The low energy action of these fields [66] is expressed as the sum $S^{(10)} = S_{\rm b} + S_{\rm f} + S_{\rm int}$ where $S_{\rm b}$ includes the bosonic terms

$$S_{\rm b} = -\int d^{10}x\sqrt{-\hat{G}_{(10)}}\, e^{-2\hat{\phi}}\Big[\, \tfrac{1}{2}\,\hat{R}\ -\ 2\,\partial_M\hat{\phi}\,\partial^M\hat{\phi} \\ +\tfrac{1}{4}\,\hat{H}_{MNP}\,.\,\hat{H}^{MNP}\ +\ \tfrac{1}{8}\sum_{n=0,1}^{8,9}\hat{F}_n.\hat{F}_n\Big]\ . \qquad (2.9)$$

Note that the action $S_{\rm b}$ is given in the string frame and hence the ten-dimensional Ricci scalar $\hat{R}$ is expressed in that frame. $\hat{H}_3 = d\hat{B}_2$ is the field strength of the B-field and the R-R field strengths $\hat{F}_n$ are defined as

$$\hat{F}_n = d\hat{C}_{n-1} - \hat{H}_3\wedge\hat{C}_{n-3}\ , \qquad (2.10)$$

where we use the democratic formulation of ref. [66]. Thus n runs from 0 to 8 for type IIA and from 1 to 9 for type IIB. $\hat{F}_n$ satisfy a self-duality condition

$$*\hat{F}_n = \lambda(\hat{F}_{10-n})\ , \qquad (2.11)$$

where λ is a parity operator acting on even forms $\mathcal{C}_{2n}$ and odd forms $\mathcal{C}_{2n-1}$ as

$$\lambda(\mathcal{C}_{2n}) = (-1)^n\,\mathcal{C}_{2n}\ , \qquad \lambda(\mathcal{C}_{2n-1}) = (-1)^n\,\mathcal{C}_{2n-1}\ . \qquad (2.12)$$

The condition in eqn. (2.11) implies that half of the R-R fields carry no extra degrees of freedom. The NS field strength $\hat{H}$ and the R-R field strengths $\hat{F}_n$ give rise to the NS-NS three-form and R-R n-form fluxes $H_3,\ F_n$ respectively if their vacuum expectation values are non zero.

$S_{\rm f}$ consists of the fermionic kinetic terms[2]

$$S_{\rm f} = -\int d^{10}x\sqrt{-\hat{G}_{(10)}}\, e^{-2\hat{\phi}}\big[\hat{\bar{\psi}}_M\Gamma^{MNP}D_N\hat{\psi}_P - \hat{\bar{\lambda}}\Gamma^M D_M\hat{\lambda}\big]\ , \qquad (2.13)$$

where $\hat{\bar{\psi}}_M = \psi^\dagger_M\Gamma^0$. Finally, the interaction terms are given in $S_{\rm int}$

$$S_{\rm int} \ = \ -\int d^{10}x\sqrt{-\hat{G}_{(10)}}\Big\{e^{-2\hat{\phi}}\big[-\partial^M\hat{\phi}\,\tilde{\Psi}^{(1)}_M + \tfrac{1}{2}\hat{H}^{MNP}\,.\,\tilde{\Psi}^{(3)}_{MNP} \qquad (2.14) \\ +2\hat{\bar{\lambda}}\Gamma^{MN}D_M\hat{\psi}_N\big] + \tfrac{1}{4}\sum_{n=0,1}^{8,9}\hat{F}_n.\tilde{\Psi}_n\Big\} + \text{quartic fermionic terms}\ ,$$

[2]For convenience we combine the two gravitinos and the two dilatinos into doublets.

where $\tilde{\Psi}^{(1)}$, $\tilde{\Psi}^{(3)}$ and $\tilde{\Psi}_n$ are coefficients of ten-dimensional one-, three- and n-forms. They are defined as

$$\begin{aligned}
\tilde{\Psi}^{(1)}_M &= -2\hat{\bar{\psi}}_N\Gamma^N\hat{\psi}_M - 2\hat{\bar{\lambda}}\Gamma^N\Gamma_M\psi_N \ , && (2.15)\\
\tilde{\Psi}^{(3)}_{MNP} &= \tfrac{1}{2}\hat{\bar{\psi}}_Q\Gamma^{[Q}\Gamma_{MNP}\Gamma^{R]}\mathcal{P}\hat{\psi}_R + \hat{\bar{\lambda}}\Gamma^Q_{MNP}\mathcal{P}\hat{\psi}_Q - \tfrac{1}{2}\hat{\bar{\lambda}}\mathcal{P}\Gamma_{MNP}\lambda \ , && (2.16)\\
(\tilde{\Psi}_n)_{M_1\ldots M_n} &= \tfrac{1}{2}e^{-\hat{\phi}}\big(\hat{\bar{\psi}}_M\Gamma^{[M}\Gamma_{M_1\ldots M_n}\Gamma^{N]}\mathcal{P}_n\hat{\psi}_N + \hat{\bar{\lambda}}\Gamma_{M_1\ldots M_n}\Gamma^N\mathcal{P}_n\hat{\psi}_N && (2.17)\\
&\quad -\tfrac{1}{2}\hat{\bar{\lambda}}\Gamma_{[M_1\ldots M_{n-1}}\mathcal{P}_n\Gamma_{M_n]}\lambda\big) \ ,
\end{aligned}$$

where $\mathcal{P} = \Gamma_{11}$, $\mathcal{P}_n = (\Gamma_{11})^n$ for type IIA while for type IIB one has $\mathcal{P} = -\sigma^3$, $\mathcal{P}_n = \sigma^1$ for $\frac{n+1}{2}$ even and $\mathcal{P}_n = i\sigma^2$ for $\frac{n+1}{2}$ odd.

2.2 Compactification to four dimensions

String theory is consistently formulated in a ten-dimensional target space. In order to reduce to the four-dimensional observable world, we choose the background to be of the form $M_{10} = M_{(3,1)} \times \mathcal{M}_6$, as already motivated in chapter 1. Due to this ansatz the Lorentz group of M_{10} decomposes as $SO(9,1) \to SO(3,1) \times SO(6)$, where $SO(6)$ is the structure group of a generic sixfold. This in turn implies the decomposition of the spinor representation as $\mathbf{16} = (\mathbf{2},\mathbf{4}) + (\bar{\mathbf{2}},\bar{\mathbf{4}})$, where $\mathbf{2},\mathbf{4}$ are the Weyl representations of $SO(3,1)$ and $SO(6)$ respectively. The background metric is block-diagonal and reads

$$ds^2 = e^{2\Delta(y)}g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n \ , \tag{2.18}$$

where $x^\mu, \mu = 0,\ldots,3$ are the coordinates on $M_{3,1}$ while $y^m, m = 1,\ldots,6$ are the coordinates of $\mathcal{M}_6$. Here $g_{\mu\nu}$ is the metric on $M_{3,1}$ and g_{mn} is the metric on the internal manifold $\mathcal{M}_6$. Generically (2.18) includes a non-trivial warp factor $\Delta(y)$. However, in the following we restrict our analysis to a large volume regime where the supergravity is trusted. In this regime the factor Δ is constant.[3] Only in section 3.4, where we determine the supersymmetry conditions in the vacuum, we shall discuss the properties of a non-trivial warp factor.

Demanding a four-dimensional background to preserve a minimal amount of supersymmetry one has to specify a class of compactification manifolds $\mathcal{M}_6$. They have a structure group reducing to $SU(3)$. In the following we will discuss the requirements which induce minimal SUSY in $D = 4$ and the properties of the $SU(3)$ structure manifolds.

[3] Compactifications including the warp factor $\Delta(y)$ were considered, for example, in [17, 67].

2.2.1 Conditions for minimal SUSY

In a supersymmetric vacuum all fields transforming non trivially under the Lorentz group have to vanish, especially the fermionic fields and their SUSY variations. The vanishing of the gravitino SUSY transformation (2.6), where $\hat{H}_3$ has no background value, amounts to the condition

$$< \delta\hat{\psi}_M >=< D_M\hat{\epsilon} >= 0 \ , \quad M = 0, \ldots, 9 \ . \tag{2.19}$$

In compactifications of ten-dimensional supergravities on a six-dimensional manifold $\mathcal{M}_6$ as given in (2.18) the spinor $\hat{\epsilon}$ reduces, via Kaluza-Klein, to the 4-dimensional SUSY parameter $\epsilon(x^\mu)$ times a spinor on the internal manifold which we denote by $\eta(y^m)$

$$\hat{\epsilon} \ = \ \epsilon(x^\mu) \otimes \eta(y^m) \ . \tag{2.20}$$

Hence the condition of preserving supersymmetry (2.19) translates into

$$D_\mu\epsilon = 0 \ , \qquad D_m^{LC}\eta = 0 \ , \tag{2.21}$$

where D_μ is the four-dimensional derivative and D_m^{LC} denotes the Levi-Civita connection on the manifold $\mathcal{M}_6$. The equation (2.21) implies that for each covariantly constant η there is a conserved supersymmetry in the four-dimensional flat ground state. For minimal SUSY in D=4 the compactification manifold should then possess only one no-where vanishing spinor satisfying the condition (2.21). The compactifications of the heterotic and type II theories on these manifolds lead to $N = 1$ and $N = 2$ supersymmetric four-dimensional effective theories respectively.[4] Momentarily we will discuss the properties of such manifolds in more detail.

2.2.2 $SU(3)$ structure manifolds

The existence of a no-where vanishing spinor η is actually the only condition one needs in order to have some (spontaneously broken) supersymmetries in the effective four-dimensional theory[5], while the condition that η is covariantly constant guarantees the flatness of the ground state. As reviewed in appendix A.2 manifolds admitting a covariantly constant spinor have $SU(3)$ holonomy group and

[4] Note that in [68] it was argued that $N = 2$ supersymmetry can be obtained by compactifying type II theories on a manifold with two globally defined spinors, which may coincide at points in $\mathcal{M}_6$.

[5] The discussion here holds for compactifications without background fluxes. In presence of fluxes unbroken supersymmetry may be realized on the background.

are known as Calabi-Yau manifolds. Compactifications on such manifolds result in unbroken supersymmetric four-dimensional effective theories with a Minkowski background. Throughout the thesis we will relax this condition and we insist only on the existence of the spinor η. Manifolds admitting one globally defined spinor have a structure group reduced from $SO(6)$ to $SU(3)$ [45].

Conversely, the reduction of the structure group from $SO(6)$ to $SU(3)$ implies the existence of a globally defined spinor η. This is due to the fact that the spinor representation $\mathbf{4}$ of $SO(6)$ decomposes under $SU(3)$ into $\mathbf{4} \rightarrow \mathbf{3} + \mathbf{1}$. Hence an invariant spinor in the singlet depends trivially on the tangent bundle of the manifold and is then globally defined. If furthermore this spinor is covariantly constant with respect to the Levi-Civita connection, as in eqn. (2.21), the manifold has $SU(3)$ holonomy and then it satisfies the Calabi-Yau conditions [69]. In a generic $SU(3)$ structure manifold the Ricci flatness is no longer satisfied. Equivalently, the spinor η is no longer covariantly constant with respect to the Levi-Civita connection, but with respect to a more general connection D^T. The latter differs from the Levi-Civita connection by a contorsion tensor τ [40, 69, 70, 71]

$$D^T_m \eta = \big(D^{LC}_m - \tfrac{1}{4}\tau_{mnp}\gamma^{np}\big)\eta = 0 \ , \tag{2.22}$$

where we define the anti-symmetrized product of six-dimensional gamma matrices as $\gamma^{m_1 \dots m_n} = \frac{1}{n!}\gamma^{[m_1} \dots \gamma^{m_n]}$. The contorsion tensor τ parameterizes the deviation of the connection D^T from the Levi-Civita connection. Clearly, in the case where the contorsion tensor vanishes one recovers the Calabi-Yau condition.

Using the spinor η, which we choose to be normalized as $\eta^\dagger_\pm \eta_\pm = 1$, one can define no-where vanishing two-form J and three-form Ω_η

$$J^{mn} \ = \ \mp 2i\eta^\dagger_\pm \gamma^{mn}\eta_\pm \ , \qquad \Omega^{mnp}_\eta = -i2\eta^\dagger_- \gamma^{mnp}\eta_+ \ , \qquad \bar{\Omega}^{mnp}_\eta = -i2\eta^\dagger_+ \gamma^{mnp}\eta_- \ , \tag{2.23}$$

where the subscripts $\pm$ on the spinors indicates their six-dimensional chiralities and the dagger denotes the hermitian conjugation. The index η indicates the specific normalization of Ω_η.[6] In this normalization one can apply Fierz identities to derive the $SU(3)$ structure constraints

$$J \wedge J \wedge J = \frac{3i}{4}\Omega_\eta \wedge \bar{\Omega}_\eta \ , \qquad\qquad J \wedge \Omega_\eta = 0 \ . \tag{2.24}$$

The fact that the spinor η is no longer covariantly constant implies that neither J nor Ω_η are closed. The non-closure is parameterized by the torsion τ which

[6]Later on we will relate Ω_η to the three-form Ω used in the compactification by an appropriate rescaling.

decomposes under $SU(3)$ into irreducible representations. Due to the fact that τ_{mnp} is antisymmetric in its last two indices[7] it can be thought of as being a one-form taking values in $so(6)$, the Lie algebra of $SO(6)$, which is decomposed into $SU(3)$ representations. More precisely, $\tau \in \Lambda^1 \otimes \Lambda^2 \cong \Lambda^1 \otimes so(6) \cong \Lambda^1 \times (su(3) \oplus su(3)^\perp)$. However, the action of $su(3) \equiv \mathbf{8}$ is trivial on the $SU(3)$ invariant quantities including the spinor η. Therefore eqn. (2.22) depends only on torsion element of $\Lambda^1 \otimes su(3)^\perp$ which is called the intrinsic torsion.[8] It transforms in

$$(\mathbf{3} \oplus \bar{\mathbf{3}}) \otimes (\mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}) = (\mathbf{1} \oplus \mathbf{1}) \oplus (\mathbf{3} \oplus \bar{\mathbf{3}}) \oplus (\mathbf{3} \oplus \bar{\mathbf{3}}) \oplus (\mathbf{6} \oplus \bar{\mathbf{6}}) \oplus (\mathbf{8} \oplus \mathbf{8}) \ . \tag{2.25}$$

These representations are conveniently encoded by five torsion classes $\mathcal{W}_i$ defined as [45, 71]

$$\begin{aligned} dJ &= -\tfrac{3}{2}\mathrm{Im}(\mathcal{W}_1 \bar{\Omega}_\eta) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \ , \\ d\Omega_\eta &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega_\eta \ , \end{aligned} \tag{2.26}$$

with constraints

$$J \wedge J \wedge \mathcal{W}_2 = J \wedge \mathcal{W}_3 = \Omega_\eta \wedge \mathcal{W}_3 = 0 \ . \tag{2.27}$$

The pattern of vanishing torsion classes defines the properties of the manifold $\mathcal{M}_6$. For example $\mathcal{M}_6$ is complex in case $\mathcal{W}_1 = \mathcal{W}_2 = 0$. Of particular interest are half-flat manifolds since they are believed to arise as mirrors of flux compactifications [79, 80, 81]. These are defined by $\mathcal{W}_4 = \mathcal{W}_5 = 0$ and $\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = 0$.

Using the inverse metric and the real two-form J one can define an almost complex structure $I_m{}^n = J_{mp} g^{pn}$

$$I_p{}^n I_m{}^p = -\delta_m^n \ . \tag{2.28}$$

The metric g_{mn} is then hermitian with respect to the almost complex structure $I_n{}^p I_m{}^q g_{pq} = g_{mn}$. The almost complex structure can be used to define a (p, q) grading of forms. Within this decomposition the form J is of type $(1, 1)$ while Ω_η is of type $(3, 0)$.

To summarize, $SU(3)$ structure manifolds are characterized by one globally defined spinor. Equivalently they are defined by the existence of two globally defined forms, a real two-form J and a complex three-form Ω. These forms are not closed, which indicates a deviation from the Calabi-Yau case. This difference

[7] Manifolds with totally antisymmetric torsion have been considered, for example, in refs. [72]–[78].

[8] In what follows we use the term torsion having in mind the intrinsic torsion.

can also be encoded by specifying a new connection on $\mathcal{M}_6$ with torsion replacing the ordinary Levi-Civita connection. The torsion is decomposed into $SU(3)$ irreducible representations and described by five torsion classes $\mathcal{W}_i$.

In this thesis we aim to discuss $N = 1$ four-dimensional effective theories arising upon compactification on $SU(3)$ structure manifolds. Therefore we review next the characteristic data of a four-dimensional $N = 1$ supergravity.

2.2.3 $N = 1$ supergravity in $D = 4$

The fields of an $N = 1$ supergravity in four dimensions can be organized into $N = 1$ supermultiplets. All the multiplets have some bosonic degrees of freedom and an equal number of fermionic degrees of freedom. These multiplets are the gravity, vector, and chiral multiplets. In the gravity multiplet one finds the four-dimensional metric $g_{\mu\nu}$ as the bosonic component and the gravitino ψ_μ, a field with spin 3/2, as the fermionic component. While the bosonic component of the vector multiplet is a vector A^a_μ, a gauge boson transforming under the adjoint of the gauge group of the theory, its fermionic component is the gaugino χ^a which is a field with spin 1/2. The chiral multiplet contains a scalar M and a spin 1/2 field Π. In the theories we are considering there will be several copies of chiral multiplets. Their multiplicity is denoted here by $I = 1, \ldots, n_{chiral}$. The $N = 1,\ D = 4$ spectrum is summarized in table 2.1.

	Bosons	Fermions
gravity multiplet	$g_{\mu\nu}$	ψ_μ
vector multiplet	A^a_μ	χ^a
chiral multiplets	M^I	Π^I

Table 2.1: *$N = 1$ spectrum in $D = 4$.*

A generic $N = 1$ four-dimensional supergravity action encoding the dynamics of vector and chiral multiplets and carrying no more than two derivatives, can be written as the sum of three actions

$$S = S_{\rm b} + S_{\rm f} + S_{\rm int}, \tag{2.29}$$

where $S_{\rm b}$ denotes the action of the bosonic fields of the $N = 1$ supermultiplets

[82]

$$
\begin{aligned}
S_{\rm b} \;=\; & -\int d^4x\sqrt{-g_4}\Big[\frac{1}{2}R + \frac{1}{4}{\rm Re}f(M)\,{\rm Tr}F_{\mu\nu}F^{\mu\nu} \\
& \qquad -\frac{1}{4}{\rm Im}f(M)\,{\rm Tr}F\tilde{F} + g_{I\bar{J}}D_\mu M^I D^\mu \bar{M}^{\bar{J}} + V\Big] \;,
\end{aligned} \tag{2.30}
$$

where R is the four-dimensional Ricci scalar and the function $f(M)$ is the holomorphic gauge kinetic function. $F_{\mu\nu}$ is the field strength of the gauge boson A^a_μ. D_μ refers to the gauge covariant derivative. The function $V(M,\bar{M})$ is a scalar potential. We denote the metric on the space of fields M^I by $g_{I\bar{J}}$. The latter is a Kähler metric. Thus it is given by the second derivative of the Kähler potential $K(M,\bar{M})$

$$
g_{I\bar{J}} = \frac{\partial}{\partial M^I}\frac{\partial}{\partial M^{\bar{J}}}K(M,\bar{M}) \;. \tag{2.31}
$$

The kinetic terms of the fermionic fields are given in $S_{\rm f}$ [82]

$$
S_{\rm f} = \int d^4x\sqrt{-g_4}\Big[\epsilon^{\mu\nu\rho\kappa}\bar{\psi}_\mu\bar{\sigma}_\nu\tilde{D}_\rho\psi_\kappa - i g_{\bar{I}J}\bar{\Pi}^{\bar{I}}\bar{\sigma}^\mu\tilde{D}_\mu\Pi^J - i{\rm Re}f\,\bar{\chi}^a\bar{\sigma}^\mu\tilde{D}_\mu\chi^a\Big] \;, \tag{2.32}
$$

where $\epsilon^{\mu\nu\rho\kappa}$ is the totally antisymmetric epsilon tensor and σ_μ are Pauli matrices (for our spinor conventions see appendix B). The derivative $\tilde{D}_\mu$ on the fermions is given by

$$
\begin{aligned}
\tilde{D}_\mu\psi_\nu &= D_\mu\psi_\nu + \tfrac{1}{4}(K_I D_\mu M^I - K_{\bar{I}}D_\mu\bar{M}^{\bar{I}})\psi_\nu + \dots \;, && (2.33)\\
\tilde{D}_\mu\chi^a &= D_\mu\chi^a + \tfrac{1}{4}(K_I D_\mu M^I - K_{\bar{I}}D_\mu\bar{M}^{\bar{I}})\chi^a + \dots, && (2.34)
\end{aligned}
$$

D_μ is the covariant derivative including the spin connection ω_μ and reads ${\rm D}_\mu\psi_\nu = (\partial_\mu + \omega_\mu)\psi_\nu$ for the gravitino while for the gaugino ${\rm D}_\mu\chi = (\partial_\mu+\omega_\mu)\chi - f^{abc}A^b_\mu\chi^c$ where f^{abc} are the constant structures of the gauge group. $K_I = \frac{\partial K}{\partial M^I}$ are derivatives of the Kähler potential K with respect to the scalars M^I. In eqs. (2.33) and (2.34) we didn't display terms involving quantities transforming in the adjoint representation of the four-dimensional gauge group contracted in an appropriate way.

Finally the action $S_{\rm int}$ includes the terms [82]

$$
S_{\rm int} = -\int d^4x\sqrt{-g_4}\Big[e^{\frac{K}{2}}(W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + \frac{i}{\sqrt{2}}D_I W\,\Pi^I\sigma^\mu\bar{\Psi}_\mu) + \frac{1}{2}\mathcal{D}_a\psi_\mu\sigma^\mu\bar{\chi}^a + \dots\Big] \;, \tag{2.35}
$$

where $\mathcal{D}_a$ denotes the auxiliary field of the vector multiplets which yields the D-term $(\mathcal{D}^a\mathcal{D}_a)$. The holomorphic function $W(M)$ is the superpotential and its Kähler covariant derivatives are given by

$$D_I W = \frac{\partial W}{\partial M^I} + \frac{\partial K}{\partial M^I} W \ . \tag{2.36}$$

In $N = 1$ four-dimensional supergravity the scalar potential V can be given in terms of the superpotential W as follows

$$V(M, \bar{M}) = e^K \big(D_I W G^{I\bar{J}} \bar{D}_{\bar{J}} \bar{W} - 3|W|^2 \big) + \frac{1}{2} (\mathrm{Re} f)^{-1} \mathcal{D}^a \mathcal{D}_a \ . \tag{2.37}$$

The $N = 1,\ D = 4$ action is invariant under the supersymmetry transformation of its fields. Below we give the scalar parts of the fermionic fields SUSY transformations

$$\begin{aligned} \delta\psi_\mu &= D_\mu \epsilon + e^{K/2} W \sigma_\mu \bar{\epsilon} \ , && (2.38) \\ \delta\chi^a &= F^a_{\mu\nu} \sigma^{\mu\nu} \epsilon - i \mathcal{D}^a \epsilon \ , && (2.39) \\ \delta\Pi^I &= \sqrt{2} e^{K/2} g^{I\bar{J}} D_{\bar{J}} \bar{W} \epsilon \ , && (2.40) \end{aligned}$$

where we denote the four-dimensional SUSY spinor by ϵ.

To summarize, the $N = 1$ four-dimensional theory can be encoded into gravity, vector, and chiral multiplets. The dynamic of their fields is given by an action which is, in turn, totally determined by three characteristic functions. These are the real Kähler potential $K(M, \bar{M})$, the holomorphic superpotential $W(M)$ and the holomorphic gauge kinetic functions $f_a(M)$.

In the next chapter we will determine these functions for an $N = 1$ four-dimensional theory resulting from the compactification of the heterotic supergravity on $SU(3)$ structure manifolds.

Chapter 3

The heterotic string on $SU(3)$ structure manifolds

In this chapter we compactify the heterotic supergravity theory on $SU(3)$ structure manifolds. As discussed in section 2.2.2 this reduction leads to an $N = 1$ effective theory in four dimensions. We determine here the superpotential and the Kähler potential of the resulting effective action from fermionic terms. To start with let us discuss the $D = 4$ spectrum.

3.1 The four-dimensional spectrum

The four-dimensional spectrum arising from compactification of the heterotic $E_8 \times E_8$ supergravity on an $SU(3)$ structure manifold is determined via Kaluza-Klein reduction. This reduction is not as straightforward as it is for Calabi-Yau compactifications. In such compactifications the light modes are in one to one correspondence with the harmonic forms of the manifold. Therefore to compute the massless fields one expands the ten-dimensional fields into harmonics. For $SU(3)$ structure manifolds such a correspondence is hard to identify. Thus the distinction between heavy and light modes is more subtle. However an alternative truncation is given in [80, 68]. The expansion is then done in a set of infinite forms which are not necessarily harmonic. Moreover, since we are interested in a standard $N = 1$ effective theory a projection of all fields transforming as a triplet under $SU(3)$ is needed. In particular, this amounts to discarding all four-dimensional fields which arise in the expansion of the ten-dimensional fields into one- and five-forms on $\mathcal{M}_6$. This truncation insures keeping only one gravitino in the gravity multiplet. To see this let us decompose the fields into $SU(3)$ representations which we summarized in tables 3.1 and 3.2.

$\hat{G}_{MN}$	$g_{\mu\nu}$	$\mathbf{1_2}$
	$g_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{1}}$
	g_{mn}	$\mathbf{1_0}+(\mathbf{6}+\bar{\mathbf{6}})_{\mathbf{0}}+\mathbf{8_0}$
$\hat{B}_{MN}$	$B_{\mu\nu}$	$\mathbf{1_T}$
	$B_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{1}}$
	B_{mn}	$\mathbf{1_0}+(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{0}}+\mathbf{8_0}$
$\hat{\phi}$	ϕ	$\mathbf{1_0}$
$\hat{A}_M$	A_μ	$\mathbf{1}_1$
	A_m	$(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{0}}$

Table 3.1: *Decomposition of the NS sector in $SU(3)$ representations*

$\hat{\psi}_M$	ψ_μ	$\mathbf{1_{3/2}}+\mathbf{3_{3/2}}$
	ψ_m	$\mathbf{1_{1/2}}+\mathbf{3_{1/2}}+2\,\bar{\mathbf{3}}_{\mathbf{1/2}}+\mathbf{6_{1/2}}+\mathbf{8_{1/2}}$
$\hat{\lambda}$	λ	$\mathbf{1_{1/2}}+\mathbf{3_{1/2}}$
$\hat{\chi}$	χ	$\mathbf{1_{1/2}}+\mathbf{3_{1/2}}$

Table 3.2: *Decomposition of the fermions in $SU(3)$ representations*

We denote the $SU(3)$ representation $\mathbf{R}$ with four-dimensional spin $\mathbf{s}$ by $\mathbf{R_s}$. For example, a triplet under $SU(3)$ yielding a field with spin one (a vector) in four-dimensions is denoted by $\mathbf{3_1}$. A four-dimensional tensor (or pseudo-scalar) is indicated by an index $\mathbf{T}$.

The four-dimensional spectrum given in tables 3.1 and 3.2 can be organized in $N = 4$ supermultiplets. More specifically the fields are displayed in gravity and seven vector supermultiplets. Only one of the vector multiplets is transforming in the adjoint of the four-dimensional gauge group while the remaining six are gauge neutral. In the gravity multiplet one finds the metric, four gravitinos, six vectors, four spin 1/2 fields while in each vector multiplet one finds one vector, four spin 1/2 fields and six scalars. A possible reduction to the standard $N = 1$ is achieved by truncating all triplets of $SU(3)$. This keeps only one gravitino in the spectrum transforming as a singlet. The latter will be the fermionic component of the $N = 1$ gravity multiplet.

So far we discussed the $SU(3)$ decomposition of the ten-dimensional fields without referring to the gauge group representations which are carried by the fields of the vector multiplet. The discussion of the gauge group decomposition inquires specifying the structure of the gauge bundle of $\mathcal{M}_6$. This determines the unbroken gauge group G in the four-dimensional effective action. However the aim here is not the construction of such bundles, therefore we will keep the precise multiplet structure largely unspecified. We will rather discuss a generalized version of the standard embedding where the spin connection is identified with an $SO(6)$ subgroup of the E_8 gauge connection [43]. Before doing that let us recall the standard embedding [5]. In this case the compactification manifold is identified with the Calabi-Yau manifold, where the spin connection takes values in $SU(3)$ The Bianchi identity for the field strength $\hat{H}_3$ defined in (2.2) leads to the relation between the two-form curvature $\hat{R}_2$ and the field strength $\hat{F}$ of the gauge boson

$$d\hat{H} = \tfrac{1}{4}\big(\mathrm{tr}(\hat{\mathrm{R}}_2 \wedge \hat{\mathrm{R}}_2) - \mathbf{Tr}(\hat{\mathrm{F}} \wedge \hat{\mathrm{F}})\big) \ . \tag{3.1}$$

As a condition for the Bianchi identity to be solvable, the right hand side of (3.1) should be cohomologically trivial

$$\big[\mathrm{tr}(\hat{\mathrm{R}}_2 \wedge \hat{\mathrm{R}}_2)\big] = \big[\mathbf{Tr}(\hat{\mathrm{F}} \wedge \hat{\mathrm{F}})\big] \ , \tag{3.2}$$

where the bracket $[\ldots]$ denotes the cohomology class. The simplest solution to this equation is given by the standard embedding. It amounts to set the spin connection of the manifold equal to the internal Yang-Mills connection which, then, takes values in a $SU(3)$ subset of one of the E_8's. The surviving gauge group is the maximal commutant of $SU(3)$ within $E_8 \times E_8$ which is, in this case, $E_6 \times E_8$. The chiral fermionic fields are found to be transforming in the $\mathbf{27}$ and $\bar{\mathbf{27}}$ of E_6.

On a generic sixfold the spin connection takes values in $SO(6)$. As a generalization of the standard embedding we take here the gauge connection to be $SO(6)$ valued as well. The identification of gauge and spin connections enables us to further decompose the $SO(6)$ representations in terms of the structure group

$SU(3)$. In this case the gauge group is decomposed as follows

$$
\begin{aligned}
E_8 &\rightarrow SO(6) \times SO(10) \\
&\rightarrow SU(3) \times U(1) \times SO(10) \ ,
\end{aligned} \tag{3.3}
$$

and its adjoint representation **248** decomposes accordingly as

$$
\mathbf{248} \rightarrow (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{15}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\bar{\mathbf{4}}, \bar{\mathbf{16}}) \ . \tag{3.4}
$$

Projecting out the triplets arising from the decomposition of $\mathbf{4} \rightarrow \mathbf{3} \oplus \mathbf{1}$ and decomposing the **15** of $SO(6)$ under the $SU(3)$ structure of $\mathcal{M}_6$ reveals that the field A_m and the the gaugino $\hat{\chi}$ are actually transforming in the representations $\mathbf{6},\ \mathbf{8},\ \mathbf{1}$ as well. Hence these fields will not be projected out but are kept in the spectrum.

The resulting field content after the truncation of the triplets of $SU(3)$ has the structure of four-dimensional $N = 1$ multiplets. The gravity multiplet consists of the metric $g_{\mu\nu}$ and the gravitino ψ_μ which are singlets under $SU(3)$. The vector A_μ and the singlet of the gaugino $\hat{\chi}$ are the components of the vector multiplet. In the linear multiplet one finds the four-dimensional two-form $B_{\mu\nu}$ and the singlet in the dilatino $\hat{\lambda}$. The fields among $g_{mn},\ B_{mn},\ \psi_m$ which are transforming as $\mathbf{6},\ \mathbf{8},\ \mathbf{1}$ under $SU(3)$ will be the components of the chiral moduli multiplets. The chiral matter multiplets are composed by the fields transforming as $\mathbf{6},\ \mathbf{8},\ \mathbf{1}$ under $SU(3)$ among A_m and $\hat{\chi}$. All the other fields are projected out due to the truncation of the triplets. The $N = 1, D = 4$ spectrum is summarized in table 3.3.

The linear multiplet can be dualized to a chiral multiplet if the B-field is massless. This is due to the fact that in four dimensions a massless two tensor is dual to a scalar. In case of the massless B-field the dual scalar 'a' is called the axion.

The fields of the vector multiplet carry an index a referring to the adjoint of the unbroken four-dimensional gauge group $G \times E_8$ in which they transform. The charged fields of the chiral multiplets transform in some appropriate representations of G. In the example discussed above where $G = SO(10)$ these fields are transforming in the $\mathbf{10}, \mathbf{16}$ or $\bar{\mathbf{16}}$. Additional singlets might also be present. In the following the precise (gauge) representation will play no role and we will only need the $SU(3)$ representations in which the fields transform.

After giving the decomposition of the fields under the $SU(3)$ structure of the manifold $\mathcal{M}_6$, keeping all fields transforming as $\mathbf{1},\ \mathbf{6},\ \mathbf{8}$ and only projecting out the triplets of the $SU(3)$, we give the Kaluza-Klein reduction of the fields. In order

multiplet	$SU(3)$ rep.	field content
gravity multiplet	$\mathbf{1}$	$(g_{\mu\nu}, \psi_\mu)$
linear multiplet	$\mathbf{1}$	$(B_{\mu\nu}, \lambda)$
vector multiplets	$\mathbf{1}$	(A_μ, χ)
chiral moduli multiplets	$\mathbf{6}$	(g_{mn}, ψ_m)
	$\mathbf{8+1}$	(g_{mn}, B_{mn}, ψ_m)
chiral matter multiplets	$\mathbf{6}$	(A_m, χ)
	$\mathbf{8+1}$	(A_m, χ)

Table 3.3: *N=1 multiplets*

to perform this reduction one needs to specify a basis of forms on the internal manifold $\mathcal{M}_6$ used to expand the fields. This expansion yields the light fields in the spectrum of the four-dimensional theory. In Calabi-Yau compactifications, for example, the basis is composed of harmonic forms. For $SU(3)$ structure manifolds such a correspondence between the harmonics and the light modes is not clear. The basis, then, consists of infinite number of forms which are not necessarily harmonics. In other words the expansion takes into account all the modes of the Kaluza-Klein tower. Luckily much of the analysis can be performed independently from the basis. For the case at hand, the heterotic compactification on $SU(3)$ structure manifolds, we nevertheless restrict to a basis of finite set of forms Δ_{finite}. This enables us to perform the calculations and determine the four-dimensional effective action explicitly.

The finite basis of forms Δ_{finite} on the $SU(3)$ structure manifold is chosen to be slightly extending the Calabi-Yau basis [80, 68]. The major difference from the CY case is the fact that the forms of Δ_{finite} are not necessarily harmonics. The explicit construction of such finite set of forms is difficult. However we can specify its properties [68]. To do that let us first introduce an additional structure known as the Mukai pairing on the space of real n-forms on $\mathcal{M}_6$ which we denote by $\Lambda^n T^*$. It is defined as

$$\langle\varphi,\psi\rangle = \big[\lambda(\varphi)\wedge\psi\big]_6 = \begin{cases} \varphi_0\wedge\psi_6 - \varphi_2\wedge\psi_4 + \varphi_4\wedge\psi_2 - \varphi_6\wedge\psi_0 & \text{for even forms,} \\ -\varphi_1\wedge\psi_5 + \varphi_3\wedge\psi_3 - \varphi_5\wedge\psi_1 & \text{for odd forms,} \end{cases} \tag{3.5}$$

where $[\ldots]_6$ denotes the forms of degree 6 and the parity operator λ is defined in eqn. (2.12).

Let us denote the finite set of forms in $\Lambda^n T^*$ by Δ^n, with dimensions $\dim\Delta^n$. As a first condition we demand that $\dim\Delta^0 = \dim\Delta^6 = 1$ and assume that Δ^0 consists of the constant functions while Δ^6 contains volume forms $\epsilon \propto J\wedge J\wedge J$. One can give a (canonical) symplectic basis on the space $\Delta^{\text{ev}} = \Delta^0 \oplus \Delta^2$ as $\omega_{\hat{A}} = (1, \omega_A)$. The dual basis of $\Delta^4 \oplus \Delta^6$ is given as $\tilde{\omega}^{\hat{A}} = (\tilde{\omega}^A, \epsilon)$

$$\int_{\mathcal{M}_6} \langle\omega_{\hat{A}}, \tilde{\omega}^{\hat{B}}\rangle = \delta^{\hat{B}}_{\hat{A}}\ , \qquad \hat{A},\hat{B} = 0,\ldots,\dim\Delta^2\ , \tag{3.6}$$

with all other intersections vanishing. Turning to the odd forms Δ^{odd} we follow a similar strategy to define a symplectic basis. However, in accord with our assumption above of truncating the triplets of the $SU(3)$, we will set $\dim\Delta^1 = \dim\Delta^5 = 0$. A symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ of Δ^3 can be defined as

$$\int_{\mathcal{M}_6} \langle\alpha_{\hat{L}}, \beta^{\hat{K}}\rangle = \delta^{\hat{K}}_{\hat{L}}\ , \qquad \hat{K},\hat{L} = 0,\ldots,\tfrac{1}{2}\dim\Delta^3\ , \tag{3.7}$$

with all other intersections vanishing.

In a manifold with $SU(3)$ structure there always exists an almost complex structure I as we reviewed in section 2.2.2. With respect to I one can define (p,q) grading of forms, for instance the two-forms ω_A are $(1,1)$-forms and their duals $\tilde{\omega}^B$ are then $(2,2)$-forms.

From the discussion above it is clear that the four-dimensional spectrum arises from expansion of the ten-dimensional fields in $(1,1)$-, $(2,2)$- and three-forms only. Let us now turn to this expansion in some more detail starting with the bosonic fields in 3.1.1 and discussing the fermions in 3.1.2

3.1.1 Bosonic Spectrum

Since the dilaton $\hat{\phi}$ is already a scalar in $D = 10$ it trivially descends to the four dimensional theory $\hat{\phi}(x,y) = \phi(x)$. The antisymmetric tensor $\hat{B}_{MN}$ is decomposed on a basis of two-forms $\Lambda^2 T^*$. Restricting to the finite set of forms Δ_{finite} the B-field $\hat{B}_{MN}$ is expanded into the basis of $\Delta^0 \oplus \Delta^2$ which we denoted by $\omega_{\hat{A}} = (1, \omega_A)$

$$\hat{B}_2 = B_2(x) + b^A(x)\,\omega_A \ , \qquad A = 1, \ldots, \dim \Delta^2 \ , \tag{3.8}$$

where b^A are four-dimensional real scalar fields and $B_2(x)$ is a two-form in $D = 4$. In case it is massless it can be dualized to a scalar 'a' which combines with the dilaton ϕ to form complex scalar.

The decomposition of the ten-dimensional metric $\hat{G}_{MN}$ results in the $D = 4$ metric $g_{\mu\nu}$ and the internal metric g_{mn} as it is shown in table 3.1. The deformations of the internal metric δg_{mn} give rise to two distinct classes of scalar fields corresponding to the $\mathbf{8} \oplus \mathbf{1}$ and the $\mathbf{6}$ representations. We decompose the first class into two-forms in $\Lambda^2 T^*$ and the latter into three-forms in $\Lambda^3 T^*$. In the finite basis of forms Δ_{finite} these are most easily distinguished by going to complex indices $\alpha, \bar{\beta} = 1, 2, 3$ with respect to the almost complex structure I. In this notation $\delta g_{\alpha\bar{\beta}}$ transforms in the $\mathbf{8} \oplus \mathbf{1}$ representation while $\delta g_{\alpha\beta}$ transforms in the $\mathbf{6}$. They are expanded as follows

$$\begin{aligned} \delta g_{\alpha\bar{\beta}} &= -i\tilde{v}^A(x)\,(\omega_A)_{\alpha\bar{\beta}} \ , && \alpha, \bar{\beta} = 1, 2, 3 \\ \delta g_{\alpha\beta} &= \tfrac{i}{\|\Omega\|^2}\,\bar{z}^K(x)\,(\bar{\rho}_K)_{\alpha\bar{\beta}\bar{\gamma}}\Omega^{\bar{\beta}\bar{\gamma}}_{\beta} \ , && K = 1, \ldots, \tfrac{1}{2}\dim \Delta^3 \ , \end{aligned} \tag{3.9}$$

where the ω_A are the $(1,1)$-forms transforming in $\mathbf{8} \oplus \mathbf{1}$ already used in (3.8) while the ρ_K are a set of $(1,2)$-forms transforming in the $\mathbf{6}$. Ω is the (3,0) form on $\mathcal{M}_6$ which differs from the Ω_η introduced in (2.23) by a rescaling $\Omega = ||\Omega||\Omega_\eta$ with $||\Omega||^2 \equiv \frac{1}{3!}\Omega_{\alpha\beta\gamma}\bar{\Omega}^{\alpha\beta\gamma}$. $||\Omega||$ is constant on the manifold but as reviewed in appendix C does depend on the scalar fields. It is introduced in (3.9) for later convenience to ensure a properly normalized metric on the space of metric deformations [55].

The expansions (3.9) are in analogy with the Calabi-Yau case where $\tilde{v}^A$ are real scalars parameterizing the Kähler-form deformations, while the z^K are complex scalars associated to the complex structure deformations (for more details see appendix C).

The expansion given in (3.9) features the metric in the Einstein frame but, as we will see in the next section, the correct four-dimensional field variables v^A arise from the expansion of the metric in the string frame. The two metrics differ by a dilaton dependent factor which relates the scalar fields as follows

$$\tilde{v}^A = v^A e^{-\phi/2} \ . \tag{3.10}$$

The real scalars v^A combine with the real scalars b^A arising in the expansion of the B-field introduced in (3.8). We denote the resulting complex scalars by $t^A = b^A + iv^A$.

The reduction of ten-dimensional gauge field $\hat{A}_M$ gives rise to the gauge field A_μ and A_m in $D = 4$. The four-dimensional gauge boson is a singlet under $SU(3)$ but transforms in the adjoint representation of $G \times E_8$. The fields A_m are expanded into a basis of $\Lambda^2 T^*$ and $\Lambda^3 T^*$ transforming either in the representation $\mathbf{8} \oplus \mathbf{1}$ of the $SU(3)$ or in the $\mathbf{6}$. The coefficients of the expansion are charged fields under G and transform in some of its representations. Since we restrict here to the finite basis of forms Δ_{finite} we give explicitly the expansions of A_m. These are best represented in complex indices

$$\hat{A}_{\alpha\beta} \;=\; \tfrac{1}{\|\Omega\|^2} A^K(x) (\bar{\rho}_K)_{\alpha\bar{\beta}\bar{\gamma}} \Omega_{\beta}^{\bar{\beta}\bar{\gamma}}, \qquad \hat{A}_{\alpha\bar{\beta}} = A^A(x)(\omega_A)_{\alpha\bar{\beta}} \; . \tag{3.11}$$

We summarize the four-dimensional light spectrum in table 3.4.

3.1.2 Fermionic Spectrum

Here we give the decomposition of the ten-dimensional fermions in the background $M_{(3,1)} \times \mathcal{M}_6$. Before doing that let first look at the decomposition of the ten-dimensional SUSY parameter $\hat{\epsilon}$

$$\hat{\epsilon} = \epsilon \otimes \eta_- + (\epsilon)^* \otimes (\eta_-)^* = \epsilon \otimes \eta_- + \bar{\epsilon} \otimes \eta_+ \; , \tag{3.12}$$

where $*$ stands for complex conjugation. The spinors ϵ and $\bar{\epsilon}$ are four-dimensional Weyl spinors parameterizing the $N = 1$ supersymmetry in $M_{(3,1)}$. They have positive and negative four-dimensional chiralities respectively. $\eta_\pm$ are Weyl spinors in the internal manifold $\mathcal{M}_6$, and their six-dimensional chiralities are denoted by $\pm$. The spinor conventions are summarized in appendix B.

In the same spirit we decompose the singlets in $\hat{\psi}_\mu$, $\hat{\chi}$, $\hat{\lambda}$. This reads as follows

$$\hat{\psi}_\mu \;=\; \psi_\mu \otimes \eta_- + \bar{\psi}_\mu \otimes \eta_+ \; , \tag{3.13}$$
$$\hat{\chi} \;=\; \chi \otimes \eta_- \; + \; \bar{\chi} \otimes \eta_+ \; , \tag{3.14}$$
$$\hat{\lambda} \;=\; \lambda \otimes \eta_+ \; + \; \bar{\lambda} \otimes \eta_- \; , \tag{3.15}$$

where ψ_μ, $\bar{\psi}_\mu$ are Weyl spinors corresponding to the four-dimensional gravitino, while the Weyl spinors χ, $\bar{\chi}$ refer to the four-dimensional gaugino. Finally λ, $\bar{\lambda}$ denote Weyl spinors corresponding to the four-dimensional dilatino. Here ψ_μ, χ, λ have positive four-dimensional chiralities while $\bar{\psi}_\mu$, $\bar{\chi}$, $\bar{\lambda}$ have negative chiralities.

Note that we suppressed the gauge index of the gaugino referring to the adjoint of the four-dimensional gauge group $G \times E_8$.

In the decomposition of $\hat{\psi}_m$ we only want to keep the **6** and **8** $\oplus$ **1** representations of $SU(3)$ (see table 3.2). These are expanded into the basis of two- and three-forms. In the finite basis this is given by

$$\hat{\psi}_\alpha \quad = \quad \xi^A \otimes (\omega_A)_{\alpha\bar{\beta}}\gamma^{\bar{\beta}}\eta_+ \; + \; \tfrac{1}{\|\Omega\|^2}\bar{\zeta}^K \otimes (\bar{\rho}_K)_{\alpha\bar{\beta}\bar{\gamma}}\Omega_\beta^{\bar{\beta}\bar{\gamma}}\gamma^\beta\eta_- \; , \tag{3.16}$$

where ζ^K and ξ^A are gauge neutral Weyl fermions.

Finally, the chiral matter arise in the decomposition of the ten-dimensional gaugino into the basis of $\Lambda^2 T^*$ and $\Lambda^3 T^*$. Analogously to (3.16) we give the expansion of the chiral matter in the finite basis in terms of forms which are in the **6** or in the **8** $\oplus$ **1** of $SU(3)$

$$\hat{\chi}_\beta \quad = \quad \chi^A \otimes (\omega_A)_{\beta\bar{\alpha}}\gamma^{\bar{\alpha}}\eta_+ \; + \; \tfrac{1}{\|\Omega\|^2}\bar{\chi}^K \otimes (\bar{\rho}_K)_{\beta\bar{\gamma}\bar{\alpha}}\Omega_\delta^{\bar{\gamma}\bar{\alpha}}\gamma^\delta\eta_- \; . \tag{3.17}$$

Here χ^A and χ^K are four-dimensional chiral matter fermions. Together with the bosonic fields of the previous section the fermions combine into supermultiplets as summarized in table 3.4.

Multiplets	multiplicity	bosonic components	fermionic components
gravity multiplet	1	$g_{\mu\nu}$	ψ_μ
vector multiplet	$\dim G + \dim E_8$	A_μ^a	χ^a
chiral moduli multiplet	$\dim \Delta^2$	t^A	ξ^A
	$1/2 \dim \Delta^3$	z^K	ζ^K
linear multiplet	1	B_2, ϕ	λ
chiral matter multiplet	$\dim \Delta^2$	A^B	χ^B
	$1/2 \dim \Delta^3$	A^K	χ^K

Table 3.4: $N = 1$ *spectrum in* $D = 4$.

After we specified the decomposition of the ten-dimensional fields and analyzed the four-dimensional spectrum we next discuss the resulting four-dimensional effective action encoding the dynamics of the fields.

3.2 The low energy effective action

In this section we compute the four-dimensional low energy effective action by a Kaluza-Klein reduction. We start from the ten-dimensional $N = 1$ supergravity action of the heterotic string $S^{(10)} = S_\mathrm{b}+S_\mathrm{f}+S_\mathrm{int}$ given in (2.1), (2.4) and (2.5). To determine the $N = 1$ four-dimensional effective action it is sufficient to compute the Kähler potential, the superpotential and the gauge kinetic function. The aim of this section is to compute these functions entirely from fermionic terms. These are the gravitino mass term and the gravitino coupling to the chiral fermion terms. Thus we will mainly focus on the computation of the fermionic actions S_f and S_int. However for completeness next we review briefly the compactification of the bosonic part of $S^{(10)}$.

3.2.1 The kinetic terms in the $D = 4$ bosonic action

The compactification of the bosonic action S_b was studied extensively in the literature. For Calabi-Yau compactifications this has been computed for example in refs. [53, 54, 55] and for the analog computation on manifolds with $SU(3)$ structure see for instance [80, 68, 56]. Therefore we can be brief and basically just recall the results found for the Kähler potential K, the holomorphic gauge kinetic functions f and the holomophic superpotential W.

The compactification of the bosonic action S_b involves the insertion of (3.8) – (3.11) into (2.1) and performing a Weyl rescaling of the four-dimensional metric

$$g_{\mu\nu} \ \rightarrow \ e^{\frac{3}{2}D}\mathcal{K}^{-1}g_{\mu\nu} \ , \tag{3.18}$$

where D is the four-dimensional dilaton and $\mathcal{K}$ is the volume of $\mathcal{M}_6$ given by

$$D \ = \ \hat{\phi} - \frac{1}{2}\mathrm{ln}\mathcal{K} \ , \qquad \mathcal{K} = \frac{1}{6}\int_{\mathcal{M}_6} J\wedge J\wedge J \ , \qquad J = v^A\omega_A \ . \tag{3.19}$$

The Weyl rescaling is needed to bring the Einstein-Hilbert action to the standard form. Resulting from this rescaling some terms involving the volume of $\mathcal{M}_6$ appear. These are absorbed into the kinetic term of the four-dimensional dilaton D. The resulting bosonic effective action is an $N = 1$ action of the form (2.30). It is then specified by a Kähler potential, a superpotential and a gauge kinetic function. The latter is given by a complex scalar combining the four-dimensional dilaton D and the axion a dual to the B-field

$$f = S \ , \qquad S = \tfrac{1}{2}\, e^{-2D} + \tfrac{i}{2}\, a \ . \tag{3.20}$$

Before giving the Kähler potentials let us summarize the results of the Kähler metrics $g_{I\bar{J}}$ on the field space for the chiral moduli multiplets. It is block-diagonal with the non-trivial entries being the metric for the dilaton-axion complex scalar $g_{S\bar{S}}$, the metric for the chiral multiplets (t^A, ξ^A) denoted by g_{AB} and finally g_{KL} the metric for the chiral multiplets (z^L, ζ^L). These are summarized as

$$g_{S\bar{S}} = \frac{1}{(S+\bar{S})^2} \ , \qquad g_{AB} = \frac{1}{4\mathcal{K}} \int_{\mathcal{M}_6} \omega_A \wedge * \omega_B \ , \qquad g_{KL} = \frac{\int_{\mathcal{M}_6} \rho_K \wedge \overline{\rho}_L}{\int_{\mathcal{M}_6} \Omega \wedge \overline{\Omega}} \ . \quad (3.21)$$

Each metric can be shown to be a Kähler metric so that the Kähler potential is the sum of three terms

$$K = K^S + K^{\mathrm{K}} + K^{\mathrm{cs}} \ , \quad (3.22)$$

where

$$\begin{aligned} e^{-K^S} &= (S+\bar{S}) \ , \\ e^{-K^{\mathrm{K}}} &= -i \int_{\mathcal{M}_6} \langle \Pi^{\mathrm{ev}}, \bar{\Pi}^{\mathrm{ev}} \rangle = \mathcal{K}, \\ e^{-K^{\mathrm{cs}}} &= i \int_{\mathcal{M}_6} \langle \Omega, \bar{\Omega} \rangle \ . \end{aligned} \quad (3.23)$$

In fact both K^{K} and K^{cs} define a special Kähler manifolds in that they can be derived from a holomorphic prepotential. Further details can be found in appendix C.

For the scalars in the chiral matter multiplet one computes the moduli dependent metrics Z_{AB} and Z_{KL}. From the Kaluza Klein reduction one finds straightforwardly by inserting (3.11) into the last term in (2.1)

$$Z_{AB} = 4e^{-\frac{\phi}{2}} \tilde{g}_{AB} \ , \qquad Z_{KL} = 4e^{-\frac{\phi}{2}} g_{KL} \ . \quad (3.24)$$

However, in order to display the metric in the standard supergravity form (2.29) one needs a further rescaling

$$A^A \to \tfrac{1}{2} e^{\frac{-\hat{\phi}}{2}} e^{\frac{1}{6}(K^{\mathrm{cs}} - K^{\mathrm{K}})} A^A \ , \qquad A^K \to \tfrac{1}{2} e^{\frac{\hat{\phi}}{4}} e^{\frac{1}{6}(K^{\mathrm{K}} - K^{\mathrm{cs}})} A^K \ , \quad (3.25)$$

which results in the metrics [54]

$$Z_{AB} = e^{\frac{1}{3}(K^{\mathrm{cs}} - K^{\mathrm{K}})} g_{AB} \ , \qquad Z_{KL} = e^{\frac{1}{3}(K^{\mathrm{K}} - K^{\mathrm{cs}})} g_{KL} \ . \quad (3.26)$$

After this brief review on the compactification of the bosonic action S_b of the heterotic supergravity let us now turn to the computation of the fermionic actions

S_{f} and S_{int}. Due to supersymmetry this will not add any new informations to the bosonic compactification. Our aim in doing this further compactification is to provide a consistent reduction of the fermionic part which, to our surprise, is missing in the literature even for Calabi-Yau. To start with let us discuss the fermionic kinetic terms resulting from the compactification of S_{f} and then we turn to the couplings of S_{int}.

3.2.2 The kinetic terms in the $D = 4$ fermionic action

In this section we will compute the four-dimensional fermionic kinetic terms resulting from the compactification of the ten-dimensional fermionic action S_{f}. Therefore we only focus in (2.4) on terms which contain a spacetime derivative $D_\mu = \partial_\mu + \omega_\mu$ where ω_μ is the spin connection.

The Γ-matrices are decomposed as in (B.17). As a consequence of (B.24) terms like $\eta_\pm^\dagger \gamma^\alpha \dots \gamma^{\bar\beta} \eta_\pm$ vanish unless they have an equal number of holomorphic and antiholomorphic gamma matrices [83, 84]. These terms in turn can be simplified by using (B.23). For example one computes

$$\eta_+^\dagger \gamma^\gamma \gamma^{\bar\alpha\beta} \gamma^{\bar\lambda} \eta_+ = g^{\gamma\bar\alpha} g^{\beta\bar\lambda} - \frac{1}{2} g^{\beta\bar\alpha} g^{\gamma\bar\lambda} \ . \tag{3.27}$$

The kinetic terms of ψ_μ, λ and χ follow straightforwardly by inserting (3.13)-(3.15) into (2.4). The only complication arises from terms involving $\hat\psi_\alpha$. In their reduction one encounters the integrals

$$\begin{aligned} \int d^6y \sqrt{g_6}\, (\omega_A)_{\bar\alpha\delta} (\omega_B)_{\beta\bar\delta} (2 g^{\delta\bar\alpha} g^{\beta\bar\delta} - g^{\beta\bar\alpha} g^{\delta\bar\delta}) &= 2\tilde{\mathcal{K}}_{AB} + 4\tilde{\mathcal{K}} g_{AB} \ , \\ \tfrac{1}{\|\Omega\|^4} \int d^6y \sqrt{g_6} (\rho_K)_{\bar\alpha\epsilon\gamma} \bar\Omega^{\epsilon\gamma}_{\bar\gamma} (\bar\rho_L)_{\beta\bar\delta\bar\lambda} \Omega^{\bar\delta\bar\lambda}_{\lambda} (g^{\bar\alpha\beta} g^{\bar\gamma\lambda} - 2 g^{\bar\gamma\beta} g^{\bar\alpha\lambda}) &= -4\tilde{\mathcal{K}} g_{KL} \ , \end{aligned} \tag{3.28}$$

where we used (3.21), (3.27) and abbreviated $\tilde{\mathcal{K}}_{AB} = \int_{\mathcal{M}_6} \omega_A \wedge \omega_B \wedge \tilde J$. The holomorphic three-form Ω is related to the three-form Ω_η via a scalar fields dependent factor (C.41). We denote the quantities in the Einstein frame with a tilde. Later on we need to compare with results computed in the string frame. To insure that the comparison is made in the same frame we will use the rescaling (3.10).

Inserting (3.16) and (3.17) into (2.4) and using (3.28) one arrives at

$$\begin{aligned} S_{\rm f} \;\; = & \int d^4x\sqrt{-g_4}\Big[\tilde{\mathcal{K}}\,\epsilon^{\mu\nu\rho\xi}\,\bar\psi_\mu\bar\sigma_\xi D_\nu\psi_\rho + \tfrac{i}{2}\,\tilde{\mathcal{K}}_i\,\bar\psi_\mu\bar\sigma^{[\mu}\sigma^{\nu]}D_\nu\bar\xi^i - \tfrac{i}{2}\,\tilde{\mathcal{K}}_A\,\xi^A\sigma^{[\mu}\bar\sigma^{\nu]}D_\mu\psi_\nu \\ & +i\,\bar\xi^A\bar\sigma^\mu D_\nu\xi^B(\tilde{\mathcal{K}}_{AB}+2\tilde{\mathcal{K}}\tilde g_{AB}) - 2i\,\bar\zeta^K\bar\sigma^\nu D_\nu\zeta^L\,\tilde{\mathcal{K}}g_{KL} - i\,\tilde{\mathcal{K}}\,\bar\lambda\bar\sigma^\mu D_\mu\lambda \\ & -i\,\tilde{\mathcal{K}}\,\bar\chi\bar\sigma^\mu D_\mu\chi - 4i\,\tilde{\mathcal{K}}\,\tilde g_{AB}\,\bar\chi^A\bar\sigma^\mu D_\mu\chi^B - 2i\,\tilde{\mathcal{K}}\,g_{KL}\,\bar\chi^K\bar\sigma^\mu D_\mu\chi^L\Big]\ , \end{aligned} \tag{3.29}$$

where $\tilde{\mathcal{K}}_A = \int_Y \omega_A\wedge\tilde J\wedge\tilde J$ and we rescaled all fields by $\frac{1}{\sqrt2}$. Note here that we ignored all terms where spacetime derivatives act on bosonic terms. They should combine into appropriate covariant derivatives as given in (2.33), (2.34).

The next step is to perform the Weyl rescaling of the metric as in (3.18). This will bring the four-dimensional gravitino kinetic term in (3.29) into the standard form. Since the σ^μ are defined with a vierbein they also rescale. In addition the Weyl rescaling requires a rescaling of all fermionic fields [82] as follows

$$\begin{aligned} &\sigma^\mu\to\tilde{\mathcal{K}}^{\frac12}\sigma^\mu\ , \qquad \psi_\mu\to\tilde{\mathcal{K}}^{-\frac14}\psi_\mu\ , \qquad \xi^A\to\tilde{\mathcal{K}}^{\frac14}\xi^A\ , \qquad \zeta^K\to\tilde{\mathcal{K}}^{\frac14}\zeta^K\ , \\ &\lambda\to\tilde{\mathcal{K}}^{\frac14}\lambda\ , \qquad \chi\to\tilde{\mathcal{K}}^{\frac14}\chi\ , \qquad \chi^A\to\tilde{\mathcal{K}}^{\frac14}\chi^A\ , \qquad \chi^K\to\tilde{\mathcal{K}}^{\frac14}\chi^K\ . \end{aligned} \tag{3.30}$$

Inspecting the Lagrangian (3.29) we see that the kinetic terms are not yet diagonal. This is achieved by shifting the gravitino as follows

$$\psi_\mu\to\psi_\mu+\frac{\tilde{\mathcal{K}}_A}{4\tilde{\mathcal{K}}}\,\sigma_\mu\bar\xi^A\ . \tag{3.31}$$

Inserting (3.30) and (3.31) into (3.29) we arrive at

$$\begin{aligned} S_{\rm f} \;\; = \;\; -i & \int d^4x\sqrt{-g_4}\Big[i\,\epsilon^{\mu\nu\rho\xi}\,\bar\psi_\mu\bar\sigma_\xi D_\nu\psi_\rho + \bar\chi\bar\sigma^\mu D_\mu\chi + \bar\lambda\bar\sigma^\mu D_\mu\lambda \\ & +2g_{KL}\,\bar\zeta^K\bar\sigma^\nu D_\nu\zeta^L - \bar\xi^A\bar\sigma^\mu D_\mu\xi^B\Big(\frac{\tilde{\mathcal{K}}_{AB}}{\tilde{\mathcal{K}}}+\frac38\frac{\tilde{\mathcal{K}}_A\tilde{\mathcal{K}}_B}{\tilde{\mathcal{K}}^2}+2\tilde g_{AB}\Big) \\ & + 4\tilde g_{AB}\,\bar\chi^A\bar\sigma^\mu D_\mu\chi^B + 4g_{KL}\,\bar\chi^K\bar\sigma^\mu D_\mu\chi^L\Big]\ . \end{aligned} \tag{3.32}$$

From (3.32) we see that the kinetic terms for the gravitino ψ_μ and the chiral fermions ζ^K agree already with the form of (2.32) as dictated by $N=1$ supergravity. However, the kinetic terms for the ξ^A, λ and χ are not yet in its standard forms. This requires two further field redefinitions. The first is[1]

$$\lambda\to\lambda - i\sqrt{\tfrac58}\frac{\tilde{\mathcal{K}}_A}{\tilde{\mathcal{K}}}\,\xi^A\ , \qquad \xi^A\to\xi^A - i\tfrac{1}{28}\sqrt{\tfrac58}\frac{\tilde{\mathcal{K}}^A}{\tilde{\mathcal{K}}}\,\lambda\ , \tag{3.33}$$

[1] Other rescalings of ξ^A, such as $\xi^A\to\xi^A+\alpha\tilde v^A(\xi^B\frac{\tilde{\mathcal{K}}_B}{\tilde{\mathcal{K}}})+i\beta\frac{\tilde{\mathcal{K}}^A}{\tilde{\mathcal{K}}}\,\lambda$, can be considered as well.

which can be understood as the supersymmetric analog of (3.19), and the second field redefinition is

$$\lambda \to \sqrt{\tfrac{14}{29}}\, e^{2D}\lambda\ , \qquad \xi^A \to \tfrac{1}{\sqrt{2}} e^{-\frac{\hat\phi}{2}}\xi^A\ , \qquad \zeta^K \to \tfrac{1}{\sqrt{2}}\zeta^K\ , \qquad (3.34)$$
$$\chi \to \tfrac{1}{\sqrt{2}} e^{-D}\chi\ , \qquad \chi^A \to \frac{1}{2} e^{\frac{-\hat\phi}{2}}\, e^{\frac{1}{6}(K^{\rm cs}-K^{\rm K})}\chi^A\ , \qquad \chi^K \to \frac{1}{2} e^{\frac{1}{6}(K^{\rm K}-K^{\rm cs})}\chi^K\ .$$

This brings the λ, χ, ξ kinetic terms into the standard form. Inserting (3.33) and (3.34) in (3.32) we arrive at

$$\begin{aligned} S_{\rm f} \ = \ -i\int d^4x\sqrt{-g_4}\, \Big[& i\,\epsilon^{\mu\nu\rho\xi}\,\bar\psi_\mu\bar\sigma_\xi D_\nu\psi_\rho + g_{S\bar S}\bar\lambda\bar\sigma^\mu D_\mu\lambda + {\rm Re}f\ \bar\chi\bar\sigma^\mu D_\mu\chi \qquad (3.35) \\ & + g_{KL}\bar\zeta^K\bar\sigma^\nu D_\nu\zeta^L + g_{AB}\bar\xi^A\bar\sigma^\mu D_\mu\xi^B + Z_{AB}\bar\chi^A\bar\sigma^\mu D_\mu\chi^B + Z_{KL}\bar\chi^K\bar\sigma^\mu D_\mu\chi^L \Big]\ . \end{aligned}$$

The comparison with (2.32) shows that the kinetic terms are now normalized in accord with the standard form of $N = 1$ supergravity.

Once the kinetic terms and the normalizations of the fermionic fields are determined let us now turn to the computation of the Yukawa couplings which arise in this compactification.

3.2.3 Yukawa Couplings

Yukawa couplings arise from the kinetic terms of the ten-dimensional gaugino $\hat\chi^{\mathcal{A}}$ given as the last term in (2.4). In the expansion of the covariant derivative of $\hat{\bar\chi}\Gamma^M D_M\hat\chi$ one finds $D_M\hat\chi^{\mathcal{A}} = \partial_M\hat\chi^{\mathcal{A}} + g f^{\mathcal{A}}_{\mathcal{BC}} A^{\mathcal{B}}_M\hat\chi^{\mathcal{C}}$ where $f^{\mathcal{A}}_{\mathcal{BC}}$ are the structure constants of the group E_8. The partial derivative of the gaugino $\partial_M\hat\chi^{\mathcal{A}}$ contributes to the four-dimensional kinetic term in (3.35). While the second term in $D_M\chi^{\mathcal{A}}$ gives rise to the Yukawa coupling terms. More specifically these arise from terms which have the structure

$$S_{Yuk} = \int d^{10}x\sqrt{-\hat G_{10}}\ f^{\mathcal{A}}_{\mathcal{BC}}\hat{\bar\chi}^{\mathcal{A}}\Gamma^M\hat A^{\mathcal{B}}_M\hat\chi^{\mathcal{C}}\ , \qquad (3.36)$$

where we consider only matter fields arising in the expansion of $\hat\chi$ and $\hat A_m$. To make this more concrete let us restrict to the finite basis $\Delta_{\rm finite}$ and insert the expansions (3.11) and (3.17) into (3.36). Doing that one arrives at

$$\begin{aligned} S_{Yuk} = \int d^4x\sqrt{-g_4}\ \ \Big[& \chi^K\cdot\chi^M\cdot A^L\,\tfrac{4}{\|\Omega\|^4}\int d^6y\,\sqrt{g_6}\,\Omega^{\bar\gamma\bar\alpha}_{\epsilon}(\bar\rho_K)_{\beta\bar\gamma\bar\alpha}(\bar\rho_L)^{\delta\beta}_{\alpha}(\bar\rho_M)^{\epsilon\alpha}_{\delta} \\ & + 8\chi^A\cdot\chi^C\cdot A^B\int_{\mathcal{M}_6}\omega_A\wedge\omega_B\wedge\omega_C + \dots\Big]\ , \qquad (3.37) \end{aligned}$$

where we used (C.42). The dots indicate that the four-dimensional fields are contracted with an invariant tensor of the group G which arise from the decomposition of the ten-dimensional E_8 structure constants f^A_{BC}. Under the decomposition of E_8 discussed in section 3.1 they decompose into the structure constants of G plus a product of the the invariant tensor in the fundamental representation of $SU(3)$ which is $\epsilon^{\alpha\beta\gamma}$ and an invariant tensor in the appropriate representation of G.

The first term in (3.37) can be rewritten as

$$\tfrac{1}{2\|\Omega\|^4}\int d^6y\,\sqrt{g_6}\,\Omega_{\epsilon}^{\bar\gamma\bar\alpha}(\bar\rho_K)_{\beta\bar\gamma\bar\alpha}(\bar\rho_L)^{\delta\beta}_{\alpha}(\bar\rho_M)^{\epsilon\alpha}_{\delta}=\int_{\mathcal{M}_6}\Omega\wedge\rho^{\alpha}_K\wedge\rho^{\beta}_L\wedge\rho^{\gamma}_M\Omega_{\alpha\beta\gamma}\,, \quad (3.38)$$

where (C.40) has been used and ρ^{α}_K is defined as follows

$$\rho^{\alpha}_K=\tfrac{1}{2\|\Omega\|^2}\bar\Omega^{\alpha\beta\gamma}(\rho_K)_{\beta\gamma\bar\alpha}dz^{\bar\beta}\ . \quad (3.39)$$

It is proved for Calabi-Yau manifolds in [55, 85] that (3.38) is the third derivative of the prepotential $\mathcal{G}$ given in (C.40). Since the finite basis Δ_{finite} is chosen to be slightly enlarging the CY basis we expect this relation to hold for Δ_{finite} as well. Therefore we write

$$\tfrac{1}{2\|\Omega\|^4}\int d^6y\,\sqrt{g_6}\Omega_{\epsilon}^{\bar\gamma\bar\alpha}(\bar\rho_K)_{\beta\bar\gamma\bar\alpha}(\bar\rho_L)^{\delta\beta}_{\alpha}(\bar\rho_M)^{\epsilon\alpha}_{\delta}=\frac{\partial^3\mathcal{G}}{\partial z^K\partial z^L\partial z^M}\ . \quad (3.40)$$

The second term in (3.37) involves the intersecting number $\mathcal{K}_{ABC}=\int_{\mathcal{M}_6}\omega_A\wedge\omega_B\wedge\omega_C$ which is the third derivative of the prepotential $\mathcal{F}$ as reviewed in appendix C. Finally by using (3.25), (3.34) one arrives at

$$S_{Yuk}=\int d^4x\sqrt{-g_4}e^{\frac{K}{2}}\Big[\chi^K\cdot\chi^M\cdot A^L\frac{\partial^3\mathcal{G}}{\partial z^K\partial z^L\partial z^M}+\chi^A\cdot\chi^C\cdot A^B\frac{\partial^3\mathcal{F}}{\partial t^A\partial t^B\partial t^C}\Big] \quad (3.41)$$

The Yukawa couplings are then given by the third derivatives of the prepotentials $\mathcal{G}$ and $\mathcal{F}$ with respect of the scalars in the chiral multiplets.

So far we computed the kinetic terms and the Yukawa couplings in the four-dimensional effective theory by a Kaluza-Klein reduction. In the next section we continue in the same spirit and derive the F- and D-terms related to non-trivial fluxes and/or torsion. The computation is done on the level of the fermionic action S_{int} where the F- and D-terms are appearing linearly.

3.2.4 The F- and D-terms

The computation of the F- and D-terms can be done in two ways. On the one hand, from the reduction of the bosonic ten-dimensional action (2.1) and the derivation of the scalar potential V one can infer the superpotential and the D-terms by using (2.37). However, this procedure is problematic since W and D enter quadratically in V. On the other hand in the reduction of some fermionic terms the D- and F-terms appear linearly [43, 52]. In fact the superpotential W can be computed from the gravitino mass term while its derivatives (the F-terms) can be computed from the couplings of the gravitino to the chiral fermions. The D-terms arise from the couplings of the gravitino to the gaugino.

In the following we are going to compute these three couplings from a Kaluza-Klein reduction of the fermionic actions $S_{\rm f}$ and $S_{\rm int}$ given in (2.4) and (2.5). There will be two different contributions which we will discuss in turn. The first is the contribution from the background NS-flux H_3 which is the vacuum expectation value for the NS-NS field strength $\hat{H}$. This contribution arises from the term $e^{-\frac{\hat{\phi}}{2}}\hat{H}_{MNP}(\hat{\bar{\psi}}_Q\Gamma^{QMNPR}\hat{\psi}_R)$ in (2.5) when the three-form field strength $\hat{H}$ takes a non-trivial background value on the manifold $\mathcal{M}_6$. The other contribution comes from the torsion on the manifold. It arises from the ten-dimensional gravitino kinetic term when considering only the internal derivative D_m.

Contribution form H_3-flux

As anticipated above the flux contribution comes from the ten-dimensional term $S_H = \frac{1}{4}\int d^{10}x\sqrt{-\hat{G}_{10}}e^{-\frac{\hat{\phi}}{2}}\hat{H}_{MNP}\hat{\bar{\psi}}_L\Gamma^{LMNPQ}\hat{\psi}_Q$ in (2.5). The flux contribution to the four-dimensional gravitino mass term arises then when both gravitinos $\hat{\psi}_M$ in S_H carry external indices ($\mu = 0,\ldots,3$). Inserting (3.13) and using (2.23) and (C.43) one finds

$$
\begin{aligned}
S_H &= \tfrac{1}{4}\int d^4x\sqrt{-g_4}\,\big[\bar{\psi}_\mu\bar{\sigma}^{[\mu}\sigma^{\nu]}\bar{\psi}_\nu\, e^{-\frac{\hat{\phi}}{2}}\int d^6y\sqrt{g_6}\,H_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\;\eta_-^\dagger\gamma^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\eta_+\big] + \ldots \\
&= \tfrac{1}{4}\int d^4x\sqrt{-g_4}\,\big[\bar{\psi}_\mu\bar{\sigma}^{[\mu}\sigma^{\nu]}\bar{\psi}_\nu\, e^{-\frac{\hat{\phi}}{2}}\int_{\mathcal{M}_6} H_3\wedge\Omega_\eta\big] + \ldots\ , \qquad (3.42)
\end{aligned}
$$

where we only display the contribution to the gravitino mass. Performing the Weyl rescalings (3.18), (3.30) and using (C.41) we arrive at

$$
S_H = -\tfrac{1}{2}\int d^4x\sqrt{-g_4}\big[\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu\; e^{\frac{K}{2}}\int_{\mathcal{M}_6}\langle\Omega, H_3\rangle\big] + \ldots\ , \qquad (3.43)
$$

where we define $\bar{\sigma}^{\mu\nu} = \frac{1}{4}\bar{\sigma}^{[\mu}\sigma^{\nu]}$ and the Mukai paring $\langle .,.\rangle$ is defined in (3.5). Comparing (3.43) with (2.35) one writes the flux contribution to the superpotential as

$$
W_H = \tfrac{1}{2}\int_{\mathcal{M}_6}\langle\Omega, H_3\rangle\ . \qquad (3.44)
$$

This is in complete analogy with refs. [43, 86, 87, 88, 89] where W has been computed before. Note that this derivation provides an independent check on the Kähler potentials (3.22), (3.23) which we explicitly used in (3.43).

The flux contribution to the F-terms arises as well from the KK reduction of S_H and the ten-dimensional gravitino-dilatino coupling $S_{\psi\lambda}$ in (2.5). The S_H contribution arises when inserting (3.31) and when choosing one of the $\hat{\psi}_M$ to carry an internal index $\hat{\psi}_m$. Inserting (3.13), (3.31) one finds

$$\begin{aligned} S_H = \quad & \tfrac{1}{4}\int d^4x\sqrt{-g_4}\Big[-\xi^A\sigma_\mu\bar{\sigma}^{[\mu}\sigma^{\nu]}\bar{\psi}_\nu\ \tfrac{\tilde{\mathcal{K}}_A}{4\tilde{\mathcal{K}}}e^{-\frac{\hat{\phi}}{2}}\int d^6y\sqrt{g_6}\,H_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\ \eta_-^\dagger\gamma^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\eta_+ \\ & +i\zeta^K\sigma^\mu\bar{\psi}_\mu e^{-\frac{\hat{\phi}}{2}}\int d^6y\sqrt{g_6}\,(\rho_K)_{\bar{\alpha}\beta\gamma}\tfrac{\bar{\Omega}_{\bar{\beta}}^{\ \beta\gamma}}{\|\Omega\|^2}\eta_-^\dagger\gamma^{\bar{\delta}\bar{\epsilon}\theta\bar{\alpha}}\gamma^{\bar{\beta}}\eta_+H_{\bar{\delta}\bar{\epsilon}\theta} \\ & -\xi^A\sigma^\mu\bar{\psi}_\mu e^{-\frac{\hat{\phi}}{2}}\int d^6y\sqrt{g_6}\,H_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\,(\omega_A)_{\bar{\delta}\epsilon}\eta_-^\dagger\gamma^\epsilon\gamma^{\bar{\delta}\bar{\alpha}\bar{\beta}\bar{\gamma}}\eta_+\Big]+\dots\ . \end{aligned} \tag{3.45}$$

Using (2.23), (3.33), (3.34), (C.43) and (C.41) and Weyl rescaling according to (3.30) and (3.18) one finds

$$\begin{aligned} S_H \quad = \quad & -i\tfrac{1}{8\sqrt{2}}\int d^4x\,\sqrt{-g_4}\,e^{\frac{K}{2}}\Big[\xi^A\,\sigma^\nu\,\bar{\psi}_\nu\,(\frac{i\mathcal{K}_A}{4\mathcal{K}})\int_{\mathcal{M}_6}\langle\,\Omega,H_3\,\rangle \\ & +\tfrac{3}{14}\sqrt{\tfrac{35}{29}}\,\lambda\,\sigma^\nu\,\bar{\psi}_\nu\ e^{2D}\int_{\mathcal{M}_6}\langle\,\Omega,H_3\,\rangle \\ & +4\,\zeta^K\,\sigma^\mu\,\bar{\psi}_\mu\int_{\mathcal{M}_6}\langle\,\rho_K,H_3\,\rangle\Big]+\dots\ . \end{aligned} \tag{3.46}$$

This was the H_3-flux contributions to the F-terms arising from S_H. In addition to these there will be a contribution from the ten dimensional gravitino-dilatino coupling which arises by inserting (3.13), (3.14) and (3.33)

$$\begin{aligned} S_{\psi\lambda} \quad &= \quad -\frac{1}{8}\int d^{10}x\sqrt{-\hat{G}_{10}}\,e^{-\frac{\hat{\phi}}{2}}\hat{H}_{\hat{\alpha}\hat{\beta}\hat{\gamma}}\bar{\hat{\psi}}_\mu\Gamma^{\hat{\alpha}\hat{\beta}\hat{\gamma}}\Gamma^\mu\hat{\lambda} && (3.47) \\ &= \quad -\frac{i}{8}\int d^4x\sqrt{-g_4}e^{-\frac{\hat{\phi}}{2}}\{\bar{\psi}_\mu\bar{\sigma}^\mu\lambda-i\sqrt{\tfrac{5}{8}}\bar{\psi}_\mu\bar{\sigma}^\mu\xi^A\frac{\tilde{\mathcal{K}}_A}{\tilde{\mathcal{K}}}\}\int d^6y\sqrt{\hat{g}_6}H_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\eta_-^\dagger\gamma^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\eta_+\ . \end{aligned}$$

After a Weyl rescaling (3.30) and using (C.43), (C.41), (3.34) one finds

$$\begin{aligned} S_{\psi\lambda} \quad = \quad & -\tfrac{i}{16\sqrt{2}}\int d^4x\,\sqrt{-g_4}\,e^{\frac{K}{2}}\,\{\sqrt{\tfrac{14}{29}}\,\lambda\,\sigma^\mu\,\bar{\psi}_\mu\,e^{2D} \\ & -i\,\sqrt{\tfrac{5}{8}}\,\xi^A\,\sigma^\mu\,\bar{\psi}_\mu\,\frac{\mathcal{K}_A}{\mathcal{K}}\,\}\int_{\mathcal{M}_6}\langle\,\Omega,H_3\,\rangle\ . \end{aligned} \tag{3.48}$$

In order to compare this expressions with the standard supergravity Lagrangian (2.29) let us compute the derivatives of W_H. We find

$$D_A W_H = \frac{i\mathcal{K}_A}{4\mathcal{K}}\, W_H\,, \quad D_K W_H = \tfrac{1}{2}\int_{\mathcal{M}_6} \left\langle\, \rho_K, H_3 \,\right\rangle, \quad D_s W_H = -e^{2D} W_H \ , \tag{3.49}$$

where we used (C.38). Inserted into $S_H + S_{\psi\lambda}$ we arrive at

$$\begin{aligned} S_H + S_{\psi\lambda} \;\; = \;\; -\tfrac{i}{\sqrt{2}} \int d^4x \sqrt{-g_4}\; e^{\frac{K}{2}} \Big[& \, (\tfrac{1}{4} - \tfrac{1}{4}\sqrt{\tfrac{5}{8}})\xi^A \sigma^\nu \bar\psi_\nu \; D_A W_H \\ & + \zeta^K \sigma^\mu \bar\psi_\mu \; D_K W_H \; + \; (\tfrac{1}{8}\sqrt{\tfrac{14}{29}} - \tfrac{3}{14}\sqrt{\tfrac{35}{29}})\lambda \sigma^\nu \bar\psi_\nu \; D_S W_H \Big] \ , \end{aligned} \tag{3.50}$$

which shows the consistency with supergravity (2.35) up to numerical factors.

Contribution from torsion

Apart from H_3-flux the superpotential and the F-terms also receive contributions due to the torsion of the manifolds $\mathcal{M}_6$. These arise from the ten-dimensional gravitino kinetic term in (2.4) when the derivative D_M carries an internal index and acts on the spinor η. Before we do the reduction let us briefly recall the structure of these derivatives as determined in [90]. One decomposes $D_m \eta_\pm$ into a basis $(\eta, \gamma^7\eta, \gamma^n\eta)$ and defines the tensors q_m, q'_m, q_{mn}via

$$D_m \eta_+ \;\; = \;\; q_m \eta_+ + i q'_m \gamma^7 \eta_+ + i q_{mn}\gamma^n \eta_- \ , \tag{3.51}$$
$$D_m \eta_- \;\; = \;\; q_m \eta_- + i q'_m \gamma^7 \eta_- - i q_{mn}\gamma^n \eta_+ \ . \tag{3.52}$$

All q are real with q_m, q'_m transforming in the $\mathbf{3}\oplus\bar{\mathbf{3}}$ of the $SU(3)$ while q_{mn} contains the representations $\mathbf{36} = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{8} \oplus \mathbf{8}$. Going to complex indices and using (2.23), (2.26) one can express q_{mn} via the torsion classes [90]

$$q_{\alpha\beta} \;\; = \;\; -\frac{i}{4}\, \mathcal{W}_{3\alpha\bar\gamma\bar\delta}\, (\Omega_\eta)_\beta^{\ \bar\gamma\bar\delta} \; - \; \frac{1}{4} (\Omega_\eta)_{\alpha\beta\gamma}\, \overline{\mathcal{W}}{}_4^\gamma \ , \tag{3.53}$$
$$q_{\alpha\bar\beta} \;\; = \;\; -\frac{i}{4}\, \overline{\mathcal{W}}_{2\alpha\bar\beta} \; + \; \frac{1}{4} g_{\alpha\bar\beta}\, \overline{\mathcal{W}}_1 \ . \tag{3.54}$$

Equations (3.51), (3.53) and (3.54) are the necessary ingredients to compute the contributions of the torsion to the superpotential and the F-terms. Let us start with the superpotential torsion contribution. It arises from the first term in (2.4) with the derivative being internal and the gravitinos carrying flat indices. Inserting (3.13) we find

$$\begin{aligned} S_{\mathrm{f}\psi} &= -\int d^{10}x\sqrt{-\hat{G}_{10}}\,\hat{\bar{\psi}}_M\Gamma^{MNP}D_N\hat{\psi}_P \\ &= -\int d^4x\sqrt{-g_4}\bar{\psi}_\mu\bar{\sigma}^{[\mu}\sigma^{\nu]}\bar{\psi}_\nu\int d^6y\sqrt{g_6}\,\eta_-^\dagger\gamma^{\bar{\alpha}}D_{\bar{\alpha}}\eta_+ + \dots\ , \end{aligned} \tag{3.55}$$

where the ... refer to terms which are not quadratic in the gravitinos. Here it is not necessary to make the shift (3.31) for the four-dimensional gravitino. This is due to the fact that the resulting gravitino mass term after the shift has the same structure as in (3.55). The integral on the internal manifold $\mathcal{M}_6$ in eqn. (3.55) can be performed by using (3.51) and (3.53) and it is equal to

$$\int\eta_-^+\gamma^{\bar{\alpha}}D_{\bar{\alpha}}\eta_+ = \frac{3i}{4}\int_{\mathcal{M}_6}\mathcal{W}_1 = \frac{i}{8}\int_{\mathcal{M}_6}\left\langle\,\Omega_\eta,(d\tilde{J})\,\right\rangle\ , \tag{3.56}$$

where d is the six-dimensional exterior derivative and in the second equation we used (2.26)[2]. Let us stress that in this derivation we did not restrict our analysis to the case of half-flat manifolds like it was done in [43]. Doing the Weyl rescaling (3.30) and using (C.41), (3.10) one finds

$$S_{\mathrm{f}\psi} = -\int dx^4\sqrt{-g_4}\ e^{\frac{K}{2}}\ \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu W_T + \dots\ . \tag{3.57}$$

where

$$W_T = \tfrac{i}{2}\int_{\mathcal{M}_6}\big\langle\Omega,(dJ)\big\rangle\ . \tag{3.58}$$

Together with the contribution from H_3-flux computed in (3.44) this yields

$$W = W_H + W_T = \tfrac{1}{2}\int_{\mathcal{M}_6}\big\langle\Omega,(H+idJ)\big\rangle\ . \tag{3.59}$$

After computing the torsion contribution to the superpotential, let us turn to the computation of this contribution for its derivatives. This can be computed from the gravitino-fermion couplings which arise from $S_{\mathrm{f}\psi}$ if we keep the following terms in the expansion

$$S_{\mathrm{f}\psi} = -\int d^{10}x\sqrt{-G_{10}}\ \big[\hat{\bar{\psi}}_\mu\Gamma^{\mu\hat{\alpha}\nu}D_{\hat{\alpha}}\hat{\psi}_\nu +\ \hat{\bar{\psi}}_\mu\Gamma^{\mu\hat{\alpha}\hat{\beta}}D_{\hat{\alpha}}\hat{\psi}_{\hat{\beta}}\big] + \dots . \tag{3.60}$$

[2]Since we are making the reduction from the Einstein frame, then (2.26) and the two-from $\tilde{J}$ should be expressed in the same frame.

By inserting the expansions of the $\hat{\psi}_\mu$ and $\hat{\psi}_m$ given in eqns. (3.13), (3.16) and making the shift of the gravitino (3.31) one arrives at

$$\begin{aligned} S_{\mathrm{f}\psi} \;=\; & -\tfrac{1}{\sqrt{2}}\int dx^4\sqrt{-g_4}\Big[\xi^A\sigma_\mu\sigma^{\mu\nu}\bar{\psi}_\nu\,\frac{\mathcal{K}_A}{4\mathcal{K}}\int_{\mathcal{M}_6}\eta_-^\dagger\gamma^{\bar{\alpha}}D_{\bar{\alpha}}\eta_+ \\ & \bar{\psi}_\mu\bar{\sigma}^\mu\zeta^K\int d^6y\sqrt{g_6}(\rho_K)_{\bar{\beta}\gamma\delta}\frac{\Omega^{\gamma\delta}_{\bar{\epsilon}}}{\parallel\Omega\parallel^2}\eta_-^\dagger\gamma^{\alpha\bar{\beta}}\gamma^{\bar{\epsilon}}D_\alpha\eta_+ \\ & +e^{\frac{-\hat{\phi}}{2}}\xi^A\sigma^\mu\bar{\psi}_\mu\int_{\mathcal{M}_6}(\omega_A)_{\bar{\beta}\alpha}\eta_-^\dagger\gamma^{\bar{\beta}}\gamma^{\alpha\bar{\delta}}D_{\bar{\delta}}\eta_+\Big)\Big]+\dots\,, \end{aligned} \tag{3.61}$$

where the rescaling (3.34) for the fields ξ,ζ were performed. Using the definition (3.51) of the derivative of the globally defined spinor η one writes

$$\begin{aligned} S_{\mathrm{f}\psi} \;=\; & \tfrac{-1}{\sqrt{2}}\int d^4x\sqrt{-g_4}\Big[\tfrac{i}{2}\bar{\psi}_\mu\bar{\sigma}^\mu\zeta^K\int d^6y\sqrt{g_6}(\rho_K)_{\bar{\beta}\gamma\delta}\frac{\Omega^{\gamma\delta}_{\bar{\epsilon}}}{\parallel\Omega\parallel^2}q^{\bar{\beta}\bar{\epsilon}} \\ & +\xi^A\sigma^\mu\bar{\psi}_\mu\Big(\tfrac{9i}{16}\frac{\mathcal{K}_A}{\mathcal{K}}\int_{\mathcal{M}_6}\mathcal{W}_1+\tfrac{i}{2}e^{\frac{-\hat{\phi}}{2}}\int d^6y\sqrt{g_6}\,(\omega_A)_{\bar{\beta}\alpha}q^{\alpha\bar{\beta}}\Big)\Big]\;+\dots\,. \end{aligned} \tag{3.62}$$

Using (3.53) and (C.31) yields

$$\begin{aligned} S_{\mathrm{f}\psi} \;=\; & \tfrac{1}{\sqrt{2}}\int d^4x\sqrt{-g_4}\Big[\tfrac{1}{8}\zeta^K\sigma^\mu\bar{\psi}_\mu\frac{\tilde{\mathcal{K}}^{\frac{1}{2}}}{(i\int\Omega\wedge\bar{\Omega})^{\frac{1}{2}}}\int_{\mathcal{M}_6}\rho_K\wedge\mathcal{W}_3 \\ & +\xi^A\sigma^\mu\bar{\psi}_\mu\Big(\tfrac{3i}{16}\frac{\mathcal{K}_i}{\mathcal{K}}\int_{\mathcal{M}_6}\mathcal{W}_1-\frac{1}{8}\,e^{\frac{-\hat{\phi}}{2}}\int_{\mathcal{M}_6}(\mathcal{W}_2\wedge J\wedge\omega_A+\mathcal{W}_1J^2\wedge\omega_A)\Big)\Big]\,. \end{aligned} \tag{3.63}$$

The integrals on $\mathcal{M}_6$ of the above expressions which involve the torsion classes $\mathcal{W}_i$ can be expressed in terms of dJ, $d\Omega$ by using (2.26). Using as well $\partial_i K=\frac{i\mathcal{K}_i}{4\mathcal{K}}$, (C.41) and the Weyl rescaling (3.30) eq. (3.63) implies

$$\begin{aligned} S_{\mathrm{f}\psi} \;=\; & \tfrac{-i}{8\sqrt{2}}\int d^4x\sqrt{-g_4}\;\Big[\zeta^K\sigma^\mu\bar{\psi}_\mu\{\frac{i}{\tilde{\mathcal{K}}(i\int\Omega\wedge\bar{\Omega})^{\frac{1}{2}}}\int_{\mathcal{M}_6}\langle\rho_K,d\tilde{J}\rangle\} \\ & +\xi^A\sigma^\mu\bar{\psi}_\mu\frac{1}{\tilde{\mathcal{K}}(i\int\Omega\wedge\bar{\Omega})^{\frac{1}{2}}}\{i\,\partial_iK\int_{\mathcal{M}_6}\langle\Omega,(d\tilde{J})\rangle-ie^{\frac{-\hat{\phi}}{2}}\int_{\mathcal{M}_6}\langle d\Omega,\partial_AJ\rangle\}\Big]\,. \end{aligned} \tag{3.64}$$

With the help of (3.10) we see that we can rewrite the above terms as the derivative of W_T with respect to the moduli (up to 1/2 factor)

$$S_{\mathrm{f}\psi} = \tfrac{-i}{8\sqrt{2}} \int d^4x\sqrt{-g_4}\ e^{\frac{K}{2}} \Big[\xi^A \sigma^\mu \bar{\psi}_\mu D_A \int_{\mathcal{M}_6} \big\langle \Omega, (idJ) \big\rangle + \zeta^K \sigma^\mu \bar{\psi}_\mu D_K \int_{\mathcal{M}_6} \big\langle \Omega, (idJ) \big\rangle \Big] \tag{3.65}$$

$$= \tfrac{-i}{4\sqrt{2}} \int d^4x\sqrt{-g_4}\ e^{\frac{K}{2}} \Big[\xi^A \sigma^\mu \bar{\psi}_\mu\, D_A W_T + \zeta^K \sigma^\mu \bar{\psi}_\mu\, D_K W_T \Big]\ . \tag{3.66}$$

Finally we look at the dilatino-gravitino coupling which might arise from two terms. The first term in S_{int} would contribute trivially to $\lambda - \psi$ coupling. The second contribution comes from $\xi^i \psi$ coupling, given in (3.63), by using (3.33).

$$\begin{aligned} S_{\lambda-\psi} &= \tfrac{-i}{7}\sqrt{\tfrac{35}{29}} \int d^4x\sqrt{-g_4}\, \lambda \sigma^\mu \bar{\psi}_\mu\, e^{2D} \left(i \int_{\mathcal{M}_6} \mathcal{W}_1 \right) \\ &= \tfrac{-i}{42}\sqrt{\tfrac{35}{29}} \int d^4x\sqrt{-g_4} \lambda \sigma^\mu \bar{\psi}_\mu e^{2D} \int_{\mathcal{M}_6} \big\langle \Omega_\eta, (id\tilde{J}) \big\rangle \\ &= \tfrac{-i}{42}\sqrt{\tfrac{35}{29}} \int d^4x\sqrt{-g_4} e^{\frac{K}{2}} \lambda \sigma^\mu \bar{\psi}_\mu\, D_s W_T\ , \end{aligned} \tag{3.67}$$

where we made use of (3.34), (3.10), (C.41) and the Weyl rescaling (3.30). Equations (3.65) and (3.67) are consistent with the standard N=1 supergravity (up to a numerical factor).

After computing the F-terms let us discuss the D-terms which might arise in the compactification of the heterotic on $SU(3)$ structure manifolds.

The D-terms

At the level of the fermionic action S_{int} the D-terms are arising from the coupling of the gravitino to the gaugino $\psi_\mu - \chi$. Inserting (3.13) and (3.14) in the ten-dimensional $\hat{\psi}_M - \hat{\chi}$ coupling, given in (2.5), one finds

$$\begin{aligned} S_D &= -\int d^{10}x \sqrt{\hat{G}_{10}}\, e^{-\hat{\phi}} \hat{F}^A_{MN} \hat{\bar{\chi}}^A \Gamma^Q \Gamma^{MN} \hat{\psi}_Q \\ &= i \int dx^4 \sqrt{-g_4} e^{-\hat{\phi}} \bar{\chi}^a \bar{\sigma}^\mu \psi_\mu \int dy^6 \sqrt{g_6}\, \eta_-^\dagger \gamma^{\bar{\alpha}\beta} \eta_- F^a_{\bar{\alpha}\beta} \\ &= \tfrac{1}{2} \int dx^4 \sqrt{-g_4} e^{-\hat{\phi}} \bar{\chi}^a \bar{\sigma}^\mu \psi_\mu \int_{\mathcal{M}_6} F^a \wedge *_6 \tilde{J}\ , \end{aligned} \tag{3.68}$$

where the index a refers to the adjoint representation of the four-dimensional unbroken gauge group in which the four-dimensional gauge boson and gaugino transform. In the last step in (3.68) we used the definition of the two-form J given in (2.23).

Performing the Weyl rescaling (3.30) and the rescaling of χ given in (3.34) one finally arrives at

$$S_D = \tfrac{1}{2\sqrt{2}} \int dx^4 \sqrt{-g_4}\, e^{-\frac{\hat{\phi}}{4}}\, \mathcal{K}^{-1}\, \bar{\chi}^a \sigma^\mu \psi_\mu \int_{\mathcal{M}_6} F^a \wedge *_6 J \ . \tag{3.69}$$

Comparing equation (3.69) with the $N = 1, D = 4$ supergravity action given in (2.35) one can infer that the D-term of the theory is equal to

$$\mathcal{D}^a \ = \ -\tfrac{1}{\sqrt{2}}\, e^{-\frac{\hat{\phi}}{4}}\, \mathcal{K}^{-1} \int_{\mathcal{M}_6} F^a \wedge *_6 J \ . \tag{3.70}$$

Thus far we determined the four-dimensional effective action resulting from the compactification of the heterotic supergravity on $SU(3)$ structure manifold. We gave the appropriate rescaling of the fields and we computed the superpotential and its derivatives, which receive contributions from torsion and fluxes, as well as the D-terms. Next we discuss the supersymmetry transformations of the fermionic fields under which the $D = 4$ theory is invariant.

3.3 The supersymmetry transformations

In an $N = 1$ four-dimensional supergravity theory [82] the superpotential, F- and D-terms appear in the SUSY transformations of the fermionic fields linearly. Thus an alternative way to compute these quantities and to check the results found earlier is to determine $\delta\psi_\mu,\ \delta\xi^A,\ \delta\zeta^K,\ \delta\lambda,\ \delta\chi^a$. Therefore, we turn next to the discussion of the four-dimensional supersymmetry variations of the gravitino, chiral fermions, gaugino and dilatino.

3.3.1 Gravitino supersymmetry transformation

The four-dimensional gravitino is a combination of the singlet of ψ_μ and chiral fermions arising in the expansion of ψ_m. This combination is given in (3.31) as $\psi^s_\mu = \psi_\mu + \frac{\tilde{\mathcal{K}}_A}{4\tilde{\mathcal{K}}} \sigma_\mu \bar{\xi}^A$. It turns out that the multiplication of ψ^s_μ by $\bar{\epsilon}\sigma^\mu$, where ϵ is the SUSY spinor, can be written as the sum

$$\frac{1}{\tilde{\mathcal{K}}}\int d^6y\sqrt{g_6}\,\hat{\bar{\epsilon}}\,(\sigma^\mu\hat{\psi}_\mu+2\gamma^\alpha\hat{\psi}_\alpha+2\gamma^{\bar{\alpha}}\hat{\psi}_{\bar{\alpha}})=\hat{\bar{\epsilon}}\sigma^\mu\hat{\psi}^s_\mu\ , \tag{3.71}$$

where $\hat{\psi}^s_\mu = \psi^s_\mu\otimes\eta_- + \bar{\psi}^s_\mu\otimes\eta_+$. The supersymmetry variation of the four-dimensional gravitino is given then as

$$\frac{1}{\tilde{\mathcal{K}}}\int d^6y\sqrt{g_6}\,\hat{\bar{\epsilon}}\,(\sigma^\mu\delta\hat{\psi}_\mu+2\gamma^\alpha\delta\hat{\psi}_\alpha+2\gamma^{\bar{\alpha}}\delta\hat{\psi}_{\bar{\alpha}})=\hat{\bar{\epsilon}}\sigma^\mu\delta\psi^s_\mu\ , \tag{3.72}$$

where $\delta\hat{\psi}_\mu$, $\delta\hat{\psi}_\alpha$ and $\delta\hat{\psi}_{\bar{\alpha}}$ can be computed from the SUSY transformation of the ten-dimensional gravitino given in (2.6) for $M=\mu,\ \alpha,\ \bar{\alpha}$ successively. Hence by using (2.6), (2.23) and (C.42) equation (3.72) amounts to

$$\begin{aligned}\delta\psi^s_\mu = D_\mu\epsilon + \frac{e^{-\frac{\hat{\phi}}{2}}}{2\tilde{\mathcal{K}}(i\int_{\mathcal{M}_6}\Omega\wedge\bar{\Omega})^{\frac{1}{2}}}\sigma_\mu\bar{\epsilon}\int_{\mathcal{M}_6}\Omega_3\wedge H_3 \\ +\frac{4}{\tilde{\mathcal{K}}^{\frac{3}{2}}}\sigma_\mu\bar{\epsilon}\int d^6y\sqrt{g_6}\eta_-^\dagger\gamma^{\bar{\alpha}}D_{\bar{\alpha}}\eta_+\ .\end{aligned} \tag{3.73}$$

For consistency a Weyl rescaling (3.30) was made. The third term in (3.73) was determined in (3.56). Taking into account the rescaling (3.10) for the two-form J one arrives at

$$\begin{aligned}\delta\psi^s_\mu &= D_\mu\epsilon+\frac{1}{2}\frac{1}{\tilde{\mathcal{K}}(i\int_{\mathcal{M}_6}\Omega\wedge\bar{\Omega})^{\frac{1}{2}}}\sigma_\mu\bar{\epsilon}e^{-\frac{\hat{\phi}}{2}}\int_{\mathcal{M}_6}\Omega_3\wedge(H_3+idJ) \\ &= D_\mu\epsilon^+ + \frac{1}{2}e^{\frac{K}{2}}\sigma_\mu\bar{\epsilon}\int_{\mathcal{M}_6}\big\langle\Omega_3,(H_3+idJ)\big\rangle\ .\end{aligned} \tag{3.74}$$

Comparing with (2.38) one finds that the superpotential is equal to

$$W=\tfrac{1}{2}\int_{\mathcal{M}_6}\Omega\wedge(H+idJ)\ , \tag{3.75}$$

which is in agreement with the results found in section 3.59. Let us no turn to the derivative of the superpotential (F-terms) which arise in the SUSY variation of the chiral fermions.

3.3.2 Chiral fermion supersymmetry transformations

From the decomposition of ψ_m given in (3.16) one can infer that the supersymmetry transformations of the chiral fermions ξ^A, ζ^K are given by

$$\delta\hat{\psi}_\alpha = \delta\xi^A \otimes (\omega_{\alpha\bar{\beta}})_A \gamma^{\bar{\beta}}\eta \; + \tfrac{1}{\|\Omega\|^2}\delta\zeta^K \otimes (\bar{\rho}_K)_{\alpha\bar{\beta}\bar{\gamma}}\Omega_\beta^{\bar{\beta}\bar{\gamma}}\gamma^\beta\eta_- \;, \tag{3.76}$$

where $\delta\hat{\psi}_\alpha$ is determined from (2.6) for $M = \alpha$. Therefore to find $\delta\xi^A$, $\delta\zeta^K$ one must find projectors such that either $\delta\xi^A$ or $\delta\zeta^K$ survives the projection in $\delta\hat{\psi}_\alpha$, $\delta\hat{\psi}_{\bar{\alpha}}$. The expected results for the SUSY transformations of the chiral fermions are the Kähler derivatives of the superpotential $D_A W$, $D_K W$.

The multiplication of (3.76) with $\mathbb{P}^\alpha = \bar{\epsilon}\gamma^\alpha$ projects out $\delta\zeta^K$ and only $\delta\xi^A$ is left. By using (2.6) one evaluates $\delta\psi_\alpha$ and hence $\delta\xi^A$ is determined after integrating out the six-dimensional dependence. However it turns out that the projector $\mathbb{P}^\alpha$, though it satisfies the criteria of projecting out $\delta\zeta^K$ in favor of $\delta\xi^A$, it annihilates some of the torsion terms which are present in $D_A W$ and therefore it annihilates some of the torsion contributions to $\delta\xi^A$. We will not present this calculation here but we assume that the SUSY variation of the ξ^A's is $\delta\xi^A = \sqrt{2}\, e^{\frac{K}{2}}\, g^{AB}\, D_B\bar{W}\;\epsilon$ where W is the superpotential given in (3.75).

Let us now turn to the SUSY transformations of the chiral fermions ζ^K. We follow, basically, the same strategy outlined above. The projector chosen here is $\mathbb{P}^\alpha_K = \bar{\epsilon}(\rho_K)_{\bar{\alpha}\Gamma\xi}\bar{\Omega}^{\Gamma\xi}_{\bar{\epsilon}}\gamma^{\bar{\epsilon}}\gamma^{\bar{\alpha}\alpha}$. Multiplying (3.76) by this projector and using (2.6) when $M = \alpha$ one finds after integrating over the six-dimensional manifold

$$\begin{aligned} 2i\delta\bar{\zeta}^K \int_{\mathcal{M}_6} \parallel \Omega \parallel^{-2} \rho_L \wedge \bar{\rho}_K = {} & e^{\frac{-\hat{\phi}}{2}}\tilde{\mathcal{K}}^{\frac{-3}{2}}\bar{\epsilon}\int_{\mathcal{M}_6}(\rho_L)_{\bar{\alpha}\Gamma\xi}\frac{\bar{\Omega}^{\Gamma\xi}_{\bar{\epsilon}}}{\parallel \Omega \parallel^2}H_{\alpha\bar{\beta}\bar{\gamma}}\eta_-^+\gamma^{\bar{\epsilon}}\gamma^{\bar{\alpha}}\gamma^{\alpha\bar{\beta}\bar{\gamma}}\eta_+ \\ & + \tilde{\mathcal{K}}^{\frac{-3}{2}}\bar{\epsilon}\int_{\mathcal{M}_6}(\rho_L)_{\bar{\alpha}\Gamma\xi}\frac{\bar{\Omega}^{\Gamma\xi}_{\bar{\epsilon}}}{\parallel \Omega \parallel^2}\eta_-^+\gamma^{\bar{\epsilon}}\gamma^{\bar{\alpha}\alpha}D_\alpha\eta_+ \;, \end{aligned} \tag{3.77}$$

here the Weyl rescaling (3.30) is performed. Using

$$\eta_-^+\gamma^{\bar{\epsilon}}\gamma^{\bar{\alpha}}\gamma^{\alpha\bar{\beta}\bar{\gamma}}\eta_+ \;\; = \;\; \tfrac{i}{8}g^{\alpha[\bar{\alpha}}\Omega_\eta^{\bar{\epsilon}\bar{\beta}\bar{\gamma}]} \;, \tag{3.78}$$

and the relation between Ω_η and the supergravity three-form Ω given in (C.41) then the first term in the right hand side of (3.77) is equal to

$$e^{\frac{-\hat{\phi}}{2}}\tilde{\mathcal{K}}^{\frac{-3}{2}}\bar{\epsilon}\int_{\mathcal{M}_6}(\rho_L)_{\bar{\alpha}\Gamma\xi}\,\frac{\bar{\Omega}^{\Gamma\xi}_{\bar{\epsilon}}}{\parallel \Omega \parallel^2}\,H_{\alpha\bar{\beta}\bar{\gamma}}\,\eta_-^+\gamma^{\bar{\epsilon}}\gamma^{\bar{\alpha}}\gamma^{\alpha\bar{\beta}\bar{\gamma}}\eta_+ = \tfrac{1}{8}\,e^{\frac{K}{2}}\int_{\mathcal{M}_6}\big\langle \rho_L, H\big\rangle \;. \tag{3.79}$$

The second six-dimensional integral is evaluated using (3.51) as follows

$$\begin{aligned}
\int_{\mathcal{M}_6} (\rho_L)_{\bar\alpha\Gamma\xi} \frac{\bar\Omega^{\Gamma\xi}_{\bar\epsilon}}{\parallel \Omega \parallel^2} \eta_-^+ \gamma^{\bar\epsilon}\gamma^{\bar\alpha\alpha}\gamma^\beta \eta_- q_{\alpha\beta} &= \int_{\mathcal{M}_6} (\rho_L)_{\bar\alpha\Gamma\xi} \frac{\bar\Omega^{\Gamma\xi}_{\bar\epsilon}}{\parallel \Omega \parallel^2} q_{\alpha\beta} (\tfrac{1}{2} g^{\bar\alpha\alpha} g^{\bar\epsilon\beta} - g^{\bar\alpha\beta} g^{\alpha\bar\epsilon}) \\
&= \tfrac{i}{8} e^{-\frac{1}{2}\tilde K^K + \frac{1}{2}K^{cs}} \int_{\mathcal{M}_6} \rho_L \wedge \mathcal{W}_3 \\
&= \tfrac{i}{8} e^{-\frac{1}{2}\tilde K^K + \frac{1}{2}K^{cs}} \int_{\mathcal{M}_6} \left\langle \rho_L, d\tilde J \right\rangle , \qquad (3.80)
\end{aligned}$$

where in the last step we used the definition of dJ (2.26). Finally the SUSY transformations of the chiral fermions ζ^K are given by

$$\delta\bar\zeta^K g_{KL} = \tfrac{1}{16} e^{\frac{K}{2}} \int_{\mathcal{M}_6} \left\langle \rho_L, (H_3 + idJ) \right\rangle , \qquad (3.81)$$

where the rotation (3.10) is used for the two form J. Finally using the fact that $D_L \Omega = \rho_L$ we arrive at

$$\delta\bar\zeta^K = \tfrac{1}{8} g^{KL} e^{\frac{K}{2}} D_L W \ . \qquad (3.82)$$

Comparing the result (3.82) with (2.40) one finds that the supersymmetry transformations of the chiral fermions ζ^K are the Kähler derivatives of the superpotential $W = \int_{\mathcal{M}_6} \left\langle \Omega, (H_3 + idJ) \right\rangle$ with respect to their supersymmetric partners z^K (up to 1/8 factor).

3.3.3 Dilatino supersymmetry Transformation

By inserting the decomposition of the dilatino (3.15) in (2.7), multiplying by $\hat\epsilon$ and integrating over $\mathcal{M}_6$ one finds

$$\delta\lambda = -\tfrac{1}{8}\sqrt{\tfrac{1}{2}} e^{\frac{K}{2}} \epsilon \int_{\mathcal{M}_6} H_3 \wedge \bar\Omega , \qquad (3.83)$$

where we made the Weyl rescaling (3.30), inserted the definition of Ω_η given in (3.51) and used (C.42). The first term in (2.7) yields a spacetime derivative acting on the dilaton. However, in our analysis we ignored derivatives acting on scalars, therefore, we set $\partial_\mu\phi$ to be equal to zero as well.

The four-dimensional dilatino is a combination of the singlet of $\hat\lambda$ and the chiral fermions ξ^A given in (3.33). This means to compute $D = 4$ SUSY transformation of the dialtino equation (3.83) should be corrected by the results of $\delta\xi^A$

$$\delta\lambda^s = \delta\lambda - i\sqrt{\tfrac{5}{8}} \frac{\tilde{\mathcal{K}}_A}{\tilde{\mathcal{K}}} \delta\xi^A , \qquad (3.84)$$

where $\delta\lambda$ is given in (3.83) and $\delta\xi^A = \sqrt{2}e^{K/2}g^{A\bar{B}}D_{\bar{B}}\bar{W}\epsilon$. With that at hand and using (3.34) one finds

$$\delta\lambda^s = \sqrt{\tfrac{29}{14}}e^{-2D}e^{\frac{K}{2}}\big((3\sqrt{\tfrac{5}{2}}+\tfrac{1}{8}\sqrt{\tfrac{1}{2}})\int_{\mathcal{M}_6}\big\langle\bar{\Omega},H_3\big\rangle - i3\sqrt{\tfrac{5}{2}}\int_{\mathcal{M}_6}\big\langle\bar{\Omega},dJ\big\rangle\big)\epsilon\ . \quad (3.85)$$

From equation (3.85) one can infer that the dilatino SUSY variations are equal, up to factors, to the derivative of the superpotential W given in (3.59) with respect to the dilaton-axion field S

$$\delta\lambda^s \ \propto\ e^{-2D}e^{\frac{K}{2}}\int_{\mathcal{M}_6}\big\langle\bar{\Omega},(H_3-idJ)\big\rangle\ \propto\ e^{\frac{K}{2}}g^{S\bar{S}}D_{\bar{S}}\bar{W} \quad (3.86)$$

3.3.4 Gaugino Supersymmetry transformation

Finally we come to the four-dimensional SUSY variation of the gaugino χ^a. This gives the D-terms of the four-dimensional theory as it can be seen from (2.39).

Inserting the decomposition of the ten-dimensional gaugino (3.14) in (2.8) and integrating over the six-dimensional manifold $\mathcal{M}_6$ one finds

$$\delta\chi^a = \frac{e^{-\frac{\hat{\phi}}{2}}}{\tilde{\mathcal{K}}}\big(F^a_{\mu\nu}\,\sigma^{\mu\nu}\ +\ \tfrac{i}{2}\int_{\mathcal{M}_6}F^a\wedge *_6\tilde{J}\big)\epsilon\ , \quad (3.87)$$

where (2.23) was used. Performing the rescaling (3.30), (3.34) and comparing with (2.39) one can infer that the resulting D-term is

$$\mathcal{D}^a\ =\ -\tfrac{1}{\sqrt{2}}\,e^{-\frac{\hat{\phi}}{4}}\,\mathcal{K}^{-1}\int_{\mathcal{M}_6}F^a\wedge *_6 J. \quad (3.88)$$

3.4 Supersymmetry conditions for the vacuum

In this section we will discuss the conditions which lead to a supersymmetric background in a flux compactification. In the case of the heterotic string Strominger has shown in [40] that for a supersymmetric vacuum the background must allow for a non vanishing torsion (for a review see appendix D). Moreover the internal manifold has to be complex and the fundamental two-form $J_{\alpha\bar{\beta}}$, the Yang-Mills field strength and the three form flux H_3 have to satisfy the following conditions[3]

$$J^{\alpha\bar{\beta}}F_{\alpha\bar{\beta}}=0\ , \qquad\qquad H_3 = i(\bar{\partial}-\partial)J\ . \quad (3.89)$$

[3]In the analysis of Strominger the condition of the flux includes a factor $\frac{1}{2}$. However we absorb this factor in the definition of J (2.23) and therefore it is absent from (3.89)

Strominger analysis was made on backgrounds of the form $M_{(3,1)} \times \mathcal{M}_6$ which allow for a warp factor Δ. The latter is shown to be equal to the dilaton as a result of the vanishing of the gravitino supersymmetry variation. However in our analysis we do not consider any warping. Hence our assumption that the dialton is constant is consistent with the Strominger result in the limit where Δ is constant.

Generically on a supersymmetric vacuum the supersymmetry transformations of the fermionic fields have to vanish in particular the chiral fermions variations. This amounts to solve the equations

$$\delta\Pi^I = \sqrt{2} e^{\frac{K}{2}} g^{I\bar{J}} D_{\bar{J}} \bar{W} \epsilon = 0 \ , \qquad \delta\chi = 0 \ , \qquad \delta\lambda = 0 \ , \tag{3.90}$$

where we denote the chiral fermions by Π^I and D_I is the Kähler derivative defined in (2.36).

From equation (3.87) one can conclude that a necessary condition such that $\delta\chi = 0$ is, in particular, the vanishing of $F^a_{\alpha\bar{\beta}} J^{\alpha\bar{\beta}} = 0$. Hence Strominger's condition on the Yang-Mills field strength is satisfied.

Setting equation (3.86) to zero implies that $(H_3 + idJ)$ can only be a combination of (3,0)-, (2,1)- and (1,2)- forms in a supersymmetric background. However, H_3 is real which in turn implies that this combination can only be the sum $H_3 + idJ = (H + idJ)_{(2,1)} + (H + idJ)_{(1,2)}$. This condition together with $\delta\zeta^K = 0$, where $\delta\zeta^K$ is given in eqn. (3.82), indicate that on a supersymmetric background one has to impose the following condition

$$(H_3 + idJ) = 0 \ . \tag{3.91}$$

Due to the fact that the two-form J is $(1,1)$-form and the NS-flux is $H_3 = H_{(2,1)} + H_{(1,2)}$, one can write that $H_{(2,1)} = -i\partial J$ and its conjugate for $H_{(1,2)}$ which amounts to

$$H_3 = i(\bar{\partial} - \partial) J \ . \tag{3.92}$$

This is in perfect analogy with the Strominger condition for the flux given in (3.89).

The condition $\delta\xi^A = 0$ is equivalent to the vanishing of the Kähler derivative ($D_A W = 0$) which we recall here

$$\begin{aligned} D_A W &= \tfrac{i}{8} \frac{\mathcal{K}_A}{\mathcal{K}} \int_{\mathcal{M}_6} \Omega \wedge H_3 - \Big(\tfrac{3}{16} \frac{\mathcal{K}_A}{\mathcal{K}} \int_{\mathcal{M}_6} \mathcal{W}_1 \\ &\qquad + \tfrac{i}{8} \int_{\mathcal{M}_6} \mathcal{W}_2 \wedge J \wedge \omega_A + \tfrac{i}{8} \int_{\mathcal{M}_6} \mathcal{W}_1 J^2 \wedge \omega_A \Big) \ . \end{aligned}$$

By setting this derivative to zero one can infer that the first and the second torsion classes are vanishing for a supersymmetric background. However the condition $\mathcal{W}_1 = \mathcal{W}_2 = 0$ is equivalent to saying that the compactification manifold $\mathcal{M}_6$ is complex. Hence we recover the complexity condition of the manifold found in [40].

With this we recover all the Strominger supersymmetry conditions for the heterotic string.

To end this chapter let us summarize what was done so far. Compactification of the heterotic supergravity on $SU(3)$ structure manifolds leads to $N = 1$ four-dimensional supergravity theory. This is characterized then by a superpotential and Kähler potential. We computed these functions from fermionic terms. The comparison with the results found earlier gives us a check on the consistency of our fermionic compactification ansatz.

In this compactification the superpotential W and its derivatives depend on the torsion of the manifold as well as on the NS-flux H_3. However there is no D-term appearing. For a supersymmetric background the torsion of the manifold is an important ingredient in the flux compactification. Torsion and flux then have to satisfy specific conditions. For example the first and the second torsion classes have to vanish simultaneously to insure that the compactification manifold is complex. Moreover the flux can only be a $(2,1)$- and $(1,2)$-forms identified with ∂J and $\bar{\partial} J$. Note here that the two-form J is no longer closed as in the case of Calabi-Yau manifolds. Its non-closure is parameterized by the torsion.

These conditions which are first discussed in [40] are recovered here in the limit where the warp factor is constant. This provides us with a further check on the consistency of our analysis.

In the same spirit we will next discuss the compactifications of the type II theories on $SU(3)$ structure manifolds [60]. These will result in $N = 2$ four-dimensional supergravities. However, our interest here is focused on $D = 4$ effective theories with $N = 1$ supersymmetry therefore we will impose an orientifold projections which insure the reduction of supersymmetry from $N = 2$ to $N = 1$.

Chapter 4

Type II $SU(3)$ structure orientifolds

In this chapter we present a study based on [60] of type IIA and type IIB compactifications on $SU(3)$ structure manifolds. As reviewed in section 2.2.2 these compactifications lead to four-dimensional theories with $N = 2$ supersymmetry. The inclusion of D-branes and orientifold planes further reduces the amount of supersymmetry. In order for the four-dimensional effective theories to possess $N = 1$ supersymmetry the D-branes and orientifold planes cannot be chosen arbitrarily but rather have to fulfill certain supersymmetry conditions called the BPS conditions. We show in section 4.2 that the truncated spectrum arranges indeed into $N = 1$ supermultiplets and we compute the superpotentials from fermionic terms in section 4.4.

First let us specify the orientifold projections which yield supersymmetric orientifold planes preserving half of the $N = 2$ supersymmetry.

4.1 Orientifold projection

Here we discuss orientifold projections in the type IIA and type IIB cases in turn. These projections are basically the action of the orientation reversal Ω_p of the string world-sheet together with the action of an internal symmetry σ which acts solely on $\mathcal{M}_6$ but leaves the $D = 4$ space-time untouched. We will restrict ourselves to involutive symmetries ($\sigma^2 = 1$) of $\mathcal{M}_6$.

4.1.1 Type IIA orientifold projection

The orientifold projection for type IIA $SU(3)$ structure orientifolds can be obtained in close analogy to the Calabi-Yau case [63]. Recall that for Calabi-Yau orientifolds the demand for $N = 1$ supersymmetry implies that σ has to be an anti-holomorphic and isometric involution [91, 92, 93]. This fixes the action of σ on the Kähler form J as $\sigma^* J = -J$, where σ^* denotes the pull-back of the map σ. Furthermore, supersymmetry implies that σ acts non-trivially on the holomorphic three-form Ω. This naturally generalizes to the $SU(3)$ structure case, since we can still assign a definite action of σ on the globally defined two-form J and three-form Ω defined in (2.23). Together the orientifold constraints read

$$\sigma^* J \;=\; -J \;, \qquad\qquad \sigma^*\Omega_\eta \;=\; e^{2i\theta}\bar{\Omega}_\eta \;, \tag{4.1}$$

where $e^{2i\theta}$ is a phase and we included a factor 2 for later convenience. Note that the second condition in (4.1) can be directly inferred from the compatibility of σ with the $SU(3)$ structure condition $\Omega_\eta \wedge \bar{\Omega}_\eta \propto J \wedge J \wedge J$ given in (2.24). In order that σ is a symmetry of the Einstein-Hilbert term of type IIA supergravity it is demanded to be an isometry. Hence, the first condition in (4.1) implies that σ yields a minus sign when applied to the almost complex structure $I_n^m = J_{np}g^{pm}$ introduced in section 2.2.2. This property is equivalent to the anti-holomorphicity of σ if I_n^m is integrable as in the Calabi-Yau case. In accord with condition (4.1) one has

$$\sigma^*\eta_+ = -e^{i\theta}\eta_- \;, \qquad\qquad \sigma^*\eta_- = e^{-i\theta}\eta_+ \;, \tag{4.2}$$

where θ is the phase introduced in eqn. (4.1).

The complete orientifold projection takes the form [1]

$$\mathcal{O} = (-1)^{F_L}\Omega_p\sigma \;, \tag{4.3}$$

where Ω_p is the world-sheet parity and F_L is the space-time fermion number in the left-moving sector.

The orientifold planes arise as the fix-points of σ. Just as in the Calabi-Yau case supersymmetric $SU(3)$ structure orientifolds generically contain $O6$ planes. This is due to the fact, that the fixed point set of σ in $\mathcal{M}_6$ are three-cycles Λ_{O6} supporting the internal part of the orientifold planes. These are calibrated with respect to the real form $\mathrm{Re}(e^{-i\theta}\Omega)$ such that

$$\mathrm{vol}(\Lambda_{\mathrm{O6}}) \propto \mathrm{Re}(\mathrm{e}^{-i\theta}\Omega) \;, \qquad \mathrm{Im}(\mathrm{e}^{-\mathrm{i}\theta}\Omega)|_{\Lambda_{\mathrm{O6}}} = J|_{\Lambda_{\mathrm{O6}}} = 0 \;, \tag{4.4}$$

[1] The factor $(-1)^{F_L}$ is included in $\mathcal{O}$ to ensure that $\mathcal{O}^2 = 1$ on all states.

where vol(Λ_{O6}) is the induced volume form on Λ_{O6} and the overall normalization of Ω was left undetermined. The conditions (4.4) also allow us to give a more explicit expression for the phase $e^{i\theta}$ as

$$e^{-2i\theta} = \bar{Z}(\Lambda_{O6})/Z(\Lambda_{O6}) \ , \tag{4.5}$$

where $Z(\Lambda_{O6})$ is given by $Z(\Lambda_{O6}) = \int_{\Lambda_{O6}} \Omega$. This expression determines the transformation behavior of θ under complex rescaling of Ω. Later on we include $e^{-i\theta}$ to define a scale invariant three-form $C\Omega$.

4.1.2 Type IIB orientifold projection

Let us now turn to type IIB $SU(3)$ structure orientifolds. Recall that for type IIB Calabi-Yau orientifolds [62] consistency requires σ to be a holomorphic and isometric involution of $\mathcal{M}_6$ [91, 93]. A holomorphic isometry leaves both the metric and the complex structure of the Calabi-Yau manifold invariant, such that $\sigma^* J = J$. We generalize this condition to the $SU(3)$ structure case by demanding that the globally defined two-form J defined in (2.23) transforms as

$$\sigma^* J = J \ . \tag{4.6}$$

Once again the invariance of the effective action implies that σ is an isometry, such that (4.6) translates to the invariance of the almost complex structure I_m^n. Due to this fact the involution respects the (p, q)-decomposition of forms. Hence the $(3, 0)$-form Ω defined in (2.23) will be mapped to a $(3, 0)$ form. Demanding the resulting form to be globally defined we have two possible choices

$$(1) \quad O3/O7: \quad \sigma^*\Omega = -\Omega \ , \qquad\qquad (2) \quad O5/O9: \quad \sigma^*\Omega = +\Omega \ . \tag{4.7}$$

The dimensionality of the orientifold planes is determined by the dimension of the fix-point set of σ [91]. Hence there are two set-ups the one with $O3$ and $O7$ planes and the other with $O5$ and $O9$ planes.

Equations (4.7) and (4.6) specify the transformation behavior of the $SU(3)$ structure spinor η [52]

$$(1) \quad O3/O7: \quad \sigma^*\eta_+ = i\eta_+ \ , \qquad\qquad \sigma^*\eta_- = -i\eta_- \ , \tag{4.8}$$

$$(2) \quad O5/O9: \quad \sigma^*\eta_+ = \eta_+ \ , \qquad\qquad \sigma^*\eta_- = \eta_- \ . \tag{4.9}$$

The dimensionality of the orientifold planes is determined by the dimension of the fix-point set of σ [91].

Correspondingly, depending on the transformation properties of Ω two different symmetry operations are possible [91, 93, 94, 96]

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\,\sigma\ , \qquad\qquad \mathcal{O}_{(2)} = \Omega_p\,\sigma\ , \tag{4.10}$$

where Ω_p is the world-sheet parity and F_L is the space-time fermion number in the left-moving sector. The type IIB analog of the calibration condition (4.4) involves a contribution from the NS-NS two-form $\hat{B}_2$. It states that the even cycles of the orientifold planes in $\mathcal{M}_6$ are calibrated with respect to the real or imaginary parts of $e^{-\hat{B}_2+iJ}$. The explicit form of this condition can be found, for example, in refs. [97, 98, 99].

4.2 Orientifold spectrum

The orientifold projections defined in (4.3) and (4.10) of the type IIA and type IIB truncate the $N=2$ spectrum to an $N=1$. Here we examine the invariant spectrum after the orientifolding. To do that let us first give the decomposition of the ten-dimensional fields of type II theories under $SU(3)$ representations. These are displayed in tables 3.1, 4.1, 4.2 and 4.3 [68] where we use the same notation as in the heterotic $SU(3)$ decomposition. We list only the decompositions of the $\hat{C}_1$ and $\hat{C}_3$ in type IIA and $\hat{C}_0, \hat{C}_2, \hat{C}_4$ in type IIB. The higher forms are related to these fields via Hodge duality of their field strengths. The form $\hat{C}_4$ has a self-dual field strength and hence only half of its components are physical.

$\hat{C}_1$	C_μ	$\mathbf{1_1}$
	C_m	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}$
$\hat{C}_3$	$C_{\mu\nu p}$	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{T}$
	$C_{\mu np}$	$\mathbf{1_1}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}+\mathbf{8_1}$
	C_{mnp}	$(\mathbf{1}+\mathbf{1})_\mathbf{0}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}+(\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}$

Table 4.1: *Type IIA decomposition of the RR sector in $SU(3)$ representations*

The fields arising in this decomposition can be arranged into one $N=8$ gravitational multiplet. However the truncation of all the triplets from the spectrum

$\hat{C}_0$	C_0	$\mathbf{1_0}$
$\hat{C}_2$	$C_{\mu\nu}$	$\mathbf{1_T}$
	$C_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{1}}$
	C_{mn}	$\mathbf{1_0}+(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{0}}+\mathbf{8_0}$
$\hat{C}_4$	$C_{\mu npq}$	$\frac{1}{2}\,[(\mathbf{1}+\mathbf{1})_{\mathbf{1}}+(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{1}}+(\mathbf{6}+\bar{\mathbf{6}})_{\mathbf{1}}]$
	$C_{mnpq}/C_{\mu\nu mn}$	$\mathbf{1_0}+(\mathbf{3}+\bar{\mathbf{3}})_{\mathbf{0}}+\mathbf{8_0}$

Table 4.2: *Type IIB decomposition of the RR sector in $SU(3)$ representations*

$\hat{\psi}^{1,2}_M$	$\hat{\psi}^{1,2}_\mu$	$\mathbf{1_{3/2}}+\mathbf{3_{3/2}}$
	$\hat{\psi}^{1,2}_m$	$\mathbf{1_{1/2}}+\mathbf{3_{1/2}}+2\,\bar{\mathbf{3}}_{\mathbf{1/2}}+\mathbf{6_{1/2}}+\mathbf{8_{1/2}}$
$\hat{\lambda}^{1,2}$	$\lambda^{1,2}$	$\mathbf{1_{1/2}}+\mathbf{3_{1/2}}$

Table 4.3: *Type II decomposition of the NS-R sector in $SU(3)$ representations*

results into keeping fields organized in the standard $N=2$ supermultiplets. These are the gravity, vector, hyper and tensor multiplets.

In a second step we impose the orientifold projection to further reduce to an $N=1$ supergravity theory. Independent of the properties of the internal manifold we can give the transformation behavior of all supergravity fields under the world-sheet parity Ω_p and $(-1)^{F_L}$ [6, 39]. Ω_p acts on $\hat{B}_2$ with a minus sign, while leaving the dilaton $\hat{\phi}$ and the ten-dimensional metric $\hat{G}$ invariant. On the R-R fields it is minus the parity operator λ defined in (2.12)

$$\Omega_p \hat{C}_k = -\lambda(\hat{C}_k)\ , \tag{4.11}$$

where k is odd for type IIA and even for type IIB and λ is defined in (2.12). The action $(-1)^{F_L}$ on the R-R bosonic fields of the supergravity theories yields a minus sign while leaves the NS-NS fields invariant. Finally, the world-sheet parity Ω_p acts on the NS-R and R-NS sectors by exchanging $\hat{\psi}^1_M$, $\hat{\lambda}^1$ and $\hat{\psi}^2_M$, λ^2. If the orientifold projection contains the operator $(-1)^{F_L}$ one finds an additional minus sign when applied to $\hat{\psi}^2_M$ and $\hat{\lambda}^2$. In this we asserted that $\hat{\psi}^1_M$ and $\hat{\lambda}^1$ are in the NS-R sector while $\hat{\psi}^2_M$ and $\hat{\lambda}^2$ are in the R-NS sector.

4.2.1 Type IIA orientifold spectrum

Bosonic spectrum

To determine the invariant spectrum for type IIA orientifolds it is convenient to combine the odd R-R forms $\hat{C}_{2n+1}$ as [66]

$$\hat{C}^{\text{odd}} = \hat{C}_1 + \hat{C}_3 + \hat{C}_5 + \hat{C}_7 + \hat{C}_9 \ . \tag{4.12}$$

Note that only half of the degrees of freedom in $\hat{C}^{\text{odd}}$ are physical, while the other half can be eliminated by a duality constraint [66]. Invariance under the orientifold projection $\mathcal{O}$ and the transformation of the fields under Ω_p and $(-1)^{F_L}$ imply that the ten-dimensional fields have to transform under σ as

$$\sigma^* \hat{B}_2 = -\hat{B}_2 \ , \qquad \sigma^* \hat{\phi} = \hat{\phi} \ , \qquad \sigma^* \hat{C}^{\text{odd}} = \lambda\big(\hat{C}^{\text{odd}}\big) \ , \tag{4.13}$$

where the parity operator λ is defined in (2.12) and we used (4.11). It turns out to be convenient as well to combine the forms Ω and J with the ten-dimensional dilaton $\hat{\phi}$ and $\hat{B}_2$ into new forms $\Pi^{\text{ev/odd}}$ as

$$\Pi^{\text{ev}} = e^{-\hat{B}_2 + iJ} \ , \qquad \Pi^{\text{odd}} = C\Omega \ , \tag{4.14}$$

where

$$C = e^{-\hat{\phi} - i\theta} e^{(K^{cs} - K^{\text{K}})/2} \ , \tag{4.15}$$

and K^{cs}, K^{K} are defined in (3.23). C depends on the ten-dimensional dilaton $\hat{\phi}$ and fixes the normalization of Ω such that the combination $C\Omega$ stays invariant under complex rescaling of Ω. This is due to the fact that θ depends on the three-form Ω as given in (4.5) such that C rescales with a factor e^{-f} if $\Omega \to e^{f}\Omega$ for every complex function f. Applied to the forms $\Pi^{\text{ev/odd}}$ and $\hat{C}^{\text{odd}}$ the orientifold conditions (4.1) and (4.13) are expressed as

$$\sigma^* \Pi^{\text{ev}} = \lambda\big(\Pi^{\text{ev}}\big) \ , \qquad \sigma^* \Pi^{\text{odd}} = \lambda\big(\bar{\Pi}^{\text{odd}}\big) \ . \tag{4.16}$$

In order to perform the Kaluza-Klein reduction it is necessary to study the action of the orientifold projection on the space of forms. The operator $\mathcal{P}_6 = \lambda\sigma^*$ squares to the identity and thus splits the space of two- and three-forms $\Lambda^2 T^*$ and $\Lambda^3 T^*$ on $\mathcal{M}_6$ into two eigenspaces as

$$\Lambda^2 T^* = \Lambda^2_+ T^* \oplus \Lambda^2_- T^* \ , \qquad \Lambda^3 T^* = \Lambda^3_+ T^* \oplus \Lambda^3_- T^* \ , \tag{4.17}$$

where $\Lambda^n_\pm T^*$ contains forms transforming with a $\pm$ sign under $\mathcal{P}_6$.

In performing the Kaluza-Klein reduction one expands the forms $\Pi^{\mathrm{ev/odd}}$ and $\hat{C}^{\mathrm{odd}}$ into the appropriate subset of $\Lambda^2 T^*$ and $\Lambda^3 T^*$ consistent with the orientifold projection. The coefficients arising in these expansions correspond to the fields of the four-dimensional theory. In the case at hand the compactification has to result in an $N = 1$ supergravity theory. The spectrum of this theory consists of a gravity multiplet a number of chiral multiplets and vector multiplets. Note that, as in the heterotic compactification, before the truncation to the light modes the number of multiplets is not finite, as the Kaluza-Klein tower consist of an infinite number of modes.

Let us first concentrate on the $N = 1$ chiral multiplets arising in the expansion of the forms Π^{ev}. The complex scalar fields in this multiplet span a Kähler manifold. Its complex chiral coordinates are determined upon expansion of J_c into appropriate modes of the internal manifold

$$J_c \equiv -\hat{B}_2 + iJ \qquad \in \quad \Lambda^2_+ T^*_{\mathbb{C}} \ . \tag{4.18}$$

The B-field can only be extended along the internal manifold $\mathcal{M}_6$, since due to (4.13) the four-dimensional two-form in $\hat{B}_2$ transforms with a negative sign under the orientifold symmetry σ^* and hence is projected out. In comparison to the general $SU(3)$ decomposition of $\hat{B}_2$ given in table 3.1 we only kept the $\mathbf{1_0} + \mathbf{8_0}$ representations while all other components left the spectrum. The complex form J_c is expanded in real elements of $\Lambda^2_+ T^*$ consistent with the orientifold projection (4.1), (4.13) and the definition of λ given in (2.12).[2] The coefficients of this expansion are complex scalar fields in four space-time dimensions parameterizing a manifold $\mathcal{M}^{\mathrm{K}}$ and provide the bosonic components of chiral multiplets.

Turning to the expansion of the R-R forms $\hat{C}^{\mathrm{odd}}$ we first note that $\hat{C}_1$ (and hence $\hat{C}_7$) are completely projected out from the spectrum. The four-dimensional part of $\hat{C}_1$ is incompatible with the orientifold symmetry as seen in (4.13). On the other hand the internal part of $\hat{C}_1$ is a triplet under $SU(3)$ and hence discarded following the assumptions made above. In contrast the expansion of $\hat{C}_3$ yields four-dimensional scalars, vectors and three-forms. Therefore, we decompose

$$\hat{C}_3 = C_3^{(0)} + C_3^{(1)} + C_3^{(3)} \ , \tag{4.19}$$

where $C_3^{(n)}$ are n-forms in $M_{3,1}$ times $(3-n)$-forms in $\mathcal{M}_6$. More precisely, in order to fulfill the orientifold condition (4.13) the components $C_3^{(0)}$, $C_3^{(1)}$ and $C_3^{(3)}$ are expanded in forms $\Lambda^3_+ T^*$, $\Lambda^2_- T^*$ and $\Lambda^0 T^*$ of $\mathcal{M}_6$ respectively. The coefficients

[2] Note that the eigenspaces $\Lambda^2_\pm T^*$ are obtained from the operator $\mathcal{P}_6 = \lambda\sigma^*$ and hence differ by a minus sign from the eigenspaces of σ^*.

in this expansion correspond to four-dimensional real scalars, vectors and three-forms. In summary the components kept, are the $\mathbf{1_1} + \mathbf{8_1}$ and $(\mathbf{1}+\mathbf{1})_\mathbf{0} + (\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}$ while all other representations in table 4.1 have left the spectrum.

The four-dimensional real scalars in $C_3^{(0)}$ need to combine with scalars arising in the expansion of $\Pi^{\rm odd}$ to form the components of chiral multiplets. The complex structure on the corresponding Kähler field space is defined through the complex form

$$\Pi_c^{\rm odd} \equiv C_3^{(0)} + i\mathrm{Re}(\Pi^{\rm odd}) \qquad \in \quad \Lambda^3_+ T^*_{\mathbb{C}} \ , \tag{4.20}$$

where we used that $\mathrm{Re}(\Pi^{\rm odd})$ transforms with a plus sign as seen from eqn. (4.16). The complex coefficients of $\Pi_c^{\rm odd}$ expanded in real forms $\Lambda^3_+ T^*$ are the bosonic components of chiral multiplets. Note that in the massless case theses chiral multiplets can be dualized to linear multiplets containing a scalar from $\mathrm{Re}(\Pi^{\rm odd})$ and a two-form dual to the scalar in $C_3^{(0)}$ [95]. The full bosonic $N=1$ spectrum for type IIA orientifold is summarized in table 4.4.

multiplet	bosonic fields	$\mathcal{M}_6$-forms
gravity multiplet	$g_{\mu\nu}$	
chiral multiplets	J_c	$\Lambda^2_+ T^*$
chiral/linear multiplets	$\Pi_c^{\rm odd}$	$\Lambda^3_+ T^*$
vector multiplets	$C_3^{(1)}$	$\Lambda^2_- T^*$

Table 4.4: $N=1$ *spectrum of type IIA orientifolds*

Fermionic spectrum

In the fermionic sector we keep only the singlet in $\hat{\psi}^{1,2}_\mu$, $\hat{\lambda}^{1,2}$ and the fields transforming as $\mathbf{1}$, $\mathbf{6}$, $\mathbf{8}$ in $\hat{\psi}^{1,2}_m$. In the following we discuss the reduction of these fields.

The two gravitinos in this case have different chiralities and hence decompose as

$$\hat{\psi}^1_\mu = \psi^1_\mu \otimes \eta_+ + \bar{\psi}^1_\mu \otimes \eta_- \ , \qquad\qquad \hat{\psi}^2_\mu = \psi^2_\mu \otimes \eta_- + \bar{\psi}^2_\mu \otimes \eta_+ \ , \tag{4.21}$$

where $\psi^{1,2}_\mu$ are Weyl spinors with positive chiralities and $\bar{\psi}^{1,2}_\mu$ are Weyl spinors with negative chiralities.

The orientifold projection $\mathcal{O}$ given in eqns. (4.3) reduces the four-dimensional theory to an $N = 1$ supergravity. Hence, the two four-dimensional gravitinos ψ_μ^1, ψ_μ^2 are not independent, but rather combine into one four-dimensional spinor ψ_μ which parameterizes the $N = 1$ supersymmetry. This spinor is chosen in such a way that its ten-dimensional extension $\hat{\psi}_\mu$ is invariant under the projection $\mathcal{O}$ given in (4.3). To investigate that in more detail let us recall the transformations behavior of ten-dimensional spinors under Ω_p and $(-1)^{F_L}$

$$\Omega_p\ \hat{\psi}_M^1 = \hat{\psi}_M^2 \ , \qquad \Omega_p\ \hat{\psi}_M^1 = \hat{\psi}_M^2 \ , \tag{4.22}$$

$$(-1)^{F_L}\hat{\psi}_M^1 = \hat{\psi}_M^1 \ , \qquad (-1)^{F_L}\hat{\psi}_M^2 = -\hat{\psi}_M^2 \ . \tag{4.23}$$

The geometric symmetry σ acts only on the internal space $\mathcal{M}_6$ which translates to a non-trivial transformation of the globally defined spinor η. The precise action of σ^* is given in eqn. (4.2) for type IIA orientifolds. With that at hand one can specify the action of orientifold projection $\mathcal{O} = (-1)^{F_L}\Omega_p\sigma^*$ on $\hat{\psi}_\mu^1$ and $\hat{\psi}_\mu^2$

$$\mathcal{O}\hat{\psi}_\mu^1 = -\sigma^*(\psi_\mu^2 \otimes \eta_- + \bar{\psi}_\mu^2 \otimes \eta_+) = -e^{-i\theta}\psi_\mu^2 \otimes \eta_+ + e^{i\theta}\bar{\psi}_\mu^2 \otimes \eta_- \ , \tag{4.24}$$

$$\mathcal{O}\hat{\psi}_\mu^2 = \sigma^*(\psi_\mu^1 \otimes \eta_+ + \bar{\psi}_\mu^1 \otimes \eta_-) = -e^{i\theta}\psi_\mu^1 \otimes \eta_- + e^{-i\theta}\bar{\psi}_\mu^1 \otimes \eta_+ \ , \tag{4.25}$$

this amounts to write

$$\mathcal{O}\psi_\mu^1 = -e^{-i\theta}\psi_\mu^2, \quad \mathcal{O}\bar{\psi}_\mu^1 = e^{i\theta}\bar{\psi}_\mu^2, \quad \mathcal{O}\psi_\mu^2 = -e^{i\theta}\psi_\mu^1, \quad \mathcal{O}\bar{\psi}_\mu^2 = e^{-i\theta}\bar{\psi}_\mu^1 \ . \tag{4.26}$$

Therefore, the invariant combination of the four-dimensional spinors is given by $\psi_\mu = \frac{1}{2}(e^{i\theta/2}\,\psi_\mu^1 - e^{-i\theta/2}\,\psi_\mu^2)$.

With a similar logic we discuss the reduction of the singlet parts in the ten-dimensional dilatinos. In type IIA they have opposite chiralities and decompose as

$$\hat{\lambda}^1 = \lambda^1 \otimes \eta_+ + \bar{\lambda}^1 \otimes \eta_- \ , \qquad \hat{\lambda}^2 = \lambda^2 \otimes \eta_- + \bar{\lambda}^2 \otimes \eta_+ \ , \tag{4.27}$$

where $\lambda^{1,2}$, $\bar{\lambda}^{1,2}$ are Weyl spinors corresponding to the $N = 2$ four-dimensional dilatinos with positive and negative chiralities respectively. The projection $\mathcal{O}$ outlines a combination of λ^1, λ^2 which survives the orientifolding and give rise to the $N = 1$ four-dimensional dilatino. Following the same strategy given above the combination is found to be $\lambda = \frac{1}{2}(e^{i\theta/2}\,\lambda^1 - e^{-i\theta/2}\,\lambda^2)$.

The reduction of $\hat{\psi}_m$ gives rise to the fermions in the chiral and vector multiplets. The precise decomposition will play no role in what follows.

Finite basis

The analysis so far was not restricted to a finite set of fields, however, to give explicitly the Kaluza-Klein reduction we will specify a finite basis leading to a finite number of fields in $D=4$. A finite reduction is achieved by selecting a finite basis of forms Δ_{finite} on the $SU(3)$ structure manifold defined in section 3.1. The orientifold projection $\mathcal{O}$ selects a subset in Δ_{finite} on which the ten-dimensional fields are expanded. To see that let us first understand how the orientifold symmetry acts on Δ_{finite}. Under the operator $\mathcal{P}_6=\lambda\sigma^*$ the forms Δ^n decompose into eigenspaces as

$$\Delta^n = \Delta^n_+ \oplus \Delta^n_- \ . \tag{4.28}$$

Using the properties (4.1) and (2.12) one infers $\dim\Delta^0_- = \dim\Delta^6_- = 0$. Furthermore, under the split (4.28) the basis $(\omega_{\hat{A}}, \tilde{\omega}^{\hat{A}})$ introduced in (3.6) decomposes as

$$(\omega_{\hat{A}}, \tilde{\omega}^{\hat{A}}) \quad \rightarrow \quad (1, \omega_a, \tilde{\omega}^\alpha, \epsilon) \in \Delta^{\text{ev}}_+ \ , \quad (\omega_\alpha, \tilde{\omega}^a) \in \Delta^{\text{ev}}_- \ , \tag{4.29}$$

where $\alpha = 1,\ldots,\dim\Delta^2_-$ while $a = 1,\ldots,\dim\Delta^2_+$. Using the intersections (3.6) one infers that $\dim\Delta^2_\pm = \dim\Delta^4_\mp$. Turning to the odd forms consistency requires that

$$\int_{\mathcal{M}_6} \langle \Delta^3_\pm, \Delta^3_\pm \rangle = 0 \ , \qquad *\Delta^3_\pm = \Delta^3_\mp \ , \tag{4.30}$$

where in the second equality we used the fact that σ is an orientation-reversing isometry. The first condition is a consequence of the fact that $\Delta^3_\pm \wedge \Delta^3_\pm$ transforms with a minus sign under $\mathcal{P}_6$ and hence is a subset of Δ^6_- up to an exact form. The equations (4.30) imply that $\Delta^3_\pm$ are Lagrangian subspaces of Δ^3 with respect to the integrated Mukai parings. Hence, also the symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ introduced in (3.7) splits as

$$(\alpha_{\hat{K}}, \beta^{\hat{K}}) \quad \rightarrow \quad (\alpha_k, \beta^\lambda) \in \Delta^3_+ \ , \qquad (\alpha_\lambda, \beta^k) \in \Delta^3_- \ , \tag{4.31}$$

where the numbers of α_k and β^λ in Δ^3_+ equal to the numbers of β^k and α_λ in Δ^3_- respectively. This is in accord with equation (3.7).

We are now in the position to give an explicit expansion of the fields into the finite form basis of Δ_{finite}. As discussed in the general case above the four-dimensional complex chiral fields arise in the expansion of the forms J_c and Π^{odd}_c introduced in eqn. (4.18) and (4.20). Restricted to Δ^2_+, Δ^3_+ and Δ^2_- one has

$$J_c = t^a \omega_a \ , \qquad \Pi^{\text{odd}}_c = N^k \alpha_k + T_\lambda \beta^\lambda \ , \qquad C^{(1)}_3 = A^\alpha \omega_\alpha \ , \tag{4.32}$$

where the basis decompositions (4.29) and (4.31) were used. Hence, in the finite reduction the $N = 1$ spectrum consists of $\dim \Delta_+^2$ chiral multiplets t^a and $\frac{1}{2}\dim \Delta^3$ chiral multiplets N^k, T_λ. In addition one finds $\dim \Delta_-^2$ vector multiplets, which arise in the expansion of $\hat{C}_3$. Moreover, one four-dimensional massless three-form arises in the expansion of $C_3^{(3)}$ in the form $1 \in \Delta_+^0$. It carries no degrees of freedom and corresponds to an additional flux parameter.

4.2.2 Type IIB orientifold spectrum

Bosonic spectrum

Let us next turn to the spectrum of type IIB $SU(3)$ structure orientifolds. To identify the invariant spectrum we first analyze the transformation properties of the ten-dimensional fields. In contrast to type IIA supergravity type IIB theory consists of even forms $\hat{C}_{2n}$ in the R-R sector, which we conveniently combine as [66]

$$\hat{C}^{\text{ev}} = \hat{C}_0 + \hat{C}_2 + \hat{C}_4 + \hat{C}_6 + \hat{C}_8 \ . \tag{4.33}$$

Only half of the degrees of freedom in $\hat{C}^{\text{ev}}$ are physical and related to the second half by a duality constraint [66]. Using the transformation properties of the fields under Ω_p and $(-1)^{F_L}$ the invariance under the orientifold projections $\mathcal{O}_{(i)}$ implies that the ten-dimensional fields have to transform as [3]

$$\sigma^* \hat{B}_2 = -\hat{B}_2 \ , \qquad \sigma^* \hat{\phi} = \hat{\phi} \ , \qquad \sigma^* \hat{C}^{\text{ev}} = \pm\lambda\big(\hat{C}^{\text{ev}}\big) \ , \tag{4.34}$$

where the plus sign in the last equation holds for orientifolds with $O3/O7$ planes, while the minus sign holds for $O5/O9$ orientifolds. The parity operator λ was introduced in eqn. (2.12). We combine the globally defined forms J and Ω with the fields $\hat{B}_2$, $\hat{\phi}$ and $\hat{C}^{\text{ev}}$ as

$$\Phi^{\text{odd}} = \Omega \ , \qquad \Phi^{\text{ev}} = e^{-\hat{\phi}} e^{-\hat{B}_2 + iJ} \ , \qquad \hat{A}^{\text{ev}} = e^{-\hat{B}_2} \wedge \hat{C}^{\text{ev}} \ . \tag{4.35}$$

where in comparison to (4.14) one finds that Φ^{odd} takes the role of Π^{ev} and Φ^{ev} replaces Π^{odd}. Applied to these forms the orientifold conditions (4.6), (4.7) and (4.34) read

$$\sigma^* \Phi^{\text{odd}} = \mp\lambda(\Phi^{\text{odd}}) \ , \qquad \sigma^* \Phi^{\text{ev}} = \lambda(\bar{\Phi}^{\text{ev}}) \ , \qquad \sigma^* \hat{A}^{\text{ev}} = \pm\lambda(\hat{A}^{\text{ev}}) \ , \tag{4.36}$$

[3] The transformation behavior of the R-R forms under the world-sheet parity operator Ω_p was given in eqn. (4.11).

where the upper sign corresponds to $O3/O7$ and the lower sign to $O5/O9$ orientifolds.

In a next step we have to specify the basis of forms on $\mathcal{M}_6$ used in the Kaluza-Klein reduction. We first briefly discuss the general case and later simplify the reduction to the finite set of forms Δ_{finite}. The decomposition of ten-dimensional fields into $SU(3)$ representations is given in tables 3.1 and 4.2. Also in type IIB case we will remove all triplets of $SU(3)$ from the spectrum [68].

In order to perform the reduction we first investigate the splitting of the spaces of forms on $\mathcal{M}_6$ under the operator $\mathcal{P}_6 = \lambda\sigma^*$. Since $\mathcal{P}_6$ squares to the identity operator it splits the forms as in eqn. (4.17). The decomposition of all even forms reads

$$\Lambda^{\text{ev}}T^* = \Lambda^{\text{ev}}_+ T^* \oplus \Lambda^{\text{ev}}_- T^* \ . \tag{4.37}$$

The four-dimensional fields arising as the coefficients of $\Phi^{\text{ev/odd}}$ and $\hat{A}^{\text{ev}}$ expanded in $\Lambda^3_\pm T^*$ and $\Lambda^{\text{ev}}_\pm T^*$ fit into $N=1$ supermultiplets.

Firstly, we decompose the odd form Φ^{odd} into the eigenspaces of $\mathcal{P}_6$. In accord with the orientifold constraint (4.36) we find

$$O3/O7: \quad \Phi^{\text{odd}} \quad \in \quad \Lambda^3_- T^*_{\mathbb{C}} \ , \qquad\qquad O5/O9: \quad \Phi^{\text{odd}} \quad \in \quad \Lambda^3_+ T^*_{\mathbb{C}} \ . \tag{4.38}$$

In the reduction also the ten-dimensional form $\hat{A}^{\text{ev}}$ is expanded in a basis of forms on $\mathcal{M}_6$ while additionally satisfying the orientifold condition (4.36). In analogy to (4.19) we decompose

$$\hat{A}^{\text{ev}} = A^{\text{ev}}_{(0)} + A^{\text{ev}}_{(1)} + A^{\text{ev}}_{(2)} + A^{\text{ev}}_{(3)} \ , \tag{4.39}$$

where the subscript (n) indicates the form degree in four dimensions. Note that in a general expansion of $\hat{A}^{\text{ev}}$ in forms of $\mathcal{M}_6$ it would be impossible to assign a four-dimensional form degree as done in eqn. (4.39). This is due to the fact that such a decomposition only allows to distinguish even and odd forms in four dimensions. However, the orientifold imposes the constraint (4.36) which introduces an additional splitting within the even and odd four-dimensional forms. Let us first make this more precise in the case of $O3/O7$ orientifolds where $\hat{A}^{\text{ev}}$ transforms as $\sigma^*\hat{A}^{\text{ev}} = \lambda(\hat{A}^{\text{ev}})$. Using the properties of the parity operator λ one finds that the scalars in $A^{\text{ev}}_{(0)}$ arise as coefficients of forms in $\Lambda^{\text{ev}}_+ T^*$ while the two-forms in $A^{\text{ev}}_{(2)}$ arise as coefficients of forms in $\Lambda^{\text{ev}}_- T^*$. Similarly, one obtains the four-dimensional vectors in $A^{\text{ev}}_{(1)}$ as coefficients of $\Lambda^3_+ T^*$ and the three-forms in $A^{\text{ev}}_{(3)}$ as coefficients of $\Lambda^3_- T^*$. In the case of $O5/O9$ orientifolds the ten-dimensional form $\hat{A}^{\text{ev}}$ transforms as $\sigma^*\hat{A}^{\text{ev}} = -\lambda(\hat{A}^{\text{ev}})$ and all signs in the $O3/O7$ expansions above are exchanged.

In both cases the decomposition (4.39) is well defined and we can analyze the multiplet structure of the four-dimensional theory.

The bosonic components of the chiral multiplets are the real scalars in $A^{\rm ev}_{(0)}$ which are complexified by the real scalars arising in the expansion of $\mathrm{Re}(\Phi^{\rm ev})$ or $\mathrm{Im}(\Phi^{\rm ev})$. From the orientifold constraint (4.36) one infers that $\mathrm{Re}(\Phi^{\rm ev})$ is expanded in forms of $\Lambda^{\rm ev}_{+}T^*$ while $\mathrm{Im}(\Phi^{\rm ev})$ is expanded in forms of $\Lambda^{\rm ev}_{-}T^*$. Therefore the complex forms are given as

$$O3/O7:\ \Phi^{\rm ev}_c = A^{\rm ev}_{(0)} + i\mathrm{Re}(\Phi^{\rm ev})\ , \qquad O5/O9:\ \Phi^{\rm ev}_c = A^{\rm ev}_{(0)} + i\mathrm{Im}(\Phi^{\rm ev})\ . \quad (4.40)$$

The complex scalars arising in the expansion of the forms $\Phi^{\rm ev}_c$ span a complex Kähler manifold $\mathcal{M}^{\rm Q}$. The bosonic fields of type IIB compactified on $SU(3)$ structure orientifolds are organized in the four-dimensional $N = 1$ multiplets and are summarized in table 4.5.

multiplet	bosonic fields	$\mathcal{M}_6$-forms	
		$O3/O7$	$O5/O9$
gravity multiplet	$g_{\mu\nu}$		
chiral multiplets	$\Phi^{\rm odd}$	$\Lambda^3_{-}T^*$	$\Lambda^3_{+}T^*$
chiral/linear multiplets	$\Phi^{\rm ev}_c$	$\Lambda^{\rm ev}_{+}T^*$	$\Lambda^{\rm ev}_{-}T^*$
vector multiplets	$A^{\rm ev}_{(1)}$	$\Lambda^3_{+}T^*$	$\Lambda^3_{-}T^*$

Table 4.5: $N = 1$ *spectrum of type IIB orientifolds*

Fermionic spectrum

Let us now turn to the discussion of the reduction of ten-dimensional fermionic fields. We project out all fields transforming as triplets in $\hat{\psi}_\mu,\ \hat{\psi}_m,\ \hat{\lambda}$ and we keep only fields transforming as $\mathbf{1},\ \mathbf{6},\ \mathbf{8}$.

First we examine the decomposition of the singlets in $\hat{\psi}^{1,2}_\mu,\ \hat{\lambda}^{1,2}$. In type IIB both ten-dimensional gravitinos and dilatinos have the same chirality and split as

$$\hat{\psi}^A_\mu = \psi^A_\mu \otimes \eta_- + \bar{\psi}^A_\mu \otimes \eta_+ \qquad A = 1, 2\ , \quad (4.41)$$

$$\hat{\lambda}^A = \lambda^A \otimes \eta_+ + \bar{\lambda}^A \otimes \eta_- \qquad A = 1, 2\ , \quad (4.42)$$

where η denotes the globally defined spinor introduced in eqn. (2.23) with six-dimensional chirality $\pm$. The four-dimensional spinors $\psi_\mu^{1,2}$, $\lambda^{1,2}$ and $\bar\psi_\mu^{1,2}$, $\bar\lambda^{1,2}$ are Weyl spinors with positive and negative chiralities respectively. The spinors $\psi_\mu^{1,2}$ combine into one four-dimensional spinor ψ_μ which parameterizes the $N=1$ supersymmetry. $\lambda^{1,2}$ combine as well into one spin 1/2 field λ. The ten-dimensional extensions of ψ_μ and λ are invariant under the orientifold projections $\mathcal{O}_{(1,2)}$ given in eqn. (4.10). We will discuss both cases in turn

$\underline{O3/O7:}$ The transformation of the ten-dimensional spinor under the world-sheet parity Ω_p and $(-1)^{F_L}$ is the same as in type IIA case and is given in (4.22). The action of σ^* on the internal spinor $\eta_\pm$ is given in (4.8). With these identities at hand one can determine the transformation behavior of ψ_μ^1 and ψ_μ^2

$$\begin{aligned}\mathcal{O}_{(1)}\hat\psi_\mu^1 &= -\sigma^*(\psi_\mu^2\otimes\eta_- + \bar\psi_\mu^2\otimes\eta_+) = +i\psi_\mu^2\otimes\eta_- - i\bar\psi_\mu^2\otimes\eta_+ \ , \qquad (4.43)\\ \mathcal{O}_{(1)}\hat\psi_\mu^2 &= \sigma^*(\psi_\mu^1\otimes\eta_- + \bar\psi_\mu^1\otimes\eta_+) = -i\psi_\mu^1\otimes\eta_- + i\bar\psi_\mu^2\otimes\eta_+ \ , \qquad (4.44)\end{aligned}$$

this amounts to write [52]

$$\mathcal{O}_{(1)}\begin{pmatrix}\psi_\mu^1\\ \psi_\mu^2\end{pmatrix} = -\sigma^2\begin{pmatrix}\psi_\mu^1\\ \psi_\mu^2\end{pmatrix}\ ,\mathcal{O}_{(1)}\begin{pmatrix}\bar\psi_\mu^1\\ \bar\psi_\mu^2\end{pmatrix} = -\bar\sigma^2\begin{pmatrix}\bar\psi_\mu^1\\ \bar\psi_\mu^2\end{pmatrix}\ , \qquad (4.45)$$

where $\bar\sigma^2=-\sigma^2$. The invariant combination of the four-dimensional spinors is given then by $\psi_\mu = \frac{1}{2}(\psi_\mu^1 + i\psi_\mu^2)$. With a similar analysis for the four-dimensional dilatino one finds $\lambda = \frac{1}{2}(\lambda^1 - i\lambda^2)$.

$\underline{O5/O9:}$ From (4.22) and (4.9) the spinors ψ_μ^1 and ψ_μ^2 transform under the orientifold projection $\mathcal{O}_{(2)}$ defined in eqn. (4.10) as follows

$$\mathcal{O}_{(2)}\begin{pmatrix}\psi_\mu^1\\ \psi_\mu^2\end{pmatrix} = \sigma^1\begin{pmatrix}\psi_\mu^1\\ \psi_\mu^2\end{pmatrix}\ ,\mathcal{O}_{(2)}\begin{pmatrix}\bar\psi_\mu^1\\ \bar\psi_\mu^2\end{pmatrix} = \sigma^1\begin{pmatrix}\bar\psi_\mu^1\\ \bar\psi_\mu^2\end{pmatrix}\ , \qquad (4.46)$$

and hence the invariant combination of the four-dimensional spinors is given by $\psi_\mu = \frac{1}{2}(\psi_\mu^1 + \psi_\mu^2)$. Similarly the invariant four-dimensional dilatino is found to be $\lambda = \frac{1}{2}(\lambda^1 + \lambda^2)$.

Finite basis

To end this section let us give a truncation to a finite number of the four-dimensional fields. As we have argued previously this is achieved by expanding the ten-dimensional fields on the finite set of forms on $\mathcal{M}_6$ denoted by Δ_{finite}. This is done in accord with the orientifold constraints for $O3/O7$ and $O5/O9$

orientifolds. Once again, the n-forms Δ^n split as $\Delta^n = \Delta^n_+ \oplus \Delta^n_-$, where $\Delta^n_\pm$ are the eigenspaces of the operator $\mathcal{P}_6 = \lambda\sigma^*$. However, since Δ^6 contains forms proportional to $J \wedge J \wedge J$ one infers from condition (4.6) that $\dim \Delta^6_+ = 0$. Clearly, one has $\dim \Delta^0_- = 0$ since Δ^0 contains constant scalars which are invariant under $\mathcal{P}_6$. A further investigation of the even forms in Δ^2 and Δ^4 shows that the basis introduced in eqn. (3.6) decomposes as

$$(\omega_{\hat{A}}, \tilde{\omega}^{\hat{A}}) \quad \to \quad (1, \omega_a, \tilde{\omega}^\alpha) \in \Delta^{\rm ev}_+ \ , \quad (\epsilon, \omega_\alpha, \tilde{\omega}^a) \in \Delta^{\rm ev}_- \ , \tag{4.47}$$

where $\alpha = 1, \ldots, \dim \Delta^2_-$ and $a = 1, \ldots, \dim \Delta^2_+$. Using $J \wedge J \wedge J \in \Delta^6_-$ and eqn. (3.6) one finds that $\Delta^2_\pm = \Delta^4_\mp$. Together with the fact that $\int \langle \Delta^{\rm ev}_\pm, \Delta^{\rm ev}_\pm \rangle = 0$ one concludes that $\Delta^{\rm ev}_\pm$ are Lagrangian subspaces of $\Delta^{\rm ev}$. This is the analog of the Lagrangian condition (4.30) found for the odd forms in type IIA. Let us turn to the odd forms $\Delta^3 = \Delta^3_+ \oplus \Delta^3_-$. Due to the condition (4.7) the three-form Ω is an element of Δ^3_- for $O3/O7$ orientifolds, while it is an element of Δ^3_+ for $O5/O9$ orientifolds. Note that in contrast to the even forms $\Delta^{\rm ev}_\pm$ the spaces Δ^3_- and Δ^3_+ have generically different dimensions. The basis of three-forms introduced in (3.7) splits under the action of $\mathcal{P}_6$ as

$$(\alpha_{\hat{K}}, \beta^{\hat{K}}) \quad \to \quad (\alpha_\lambda, \beta^\lambda) \in \Delta^3_+ \ , \quad (\alpha_k, \beta^k) \in \Delta^3_- \ , \tag{4.48}$$

where $\lambda = 1, ..., \frac{1}{2}\dim \Delta^3_+,\ k = 1, ..., \frac{1}{2}\Delta^3_-$.

Given the basis decompositions (4.47) and (4.48) we can explicitly determine the finite four-dimensional spectrum of type IIB orientifold theories. For orientifolds with $O3/O7$ planes one expands $\Phi^{\rm ev}_c$ and $A^{\rm ev}_{(1)}$ into $\Delta^{\rm ev}_+$ and Δ^3_+ as

$$\Phi^{\rm ev}_c = \tau + G^a \omega_a + T_\alpha \tilde{\omega}^\alpha \ , \qquad A^{\rm ev}_{(1)} = A^\lambda \alpha_\lambda \ , \tag{4.49}$$

where τ, G^a, T_α are complex scalars in four dimensions. The vector coefficients of the forms α_λ in the expansion of $A^{\rm ev}_{(1)}$ are eliminated by the duality constraint on the field strength of $\hat{A}^{\rm ev}$. In addition we find that $\Phi^{\rm odd}$ depends on $\frac{1}{2}(\dim \Delta^3_- - 2)$ complex deformations z^k. Therefore the full $N = 1$ spectrum consists of $\frac{1}{2}(\dim \Delta^3_- - 2)$ chiral multiplets z^k as well as $\dim \Delta^2 + 1$ chiral multiplets τ, G^a, T_α. Moreover, we find $\frac{1}{2}\dim \Delta^3_+$ vector multiplets A^λ.

The story slightly changes for orientifolds with $O5/O9$ planes. In this case the chiral coordinates are obtained by expanding

$$\Phi^{\rm ev}_c = t^\alpha \omega_\alpha + u_b\, \tilde{\omega}^b + S\, \epsilon \ , \qquad A^{\rm ev}_{(1)} = A^k \alpha_k \ , \tag{4.50}$$

where t^α, u_b, S are complex four-dimensional scalars and the volume form ϵ is normalized as $\int_{\mathcal{M}_6} \epsilon = 1$. Moreover, the form $\Phi^{\rm odd}$ depends on $\frac{1}{2}(\dim \Delta^3_+ - 2)$

complex deformations z^λ. In summary the complete $N=1$ spectrum consists of $\frac{1}{2}(\dim\Delta^3_+ - 2)$ chiral multiplets z^λ as well as $\dim\Delta^2+1$ chiral multiplets t^α, u_b, S. Finally, the expansion of $A^{\text{ev}}_{(1)}$ yields $\frac{1}{2}\dim\Delta^3_-$ independent vector multiplets A^k.

4.3 Kähler potential

In this section we briefly review the results about the Kähler potential encoding the kinetic terms of the chiral multiplets.[4] Recall that, as given in (2.30), the standard bosonic action for chiral multiplets with bosonic components M^I contains the kinetic term [82]

$$S_{\text{chiral}} = \int d^4x\sqrt{-g_4}\, g_{I\bar{J}}\, \partial_\mu M^I \partial^\mu \bar{M}^{\bar{J}} \ , \tag{4.51}$$

where the metric $g_{I\bar{J}} = \partial_{M^I}\partial_{\bar{M}^J} K$ is Kähler and locally given as the second derivative of a real Kähler potential $K(M,\bar{M})$. In other words, the function K determines the dynamics of the system of chiral multiplets. $\mathcal{M}_6$ and can be integrated over the manifold $\mathcal{M}_6$.

4.3.1 Type IIA Kähler potential and the Kähler metric

We found in the previous section that the complex scalars in the chiral multiplets are obtained by expanding the complex forms Π^{ev} and Π^{odd}_c into appropriate forms on $\mathcal{M}_6$. Locally, the field space takes the form

$$\mathcal{M}^{\text{K}} \times \mathcal{M}^{\text{Q}} \ , \tag{4.52}$$

where $\mathcal{M}^{\text{K}}$ and $\mathcal{M}^{\text{Q}}$ are Kähler manifolds spanned by the complex scalars arising in the expansion of Π^{ev} and Π^{odd}_c respectively. The manifold $\mathcal{M}^{\text{K}}$ directly inherits its Kähler structure from the underlying $N=2$ theory. On the other hand, $\mathcal{M}^{\text{Q}}$ is a submanifold of the quaternionic space spanned by the hyper multiplets and has half its dimension. In [60] it is shown that $\mathcal{M}^{\text{Q}}$ is indeed Kähler manifold. The Kähler potentials K^{K} and $\mathcal{M}^{\text{Q}}$ encoding the metrics on $\mathcal{M}^{\text{K}}$ and $\mathcal{M}^{\text{Q}}$ respectively are given as

$$\begin{aligned} K^{\text{K}}(J_c) &= -\ln\big[-i\int_{\mathcal{M}_6}\big\langle\Pi^{\text{ev}},\bar{\Pi}^{\text{ev}}\big\rangle\big] = -\ln\big[\tfrac{1}{6}\int_{\mathcal{M}_6} J\wedge J\wedge J\big] \ , && (4.53)\\ K^{\text{Q}}(\Pi^{\text{odd}}_c) &= -2\ln\big[i\int_{\mathcal{M}_6}\big\langle\Pi^{\text{odd}},\bar{\Pi}^{\text{odd}}\big\rangle\big] = -\ln\big[e^{-4D}\big] \ , && (4.54)\end{aligned}$$

[4] for a detailed study see ref. [60].

where $\Pi^{\rm ev} = e^{J_c}$ and $\Pi^{\rm odd}$ are introduced in (4.14). The functional appearing in the logarithm of the Kähler potentials are known as Hitchin functionals of the real two- and three forms $\mathrm{Re}(\Pi^{\rm ev})$ and $\mathrm{Re}(\Pi^{\rm odd})$

$$H\big[\mathrm{Re}(\Pi^{\rm ev})\big] = -i\int \big\langle \Pi^{\rm ev}, \bar{\Pi}^{\rm ev}\big\rangle \ , \qquad H\big[\mathrm{Re}(\Pi^{\rm odd})\big] = i\int \big\langle \Pi^{\rm odd}, \bar{\Pi}^{\rm odd}\big\rangle \ , \quad (4.55)$$

these were first introduced by Hitchin in ref. [100]. A similar result with odd and even forms exchanged is found for type IIB orientifolds to which we turn now.

4.3.2 Type IIB Kähler potential and the Kähler metric

The complex scalars in the chiral multiplets are obtained by expanding $\Phi^{\rm odd}$ and $\Phi^{\rm ev}_c$ into appropriate forms on $\mathcal{M}_6$ as introduced in eqns. (4.38) and (4.40). These complex scalars locally span the product manifold $\mathcal{M}^{\rm K}\times\mathcal{M}^{\rm Q}$, where $\mathcal{M}^{\rm K}$ contains the independent scalars in $\Phi^{\rm odd}$ while $\mathcal{M}^{\rm Q}$ contains the scalars in $\Phi^{\rm ev}_c$. As in the type IIA orientifolds the complex and Kähler structure of $\mathcal{M}^{\rm K}$ is directly inherited from the underlying $N=2$ theory. Then the Kähler potential is given as

$$K^{\rm K}(z,\bar z) = -\ln\Big[-i\int_{\mathcal{M}_6}\big\langle \Phi^{\rm odd}, \bar\Phi^{\rm odd}\big\rangle\Big] = -\ln\Big[-i\int_{\mathcal{M}_6}\Omega\wedge\bar\Omega\Big] \ , \qquad (4.56)$$

where in the second equality we used the definitions (4.35) and (3.5) of $\Phi^{\rm odd}$ and the pairings $\langle\cdot,\cdot\rangle$.

As discussed in section 4.2 the complex coordinates spanning $\mathcal{M}^{\rm Q}$ are obtained by expanding $\Phi^{\rm ev}_c$ in elements of $\Lambda^{\rm ev}_{\pm}$ depending on whether we are dealing with $O3/O7$ or $O5/O9$ orientifolds. The precise definition of $\Phi^{\rm ev}_c$ was given in eqn. (4.40). The metric on $\mathcal{M}^{\rm Q}$ is the second derivative of the Kähler potential

$$K^{\rm Q}(\Phi^{\rm ev}_c) = -2\ln\Big[i\int_{\mathcal{M}_6}\big\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\big\rangle\Big] = -\ln\big[e^{-4D}\big] \ , \qquad (4.57)$$

where in the second equality we have used the definition of $\Phi^{\rm ev}$ as given in (4.35). Note that $K^{\rm Q}$ is a function of $\mathrm{Im}(\Phi^{\rm ev}_c)$ only, such that it depends on $\mathrm{Re}(\Phi^{\rm ev})$ in $O3/O7$ orientifolds while it depends on $\mathrm{Im}(\Phi^{\rm ev})$ in $O5/O9$ orientifolds. The functionals appearing in the logarithm are the Hitchin functionals [120]

$$H[\mathrm{Re}(\Phi^{\rm ev})] = i\int_{\mathcal{M}_6}\big\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\big\rangle \ , \qquad H[\mathrm{Im}(\Phi^{\rm ev})] = i\int_{\mathcal{M}_6}\big\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\big\rangle \ , \qquad (4.58)$$

depending on whether we are dealing with $O3/O7$ and $O5/O9$ orientifolds.

4.4 Superpotentials of type II $SU(3)$ structure orientifolds

In this section we derive the superpotentials for type IIA and type IIB $SU(3)$ structure orientifolds in presence of fluxes and torsion. Here we follow the same strategy outlined while discussing the heterotic superpotential. The calculations are then performed on the level of the fermionic effective action. As reviewed earlier the superpotential W appears linearly as the mass of the four-dimensional gravitino ψ_μ. To determine W for the orientifold setups one dimensionally reduces the fermionic part of type IIA and type IIB actions. As in the bosonic part, the orientifold projections ensure that the resulting four-dimensional theories possess $N = 1$ supersymmetry.

Let us start by recalling the relevant fermionic terms for our discussion in the ten-dimensional type IIA and type IIB supergravity theories. We conveniently combine the two gravitinos into a two-vector $\hat{\psi}_N = (\hat{\psi}^1_N, \hat{\psi}^2_N)$. The effective action for the gravitinos in string frame takes the form

$$
\begin{aligned}
S_\psi \;=\; -\int d^{10}x\sqrt{-\hat{G}_{(10)}}\ \Big[e^{-2\hat{\phi}}\,\hat{\bar{\psi}}_M\,\Gamma^{MNP}\,D_N\,\hat{\psi}_P \\
+\tfrac{1}{4}\,e^{-2\hat{\phi}}\,\hat{H}_{MNP}.\Psi^{MNP} + \tfrac{1}{8}\sum_{n=0,1}^{8,9}\hat{F}_n.\Psi_n\Big]\ ,
\end{aligned}
\tag{4.59}
$$

where the R-R field strengths $\hat{F}_n$ are defined in (2.10) for n runs from 0 to 8 for type IIA and from 1 to 9 for type IIB. We denote by Ψ and Ψ_n the quadratic part in $\hat{\psi}_M$ of the ten-dimensional three- and n-forms $\tilde{\Psi}^{(3)}_M$ and $\tilde{\Psi}_n$ defined in (2.15)

$$
\begin{aligned}
(\Psi)_{M_1M_2M_3} &= \hat{\bar{\psi}}_M\Gamma^{[M}\Gamma_{M_1M_2M_3}\Gamma^{N]}\mathcal{P}\hat{\psi}_N\ ,\\
(\Psi_n)_{M_1\ldots M_n} &= e^{-\hat{\phi}}\hat{\bar{\psi}}_M\Gamma^{[M}\Gamma_{M_1\ldots M_n}\Gamma^{N]}\mathcal{P}_n\hat{\psi}_N\ .
\end{aligned}
\tag{4.60}
$$

We will discuss the reduction of the action (4.59) on the manifold $M_{3,1}\times\mathcal{M}_6$ focusing on the derivation of four-dimensional mass terms of the form given in (2.35) for type IIA and type IIB in turn and determine the induced superpotentials.

4.4.1 Type IIA superpotential

Let us first determine the superpotential for type IIA orientifolds induced by non-trivial background fluxes and torsion. We denote the background flux of $d\hat{B}_2$

by H_3 while the fluxes of the R-R forms $d\hat{C}_n$ are denoted by F_{n+1}. In type IIA supergravity we additionally allow for a scalar parameter F_0, which corresponds to the mass in the massive type IIA theory introduced by Romans [101]. In order that the background fluxes respect the orientifold projection they have to obey (4.13). It is convenient to combine the R-R background fluxes into an even form $F^{\rm ev}$ on $\mathcal{M}_6$ as

$$F^{\rm ev} = F_0 + F_2 + F_4 + F_6 \ . \tag{4.61}$$

In addition to the background fluxes also a non-vanishing intrinsic torsion of the $SU(3)$ structure manifold will induce terms contributing to the $N = 1$ superpotential. As discussed earlier these arise due to the non-closure of the globally defined two-form J and three-form Ω_η and can be parameterized as given in eqn. (2.26).

In order to actually perform the reduction of (4.59) to four dimensions we use (4.21) and the gamma matrices decomposition given in (B.17). Imposing the orientifold projection $\mathcal{O}$ amounts to combine the two spinors $\psi_\mu^{1,2}$ in a four-dimensional invariant spinor ψ_μ. In order to ensure the correct form of the four-dimensional kinetic terms for ψ_μ we restrict to the specific choice

$$\psi_\mu = e^{i\theta/2}\,\psi^1_\mu = -e^{-i\theta/2}\,\psi^2_\mu \ , \qquad\qquad \bar\psi_\mu = e^{-i\theta/2}\,\bar\psi^1_\mu = -e^{i\theta/2}\,\bar\psi^2_\mu \ . \tag{4.62}$$

These conditions define a reduction of a four-dimensional $N = 2$ to an $N = 1$ supergravity theory [102, 103]. Hence, the mass terms of the spinors ψ_μ take the standard $N = 1$ form given in eqn. (2.35)

$$\begin{aligned} S_\psi \ = \ & -\int d^4x\sqrt{-g_4}\, e^{\frac{K}{2}}\,\bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu \int_{\mathcal{M}_6}\Big[4e^{-\hat\phi+i\theta}\eta_+^\dagger\gamma^m D_m\eta_- + 4e^{-\hat\phi-i\theta}\eta_-^\dagger\gamma^m D_m\eta_+ \\ & + \tfrac{1}{3!}\, e^{-\hat\phi+i\theta}\,(\hat H_3)_{mnp}\,\eta_+^\dagger\gamma^{mnp}\eta_- \ - \ \tfrac{1}{3!}\, e^{-\hat\phi-i\theta}\,(\hat H_3)_{mnp}\,\eta_-^\dagger\gamma^{mnp}\eta_+ \\ & +\tfrac{1}{2}\sum_{k\ \text{even}} \tfrac{1}{k!}\big((\lambda\hat F_k)_{m_1\ldots m_k}\eta_+^\dagger\gamma^{m_1\ldots m_k}\eta_+ + (\hat F_k)_{m_1\ldots m_k}\eta_-^\dagger\gamma^{m_1\ldots m_k}\eta_-\big)\Big] + \ldots \ , \end{aligned} \tag{4.63}$$

where $e^{K/2} = e^{2D}e^{K^{\rm K}/2}$ with $K^{\rm K}$ as defined in eqn. (4.53). The four-dimensional dilaton e^D is introduced in (3.19). Note that after the reduction of the $D = 10$ string frame action to four space-time dimensions we performed a Weyl-rescaling to obtain a standard Einstein-Hilbert term. More precisely, in the derivation of (4.63) we made the rescaling [5]

$$g_{\mu\nu} \to e^{2D}g_{\mu\nu} \ , \qquad \sigma^\mu \to e^{-D}\sigma^\mu \ , \qquad \psi_\mu \to e^{D/2}\psi_\mu \ . \tag{4.64}$$

[5]Due to the fact that we start here from a ten-dimensional action in the string frame the Weyl rescaling (4.64) is different from (3.30).

The rescaling of ψ_μ ensures that the four-dimensional theory has a standard kinetic term for the gravitino. The superpotential can be obtained by comparing the action (4.63) with the standard $N=1$ mass term given in (2.35). We will discuss the arising terms in turn.

Let us express the result (4.63) in terms of the globally defined two-form J and Ω_η defined in (2.23). First recall that Ω_η is related to the Ω by a rescaling (C.41). The quantities in the first line of (4.63) are expressed in terms of the forms Ω_η and J by using (3.56). Using the definition of $\Pi^{\rm odd} = C\Omega$ displayed in (4.14), (4.15) one finds

$$\begin{aligned} 4\int d^6y\sqrt{g_6}\; e^{-\hat\phi}\Big[e^{i\theta}\eta_+^\dagger\gamma^m D_m\eta_- + e^{-i\theta}\eta_-^\dagger\gamma^m D_m\eta_+\Big] & \\ = -\int_{\mathcal{M}_6}\big\langle d\mathrm{Re}(\Pi^{\rm odd}), J\big\rangle\ . & \end{aligned} \tag{4.65}$$

Similarly, one expresses the remaining terms in the action (4.63) using the three-from $\Pi^{\rm odd}$ and the two-form J. More precisely, the terms in the second line of eqn. (4.63) are rewritten by applying eqns. (2.23), (C.41), (4.14) and (C.43) as

$$\begin{aligned} \tfrac{1}{3!}\int d^6y\sqrt{g_6}\;\big[e^{-\hat\phi+i\theta}(\hat H_3)_{mnp}\eta_+^\dagger\gamma^{mnp}\eta_- - e^{-\hat\phi-i\theta}(\hat H_3)_{mnp}\eta_-^\dagger\gamma^{mnp}\eta_+\big] & \\ = -i\int_{\mathcal{M}_6}\Big[\big\langle H_3\wedge\mathrm{Re}(\Pi^{\rm odd}),1\big\rangle + \big\langle d\mathrm{Re}(\Pi^{\rm odd}),\hat B_2\big\rangle\Big]\ , & \end{aligned} \tag{4.66}$$

where we have used that $\hat H_3 = d\hat B_2 + H_3$ with H_3 being the background flux. Finally, we apply gamma-matrix identities and the definition (2.23) of J to rewrite the terms appearing in the last line of (4.63) as

$$\begin{aligned} \tfrac12\sum_{k\ \text{even}}\tfrac{1}{k!}\int d^6y\sqrt{g_6}\;\Big[(\lambda\hat F_k)_{m_1\ldots m_k}\eta_+^\dagger\gamma^{m_1\ldots m_k}\eta_+ + (\hat F_k)_{m_1\ldots m_k}\eta_-^\dagger\gamma^{m_1\ldots m_k}\eta_-\Big] & \\ = \int_{\mathcal{M}_6}\Big[\big\langle F^{\rm ev}, e^{-\hat B_2+iJ}\big\rangle - \big\langle H_3\wedge C_3^{(0)},1\big\rangle - \big\langle dC_3^{(0)},\hat B_2\big\rangle + i\big\langle dC_3^{(0)},J\big\rangle\Big]\ , & \end{aligned} \tag{4.67}$$

where $C_3^{(0)}$ is defined in (4.19) as the part of $\hat C_3$ being a three-form on $\mathcal{M}_6$ yielding scalar fields in $M_{3,1}$. In deriving this identity one uses the definition of $\hat F_k$ given in eqn. (2.10) while eliminating half of the R-R fields by the duality condition (2.10).

In summary one can now read off the complete type IIA superpotential induced by background fluxes and torsion. Introducing the differential operator $d_H =$

$d - H_3\wedge$ one finds (see also refs. [68, 104])

$$W^{O6} = \int_{\mathcal{M}_6} \left\langle F^{\rm ev} + d_H \Pi_c^{\rm odd}, e^{J_c} \right\rangle \ , \tag{4.68}$$

where we used the definitions of $J_c = -\hat{B}_2 + iJ$ and $\Pi_c^{\rm odd} = C_3^{(0)} + i\mathrm{Re}(\Pi^{\rm odd})$ given in eqns. (4.18) and (4.20). The superpotential extends the results of refs. [68, 104, 105, 106, 107, 108]. Let us now determine W for the type IIB orientifold compactifications.

4.4.2 Type IIB superpotential

In the following we will determine the superpotential of type IIB orientifolds induced by the background fluxes and torsion. In type IIB theory we allow for a non-trivial NS-NS flux H_3 as well as odd R-R fluxes. Due to the fact that we do not expand in one- or five-forms on $\mathcal{M}_6$ the only non-vanishing R-R is the three-form F_3. These fluxes satisfy equation (4.34).

Since, there are some qualitative differences between $O3/O7$ set-up and $O5/O9$ set-up we will discuss them in the following separately.

$\underline{O3/O7}$: Recall that the invariant spinor under the orientifold projections $\mathcal{O}_{(1)}$ defined in eqn. (4.10) is given as the sum $\psi_\mu = \frac{1}{2}(\psi^1_\mu + i\psi^2_\mu)$ together with the conjugate expression for $\bar{\psi}_\mu$. It turns out to be sufficient to determine W for a simpler choice of the four-dimensional spinor ψ_μ given by

$$\psi_\mu = \psi^1_\mu = -i\psi^2_\mu \ , \qquad \bar{\psi}_\mu = \bar{\psi}^1_\mu = i\bar{\psi}^2_\mu \ . \tag{4.69}$$

These conditions define the reduction of the $N = 2$ theory to $N = 1$ induced by the orientifold projection. Inserting the decompositions (4.41) together with (4.69) into the ten-dimensional action (4.59) one determines the ψ_μ mass terms

$$\begin{aligned} S_\psi \ &= \ - \int d^4x \sqrt{-g_4}\, e^{\frac{K}{2}}\ \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu \\ &\qquad \int d^6y \sqrt{g_6}\ \tfrac{1}{3!} \Big[(e^{-\hat{\phi}}\, (\hat{H}_3)_{mnp} + i\, (\hat{F}_3)_{mnp})\, \Omega^{mnp} \Big] + \dots \ , \end{aligned} \tag{4.70}$$

where $e^{K/2} = e^{2D} e^{K^{\rm cs}/2}$ with $K^{\rm cs}$ as defined in eqn. (4.56). In order to derive this four-dimensional action we performed the Weyl-rescaling (4.64) to obtain a standard Einstein-Hilbert term. Moreover, we used the identities (2.23) and (C.42) to replace the gamma-matrix expressions $\eta_-^\dagger \gamma^{mnp} \eta_+$ with the complex three-form

Ω^{mnp} and absorbed a factor arising due to the Weyl-rescaling (4.64) into $e^{K/2}$. It is interesting to note that there is no contribution from the reduction of the ten-dimensional kinetic term in the action (4.59). This can be traced back to the fact that in type IIB orientifolds with $O3/O7$ planes the globally defined three- and two-forms Ω and J transform with opposite signs under the map σ^*. However, since the volume form is positive under the orientation preserving map σ the integral over terms like $d\Omega \wedge J$ vanishes. The non-closed forms dJ and $d\Omega$ nevertheless yield a potential for the four-dimensional scalars which is encoded by non-trivial D-terms.

Let us now express the action (4.70) in terms of the globally defined three-form Ω and the form Φ^{ev}. Using the definition (4.35) of Φ^{ev} one infers

$$\begin{aligned} \tfrac{1}{3!}\int d^6y\sqrt{g_6}\big[e^{-\hat{\phi}-i\theta}(\hat{H}_3)_{mnp}\Omega^{mnp}\big] &= -i\int_{\mathcal{M}_6} e^{-\hat{\phi}}\big[\langle H_3,\Omega\rangle + \langle d\hat{B}_2,\Omega\rangle\big] \qquad (4.71)\\ &= -i\int_{\mathcal{M}_6}\big[\langle H_3\wedge \mathrm{Re}(\Phi^{\text{ev}}),\Omega\rangle - \langle d\mathrm{Re}(\Phi^{\text{ev}}),\Omega\rangle\big]\ , \end{aligned}$$

where we have used $\mathrm{Re}(\Phi^{\text{ev}})_0 = e^{-\hat{\phi}}$ and $\mathrm{Re}(\Phi^{\text{ev}})_2 = -e^{-\hat{\phi}}\hat{B}_2$ as simply deduced from the definition (4.35). For the R-R term in (4.70) one derives

$$\tfrac{i}{3!}\int d^6y\sqrt{g_6}\,(\hat{F}_3)_{mnp}\Omega^{mnp} = \int_{\mathcal{M}_6}\big[\langle F_3,\Omega\rangle + \langle dA_2^{(0)},\Omega\rangle - \langle H_3\wedge A_0^{(0)},\Omega\rangle\big]\ , \qquad (4.72)$$

where $A_2^{(0)}$ and $A_0^{(0)}$ denote the two- and zero- forms in $A^{\text{ev}}_{(0)}$ defined in (4.39).[6] Together the two terms (4.71) and (4.72) combine into the superpotential

$$W^{O3/O7} = \int_{\mathcal{M}_6}\langle F_3 + d_H\Phi_c^{\text{ev}},\Omega\rangle \qquad (4.73)$$

where $d_H = d - H_3\wedge$ and Φ_c^{ev} is defined in eqn. (4.40). This superpotential contains the well-known Gukov-Vafa-Witten superpotential [15, 109] as well as contributions due to non-closed two-forms $\hat{B}_2$ and $\hat{C}_2$.

Let us complete the discussion of type IIB orientifolds by determining the $O5/O9$ superpotential.

$\underline{O5/O9:}$ To derive the superpotential for $O5/O9$ orientifolds we restrict to a specific choice for the four-dimensional invariant spinor $\psi_\mu = \frac{1}{2}(\psi^1_\mu + \psi^2_\mu)$

$$\psi_\mu = \psi^1_\mu = \psi^2_\mu\ , \qquad \bar{\psi}_\mu = \bar{\psi}^1_\mu = \bar{\psi}^2_\mu\ . \qquad (4.74)$$

[6]Expanding $A^{\text{ev}}_{(0)}$ in (4.39) one finds $A_2^{(0)} = \hat{C}_2 - \hat{C}_0\hat{B}_2$ and $A_0^{(0)} = \hat{C}_0$.

Together with the decomposition (4.41) we reduce the action (4.59) to determine the mass term of ψ_μ as

$$S_\psi = -\int d^4x\sqrt{-g_4}\; e^{\frac{K}{2}}\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu \int d^6y\sqrt{g_6}\,\big\langle -i\hat{F}_3 + d(e^{-\hat{\phi}}J),\Omega\big\rangle + \dots, \quad (4.75)$$

where we have applied (2.23), (C.42), (2.26) and performed the Weyl rescaling (4.64). Note that the term involving the NS-NS fluxes vanishes in the case of $O5/O9$ orientifolds since Ω and $\hat{H}_3$ transform with an opposite sign under the symmetry σ^* as can be deduced from eqns. (4.7) and (4.34). Inserting the definition (2.10) of $\hat{F}_3$ into (4.75) one obtains the superpotential [104]

$$W^{O5/O9} = -i\int_{\mathcal{M}_6}\big\langle F_3 + d\Phi_c^{\mathrm{ev}},\Omega\big\rangle\ , \quad (4.76)$$

where we have used $\mathrm{Im}(\Phi^{\mathrm{ev}})_2 = e^{-\hat{\phi}}J$ and the definition (4.40) of Φ_c^{ev}. $W^{O5/O9}$ is independent of the NS-NS flux H_3 which was shown in ref. [62] to contribute a D-term potential to the four-dimensional theory.

Chapter 5

Conclusion

In this thesis we determined the four-dimensional $N = 1$ low energy effective theories for a more general class of compactifications arising if the internal manifold $\mathcal{M}_6$ is no longer restricted to be Calabi-Yau. We have seen that in order for the resulting four-dimensional theory to still admit some supersymmetry $\mathcal{M}_6$ cannot be chosen arbitrarily, but rather has to admit at least one globally defined spinor. In case that $\mathcal{M}_6$ has exactly one globally defined spinor the structure group of the manifold reduces to $SU(3)$. Equivalently, these manifolds are characterized by the existence of two globally defined forms, a real two-form J and a complex three-form Ω. These forms are in general not closed, which indicates a deviation from the Calabi-Yau case. This difference can also be encoded by specifying a new connection on $\mathcal{M}_6$ with torsion replacing the ordinary Levi-Civita connection.

Compactifications of the heterotic and type II theories on such manifolds lead to (spontaneously broken) $N = 1$ and $N = 2$ theories respectively. From a phenomenological point of view $N = 1,\ D = 4$ theories are of importance. Therefore we imposed orientifold projections in the case of type II compactifications which truncate the resulting theories to $N = 1$ supergravities. These yield setups with $O6$ planes in type IIA, while for type IIB reductions two setups with $O3$ and $O7$ as well as $O5$ and $O9$ planes are encountered.

We used Kaluza-Klein reduction to determine the four-dimensional theories. In contrast to the standard Calabi-Yau compactifications the reduction on $SU(3)$ structure manifolds is more subtle. This can be traced back to the fact that in these generalized compactifications the distinction between massless or light modes and the massive Kaluza-Klein modes is not anymore straightforward. Recall that in Calabi-Yau compactifications the massless modes are in one-to-one correspondence with the harmonic forms of $\mathcal{M}_6$ and one only keeps these in the

Kaluza-Klein reduction. For manifolds with $SU(3)$ structure a similar characterization is missing so far. Therefore we kept in the reduction the whole Kaluza-Klein tower which means that we expanded in forms on $\mathcal{M}_6$ which are not necessarily harmonics. However, we specified a finite basis of forms which enabled us to do the reduction explicitly. Among the Kaluza-Klein modes we did not keep any triplets of $SU(3)$, or in other words, we did not keep any modes which arise from one-forms (or five-forms) of $\mathcal{M}_6$. Apart from this constraint we kept the analysis generic. Projecting out the triplets of $SU(3)$ is necessary to keep the standard multiplets of $N = 1$ and $N = 2$ supergravities.

In compactifications on $SU(3)$ structure manifolds a scalar potential is induced by the torsion as well as the existing background fluxes. Due to the $N = 1$ supersymmetry it can be encoded by a Kähler potential, a holomorphic superpotential and possible D-terms. We derived the general form of the superpotentials on $SU(3)$ structure manifolds for heterotic and type IIA/B orientifold setups by evaluating appropriate fermionic mass terms. The reason for doing so is the fact that in the bosonic terms the superpotential and its derivatives appear quadratically which complicates the computation. Instead they appear linearly in the fermionic couplings and can be computed straightforwardly. With a similar analysis we determined the superpotential derivatives (F-terms) for the heterotic compactification. However, in order to do so one also needs the proper normalization of the fermionic kinetic terms. Therefore, we discussed first the kinetic terms of the four-dimensional fermions. To our knowledge such an analysis has not yet been done even for Calabi-Yau compactifications.

Knowing the superpotential and the Kähler potential one can determine the conditions for four-dimensional supersymmetric vacua. It is readily checked that these conditions evaluated for the orientifold setups are in accord with the $N = 1$ conditions on ten-dimensional backgrounds derived in refs. [44, 110]. In the heterotic case we have derived explicitly these conditions by evaluating the SUSY transformations of the fermionic fields and setting them to zero. It turned out that flux and torsion have to satisfy certain constraints. These are the vanishing of the first and second torsion classes simultaneously, or in other words, the compactification manifold has to be complex. In addition, the flux should be equal to the third torsion class which amounts to allow only $(2,1) + (1,2)$-form fluxes. Furthermore, the dilaton is found to be constant. These conditions were first discussed by Strominger [40]. Here we recovered his conditions in the limit of non-warped compactifications.

We computed the Yukawa couplings arising in the heterotic compactification on $SU(3)$ structure manifolds. These appear to be third derivatives of the prepotentials extending the results of Calabi-Yau compactifications [55, 85, 53].

As mentioned earlier, our motivation to study $N = 1,\ D = 4$ theories is their phenomenological importance. However, in attempts to construct specific models for particle physics and cosmology an essential step is the study of moduli stabilization and the inclusion of matter and moduli fields due to space-time filling D-branes in type II compactifications. These can be directions for further research. Furthermore, it is an interesting task to investigate the generalization of type II $SU(3)$ structure orientifolds [60] which can be useful in the study of mirror symmetry in presence of NS-NS fluxes. This symmetry relates two Calabi-Yau manifolds Y, $\tilde{Y}$ and exchanges their odd/even cohomologies. Compactifications of type IIA on Y and type IIB on $\tilde{Y}$ lead to equivalent theories in four dimensions. Mirror symmetry extends naturally to Calabi-Yau compactification with R-R fluxes. This is due to the fact that R-R fluxes are in even and odd cohomologies in type IIA and type IIB respectively. The completion of mirror symmetry in the presence of NS-NS background fluxes is not as straightforward and it is still an area of intense current research [68, 60][111] –[119]. For compactifications with 'electric' NS-NS fluxes it was conjectured in refs. [80, 114] that the mirror geometry is a set of specific $SU(3)$ structure manifolds known as half-flat manifolds. To extend this conjecture to 'magnetic' NS-NS fluxes various more drastic deviations from the standard compactifiactions are expected [68, 115, 116, 117]. These mirrors are extensions of generalized almost complex manifolds with $SU(3) \times SU(3)$ structure.[1] In these manifolds the tangent and cotangent bundles T, T^* are no longer the central geometric objects, but rather get replaced by the generalized tangent bundle E locally given by $T \oplus T^*$. The study of type II $SU(3)$ structure orientifolds [60] permits to strength this conjecture since many of the $SU(3)$ structure results naturally generalize to the $SU(3) \times SU(3)$ structure case. For instance, in the derivation of $SU(3) \times SU(3)$ superpotentials of [60] the analysis and techniques learned in $SU(3)$ structure orientifolds were used.

[1]The notion of generalized almost complex manifold was introduced by Hitchin [120] and Gualtieri [121], while an intensive discussion of $SU(3) \times SU(3)$ structure can be found in the work of Graña, Louis and Waldram [68].

Chapter 6

Appendix

A Kaluza-Klein reduction

Kaluza-Klein theory explains Einstein gravity and electromagnetism in four dimensions as pure gravity from the five-dimensional point of view [123]. Since KK reduction plays an important role throughout the thesis, we recall it here. We give as well the compactification on a 6-dimensional manifold as a generalization of the Kaluza-Klein S^1 reduction. Finally we give the example of Calabi-Yau manifolds as the six-dimensional compactification manifolds.

A.1 The reduction on a circle

In five dimensions the Einstein-Hilbert action is given by

$$S_5 = -\int d^5x\sqrt{-\hat{G}}\,\hat{R}\ , \tag{A.1}$$

where $\hat{G}$ and $\hat{R}$ are the determinant of the 5-dimensional metric $\hat{G}_{MN}$ and the Ricci scalar of the theory, respectively.[1] Assuming that one of the space directions is periodic one can view the original theory as being reduced on a circle. The spacetime coordinates $x^M\ ,\ M = 0,\dots,4$ split into the four-dimensional spacetime coordinates $x^\mu\ ,\ \mu = 0,\dots,3$ and the internal periodic coordinate y. In this context the simplest solution of the five dimensional equation of motion $\hat{R}_{MN} = 0$

[1] To avoid another notation for the compactified fields we denote the higher dimensional fields before compactification with a hat.

can be given by a four-dimensional Minkowski space times a circle, $M_{(3,1)} \times S^1$. Fluctuations around this background can be encoded into the four-dimensional metric $g_{\mu\nu}$, a four-dimensional vector $\hat{G}_{\mu 5} = A_\mu$, and a scalar $\hat{G}_{55} = \Phi$. These fields are periodic in y and hence can be Fourier expanded in the form

$$\hat{\phi}(x^\mu, y) = \sum_n \phi_n(x^\mu) e^{iny/r} \ . \tag{A.2}$$

From a four-dimensional point of view the five dimensional massless fields generate massive Kaluza-Klein modes. The masses are of the order $m^2 \sim \frac{n^2}{r^2}$. This can be seen by applying the Laplace operator to the five dimensional scalars[2]

$$0 = \partial_M \partial^M \hat{\phi} = \big(\partial_\mu \partial^\mu \ + \partial_y \partial^y\big)\hat{\phi} = \sum_n \big(\partial_\mu \partial^\mu - (n/r)^2\big)\phi_n(x^\mu) e^{iny/r} \ . \tag{A.3}$$

The massless fields are the zero modes ϕ_0 of the Kaluza-Klein tower and all the other modes $\phi_1, \ldots, \phi_n$ are massive. The size of the compact dimension determines how heavy those modes are. In the case where r is small enough the masses get large and their modes can be discarded from the low energy approximation. The resulting four-dimensional effective action describing the massless fields ϕ_0 is found after integrating out the dependence of y and some adequate rescaling. It is written as

$$S_4 = \int d^4x \sqrt{-g}\big(- R - \frac{1}{4}\phi F_{\mu\nu} F^{\mu\nu} - \frac{1}{6\phi}\partial_\mu \phi \partial^\mu \phi\big) \ , \tag{A.4}$$

where R is the Ricci scalar of the four-dimensional theory and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the abelian gauge boson A_μ.

A.2 Reduction on a 6-dimensional manifold

The S^1 reduction of the five-dimensional theory can be easily generalized to higher dimensional theories with an arbitrary number of compact dimensions. The example given here is the reduction of a ten-dimensional massless scalar $\hat{\phi}$ on a 6-dimensional manifold $\mathcal{M}_6$. Let us denote the ten-dimensional coordinates by $x^M,\ M = 0, \ldots, 9$, the four-dimensional coordinates by $x^\mu,\ \mu = 0, \ldots, 3$ and the six-dimensional coordinates on $\mathcal{M}_6$ by $y^m,\ m = 1, \ldots, 6$. The ten-dimensional metric is block diagonal. Hence the equation of motion for this scalar $\hat{\phi}$ is written as

$$\Delta_{10}\hat{\phi} = \Delta_4 \hat{\phi} + \Delta_6 \hat{\phi} = 0 \ , \tag{A.5}$$

[2]For simplicity we consider here the case where A_μ is trivial in the background.

where $\Delta_{10},\ \Delta_6,\ \Delta_4$ are the Laplacians in 10, 6 and 4 dimensions respectively. As in the example of the reduction on a circle $\hat{\phi}$ can be expanded in a four-dimensional part $\phi^i(x^\mu)$ times a six-dimensional part $\omega_i(y^m)$

$$\hat{\phi}(x^M) = \phi^i(x^\mu)\omega_i(y^m)\ , \tag{A.6}$$

where ω_i are a set of functions on $\mathcal{M}_6$ and the index i denotes their multiplicity. For massless fields $\phi(x^\mu)$ in four-dimensions these are the harmonics of the manifold $\mathcal{M}_6$. This is easily seen from eqn. (A.5) if we insert the decomposition (A.6) of $\hat{\phi}$. The four-dimensional mass is then given by $m^2\omega_i = \Delta_6\omega_i$. Clearly the massless fields correspond to the harmonics of the manifold $\mathcal{M}_6$ satisfying the condition $\Delta_6\omega_i = 0$.

A.3 Calabi-Yau manifolds

In the literature there is a well studied example of complex compactification manifolds $\mathcal{M}_6$ known as Calabi-Yau manifolds [8]. Although we consider, in this thesis, more general manifolds we will give a brief review on the Calabi-Yau manifolds. Those are compact Kähler manifolds with vanishing first Chern class. Equivalently, they are Ricci flat Kähler manifolds with $SU(3)$ holonomy.

The Kählerity condition: That the manifold is Kähler amounts to the fact that its metric g_{mn} is given by the second derivative of some function known as the Kähler potential. The metric g_{mn} is hermitian with respect to the complex structure. In complex coordinates this means that only its mixed type components $g_{\alpha\bar{\beta}}$ are non vanishing. Given the hermitian Kähler metric g on the Calabi-Yau manifold, one can build a closed $(1,1)$-form called the Kähler form

$$J \ = \ i\, g_{\alpha\bar{\beta}}\, dz^\alpha \wedge d\bar{z}^{\bar{\beta}}\ . \tag{A.7}$$

Ricci flatness: Kähler manifolds with vanishing first Chern class admit a Ricci-flat metric

$$R_{\alpha\bar{\beta}} \ = \ 0\ , \tag{A.8}$$

where $R_{\alpha\bar{\beta}}$ is the Ricci tensor. Consequently, a Calabi-Yau manifold can also be defined as a compact Ricci-flat Kähler manifold.

$SU(3)$ **holonomy:** The condition that the manifold has an $SU(3)$ holonomy amounts to having a covariantly constant spinor η with respect to the Levi-Civita connection

$$D^{LC}\eta \ = \ 0 \ . \tag{A.9}$$

The existence of such a spinor is of importance; it allows preserving minimal supersymmetries of the compactified theories.

The Hodge diamond: As we have seen earlier the harmonics of the manifold are in one to one correspondence with the massless fields in four dimensions. As there is a unique harmonic (p,q)-form representative in each cohomology class of $H^{(p,q)}$, their multiplicity is counted by the dimension of the non-trivial cohomologies of the Calabi-Yau manifold. These are the quotients $H^{(p,q)} = \frac{\{\omega|d\omega=0\}}{\{\alpha|\alpha=d\beta\}}$, where ω is a (p,q)-form. Their dimensions $h^{(p,q)} = \dim H^{(p,q)}$ can be summarized in the Hodge diamond as follows

$$\begin{array}{ccccccc} & & & h^{(0,0)} & & & \\ & & h^{(1,0)} & & h^{(0,1)} & & \\ & h^{(2,0)} & & h^{(1,1)} & & h^{(0,2)} & \\ h^{(3,0)} & & h^{(2,1)} & & h^{(1,2)} & & h^{(0,3)} \\ & h^{(3,1)} & & h^{(2,2)} & & h^{(1,3)} & \\ & & h^{(3,2)} & & h^{(2,3)} & & \\ & & & h^{(3,3)} & & & \end{array} \ = \ \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{(1,1)} & & 0 & \\ 1 & & h^{(2,1)} & & h^{(2,1)} & & 1 \\ & 0 & & h^{(1,1)} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \ . \tag{A.10}$$

B The Clifford Algebra in 4 and 6 dimensions

In this appendix we assemble the spinor conventions in four and six dimensions used throughout the thesis. In $D = 10$ the Γ-matrices are hermitian and satisfy the Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN} \ , \qquad M, N = 0, \dots, 9 \ . \tag{B.11}$$

One defines [6]

$$\Gamma^{11} = \Gamma^0 \dots \Gamma^9 \ , \tag{B.12}$$

which has the properties

$$(\Gamma^{11})^2 = 1 \ , \quad \{\Gamma^{11}, \Gamma^M\} = 0 \ . \tag{B.13}$$

This implies that the Dirac representation can be split into two Weyl representations

$$\mathbf{32}_{Dirac} = \mathbf{16} + \mathbf{16}' \tag{B.14}$$

with eigenvalue $+1$ and -1 under Γ^{11}.

In backgrounds of the form $\mathcal{M}_{10} = M_{(3,1)} \times \mathcal{M}_6$ the 10-dimensional Lorentz group decomposes as

$$SO(9,1) \to SO(3,1) \times SO(6) \ , \tag{B.15}$$

implying a decomposition of the spinor representations as

$$\mathbf{16} = (\mathbf{2}, \mathbf{4}) + (\bar{\mathbf{2}}, \bar{\mathbf{4}}) \ . \tag{B.16}$$

Here $\mathbf{2}, \mathbf{4}$ are the Weyl representations of $SO(3,1)$ and $SO(6)$ respectively.

In this background the ten-dimensional Γ-matrices can be chosen block-diagonal as

$$\Gamma^M = (\gamma^\mu \otimes \mathbf{1}, \gamma^5 \otimes \gamma^m), \quad \mu = 0, \ldots, 3, \ m = 1, \ldots, 6 \ , \tag{B.17}$$

where γ^5 defines the Weyl representation in $D = 4$. In this basis Γ^{11} splits as [6]

$$\Gamma^{11} = -\gamma^5 \otimes \gamma^7 \ , \tag{B.18}$$

where γ^7 defines the Weyl representations in $D = 6$.

Let us now turn to our spinor conventions in $D = 6$ and $D = 4$ respectively.

B.1 Clifford algebra in 6-dim

In $D = 6$ the gamma matrices are chosen hermitian, $\gamma^{m\dagger} = \gamma^m$, and they obey the Clifford algebra

$$\{\gamma^m, \gamma^n\} = 2g^{mn} \ , \quad m, n = 1, \ldots, 6 \ . \tag{B.19}$$

The Majorana condition on a spinor η reads

$$\eta^\dagger = \eta^T C \ , \tag{B.20}$$

where C is the charge conjugation matrix

$$C^T = C \ , \quad \gamma_m^T = -C\gamma_m C^{-1} \ . \tag{B.21}$$

One can regroup these six gamma-matrices into three sets of anticommuting raising and lowering operators [6]

$$\gamma^{\alpha} = \tfrac{1}{2}(\gamma^m + i\gamma^{m+3}) \ , \qquad \gamma^{\bar\alpha} = \tfrac{1}{2}(\gamma^m - i\gamma^{m+3}) \ , m = 1,2,3 \ , \tag{B.22}$$

satisfying

$$\{\gamma^{\alpha}, \gamma^{\bar\beta}\} = g^{\alpha\bar\beta} \ , \qquad \{\gamma^{\bar\alpha}, \gamma^{\bar\beta}\} = 0 \ , \qquad \{\gamma^{\alpha}, \gamma^{\beta}\} = 0 \ . \tag{B.23}$$

In this basis the two chiral spinors $\eta_\pm$ are annihilated by $\gamma^\alpha, \gamma^{\bar\alpha}$ respectively, and one has

$$\gamma^{\alpha}\eta_+ = 0 \ , \qquad \gamma^{\bar\alpha}\eta_- = 0 \ . \tag{B.24}$$

B.2 Clifford algebra in 4-dim

In $D = 4$ we adopt the conventions of [82] and choose

$$\gamma^{\mu} = -i \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \ , \tag{B.25}$$

where the σ^μ are the 2×2 Pauli matrices

$$\begin{aligned} \sigma^0 &= \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \\ \sigma^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \end{aligned} \tag{B.26}$$

and $\bar\sigma^0 = \sigma^0, \qquad \bar\sigma^{1,2,3} = -\sigma^{1,2,3}$.

C The geometry of the scalar manifold in $SU(3)$ compactifications

In this appendix we collect the results of the geometry of the scalar manifold arising in compactifications on manifolds with $SU(3)$ structure. For the special case of Calabi-Yau manifolds this geometry coincides with the geometrical moduli space of the deformations of the Calabi-Yau metric [55]. For more general manifolds one can still define metric deformations and a metric on the space of metric deformations. The resulting geometry has been discussed in refs. [80, 68] and shown to be a product manifold of the form

$$\mathcal{M} = \mathcal{M}^{\mathrm{K}} \times \mathcal{M}^{\mathrm{cs}} \ , \tag{C.27}$$

where $\mathcal{M}^{\mathrm{K}}$ corresponds to the deformations of J while $\mathcal{M}^{\mathrm{cs}}$ corresponds to the deformations of the three-form Ω. $N = 1$ supersymmetry constrains this product to be a Kähler manifold. However, for compactifications on manifolds with $SU(3)$ structure each factor has a special Kähler geometry in that the Kähler potential is a sum of two terms

$$K = K^{\mathrm{K}} + K^{\mathrm{cs}} \ , \tag{C.28}$$

and both Kähler potentials can be derived from a holomorphic prepotential. Let us discuss this in more detail.

C.1 The $\mathcal{M}^{\mathrm{K}}$ component

The coordinates of $\mathcal{M}^{\mathrm{K}}$ are the scalars $t^A = b^A + iv^A$ which arise from expanding $B_2 + iJ$ in a set of two forms ω_A as done in (3.8), (3.9). The metric on this space is defined as

$$g_{AB} = \frac{1}{4\mathcal{K}} \int_{\mathcal{M}_6} \omega_A \wedge *_6\omega_B \ , \qquad A, B = 1, \dots, \dim \Delta^2 \ , \tag{C.29}$$

where $\mathcal{K}$ is the volume of $\mathcal{M}_6$

$$\mathcal{K} = \frac{1}{6} \int_{\mathcal{M}_6} J \wedge J \wedge J \ . \tag{C.30}$$

$*_6\omega_B$ denotes a set of four-forms which are dual to the set of two-forms ω_A. In [68] it was shown that the KK-reduction has to be such that their existence is ensured and furthermore that the metric defined by (C.29) is non-degenerate. Repeating the arguments of ref. [53] for the more general class of manifolds Y one derives

$$*_6\omega_A = -J \wedge \omega_A + \frac{\mathcal{K}_A}{4\mathcal{K}} J \wedge J \ , \tag{C.31}$$

which holds not only for Calabi-Yau manifolds but also for the more general manifolds of $SU(3)$ structure. Inserting into (C.29) we arrive at

$$g_{AB} = -\frac{1}{4\mathcal{K}} (\mathcal{K}_{AB} - \frac{1}{4\mathcal{K}} \mathcal{K}_A \mathcal{K}_B) \ , \tag{C.32}$$

where we abbreviate

$$\mathcal{K}_A = \int_{\mathcal{M}_6} \omega_A \wedge J \wedge J \, , \quad \mathcal{K}_{AB} = \int_{\mathcal{M}_6} \omega_A \wedge \omega_B \wedge J \, , \quad \mathcal{K}_{ABC} = \int_{\mathcal{M}_6} \omega_A \wedge \omega_B \wedge \omega_C \, . \tag{C.33}$$

g_{AB} is a Kähler metric of the Kähler potential $K^{\mathrm{K}} = -\ln \mathcal{K}$, i.e.

$$g_{AB} = \partial_{t^A} \partial_{\bar{t}^B} K \, , \qquad K = -\ln \tfrac{1}{6} \int_{\mathcal{M}_6} J \wedge J \wedge J \, . \tag{C.34}$$

In fact g_{AB} is even a special Kähler metric in that K can be derived from a holomorphic prepotential $\mathcal{F}$ via

$$K = -\ln \left[X^{\hat{A}} \overline{(\mathcal{F}_{\hat{A}})} - \bar{X}^{\hat{A}} \mathcal{F}_{\hat{A}} \right] \, , \qquad \mathcal{F}_{\hat{A}} = \partial_{X^{\hat{A}}} \mathcal{F} \, , \qquad \hat{A} = 0, \ldots, \dim \Delta^2 \, , \tag{C.35}$$

for

$$\mathcal{F} = (X^0)^2 \mathcal{K}_{ABC} t^A t^B t^C \, , \qquad t^A \equiv \frac{X^A}{X^0} \, . \tag{C.36}$$

C.2 The $\mathcal{M}^{\mathrm{cs}}$ component

$\mathcal{M}^{\mathrm{cs}}$ is spanned by the complex scalars $z^{\hat{K}}$ with a metric [55, 80, 68]

$$g_{KL} = -\frac{\int_{\mathcal{M}_6} \rho_K \wedge \bar{\rho}_L}{\int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}} \, , \tag{C.37}$$

where ρ_K and Ω are (2,1)-forms and (3,0)- form respectively satisfying the relations

$$\frac{\partial}{\partial z^K} \Omega = -K_K \Omega + \rho_K \, . \tag{C.38}$$

This metric is special Kähler with a Kähler potential given by

$$K^{\mathrm{cs}} = -\ln(i \int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}) \, . \tag{C.39}$$

Furthermore Ω can be expanded in terms of a real symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{L}})$ of three-forms

$$\Omega = Z^{\hat{K}}(z) \alpha_{\hat{K}} - \mathcal{G}_{\hat{L}}(z) \beta^{\hat{L}} \, , \qquad \hat{K}, \hat{L} = 0, \ldots, \dim \Delta^3 \, , \tag{C.40}$$

where $\mathcal{G}_{\hat{K}} = \partial_{Z^{\hat{K}}} \mathcal{G}$ is the derivative of the prepotential $\mathcal{G}$ with respect to the period $Z^{\hat{K}}$. The z^K used in (3.9) are the special coordinates defined as $z^K = Z^K / Z^0$.

The holomorphic three-form Ω is related to Ω_η of (2.23) by a scale factor

$$\Omega = \Omega_\eta ||\Omega|| \ , \qquad ||\Omega|| = e^{\frac{1}{2}K^{\mathrm{cs}} - \frac{1}{2}K^{\mathrm{K}}} \ , \tag{C.41}$$

where $K^{\mathrm{K}} = -\mathrm{ln}\mathcal{K}$ depends on the volume $\mathcal{K}$ in the appropriate frame. In terms of the components of Ω one thus has

$$\Omega^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}||\Omega|| \ . \tag{C.42}$$

The holomorphic three form Ω and the $(2,1)$-form ρ satisfy

$$*_6\Omega = -i\Omega \ , \quad *_6\rho_a = i\rho_a \ . \tag{C.43}$$

D Strominger conditions

Here we review the results of [40]. Consider a warped compactification where the metric takes the form

$$ds^2 = g^0_{MN} dx^M \otimes dx^N = e^{2\Delta(y)}(\hat{g}_{\mu\nu} dx^\mu \otimes dx^\nu + \hat{g}_{mn} dy^m \otimes dy^n) \ . \tag{D.44}$$

Making the following rescaling

$$g_{MN} = e^{-2\phi} g^0_{MN} \ , \ \psi_M = e^{-\frac{\phi}{2}}(\psi^0_M - \frac{\sqrt{2}}{4}\Gamma^0_M \lambda^0) \ , \tag{D.45}$$

the SUSY variation equations for the gravitino, gaugino, and the chiral fermions are, then, rewritten as the following

$$\delta\psi_M = D_M\epsilon - \frac{1}{4}H_M\epsilon \ , \tag{D.46}$$

$$\delta\chi = -\frac{1}{4}\Gamma^{MN}F_{MN}\epsilon \ , \tag{D.47}$$

$$\delta\lambda = \Gamma^M D_M\phi + \frac{1}{24}H\epsilon \ , \tag{D.48}$$

where $H = \Gamma^{MNP}H_{MNP}$, $H_M = \Gamma^{NP}H_{MNP}$. Then the supersymetric condition, namely the vanishing of the SUSY variation of the gravitino rescales as,

$$\delta\psi_\mu = D_\mu\epsilon = 0 \ \rightarrow \ \hat{D}_\mu\epsilon + \frac{1}{2}\gamma_\mu\gamma^m\partial_m \mathrm{ln}(\Delta - \phi)\epsilon = 0 \ , \tag{D.49}$$

where the hatted quantities refer to $\hat{g}$defined in (D.44). The integrability condition for this is

$$\hat{\gamma}^{\mu\nu}\hat{D}_\mu\delta\psi_\mu = -\tfrac{1}{4}\hat{R}^4\epsilon - \hat{3}D_m(\Delta-\phi)\hat{D}^m(\Delta-\phi)\epsilon = 0\ , \qquad \text{(D.50)}$$

whose consequences for compact six dimensional manifold are the equality of the warp factor and the dilaton, and the vanishing of the Ricci scalar of the six dimensional manifold,

$$\Delta(y) = \phi(y) + cont\ ,\ \ \hat{R} = 0\ . \qquad \text{(D.51)}$$

It is possible to construct a two form which is closed with respect to a new connection $\mathcal{D}$,

$$J_m^n = i\eta_+^+\Gamma^n_m\eta_+\ , \qquad \text{(D.52)}$$

$$\mathcal{D}J_m^n = D_m J_n^p - H^p_{sm}J^s_n - H^s_{mn}J^p_s = 0\ , \qquad \text{(D.53)}$$

$$\mathcal{D}_m = D_m - \frac{1}{4}H^{np}_m\Gamma_{np}\ . \qquad \text{(D.54)}$$

The integrability condition for J_m^n to be a complex structure is the vanishing of the Nijenhuis tensor N_{mnp} to vanish

$$N^p_{mn} = J^q_m J_{[n\ \ ,q]}^{\ \ p} - J^q_n J^s_{[m\ \ ,q]}\ \ p\ . \qquad \text{(D.55)}$$

Using (D.53) one finds

$$N_{mnp} = H_{mnp} - 3J^q_{[m}J^r_n H_{p]qr} = 0\ , \qquad \text{(D.56)}$$

which in turn gives

$$H = \frac{i}{2}(\bar{\partial}-\partial)J\ . \qquad \text{(D.57)}$$

Constructing a holomorphic closed three form, one finds that its norm is given via the dilaton,

$$\Omega = e^{8\phi}\eta_-^+\gamma_{abc}\eta_+ dz^a\wedge dz^b\wedge dz^c\ , \qquad \parallel\Omega\parallel = e^{8(\phi-\phi_0)}\ . \qquad \text{(D.58)}$$

Setting equation (D.47) to zero yields a condition on the Yang-mills field strength and the two-form J

$$J^{a\bar{c}}F_{a\bar{c}} = 0\ . \qquad \text{(D.59)}$$

Finally let's summaries the conditions for a supersymmetric vacuum:

- The compactification manifold is complex
- $H = \frac{i}{2}(\bar{\partial}-\partial)J$
- $J^{a\bar{c}}F_{a\bar{c}} = 0$
- $\parallel\Omega\parallel = e^{8(\phi-\phi_0)}$

E Stable forms and the Hitchin functional

In this appendix we collect some basic facts about the geometry of stable forms on a six-dimensional manifold $\mathcal{M}_6$. The definition of the Hitchin functionals will be recalled. A more exhaustive discussion of these issues can be found in refs. [100, 120, 122, 121]. Consider a six-dimensional manifold $\mathcal{M}_6$ with a real globally defined three-form $\rho \in \Lambda^3 T^*$. A natural notion of non-degeneracy is that the form ρ is stable. From an abstract point of view a stable form ρ is defined by demanding that the natural action of $GL(6)$ on ρ spans an open orbit in $\Lambda^3_p T^*$ at each point p of $\mathcal{M}_6$. This condition can also be formulated in terms of the map $q: \ \Lambda^3 T^* \ \rightarrow \ \Lambda^6 T^* \otimes \Lambda^6 T^*$ defined as [100]

$$q(\rho) = \big\langle e^m \wedge f_n \lrcorner \rho, \rho \big\rangle \big\langle e^n \wedge f_m \lrcorner \rho \wedge \rho \big\rangle \ , \tag{E.60}$$

where e^m is a basis of $T^*\mathcal{M}_6$ and f_m is a basis of $T\mathcal{M}_6$. The set of stable three-forms on $\mathcal{M}_6$ is then shown to be

$$U^3 = \big\{ \rho \in \Lambda^3 T^* : q(\rho) < 0 \big\} \ , \tag{E.61}$$

where $q(\rho) < 0$ holds if $q(\rho) = -s \otimes s$ for some $s \in \Lambda^6 T^*$. Clearly, since $\Lambda^6 T^* \cong \mathbb{R}$ this means that the product of the coefficients of the volume forms in (E.60) is negative.

It was shown in ref. [100] that each real stable form $\rho \in U^3$ is written as

$$\rho = \tfrac{1}{2}(\Omega + \bar{\Omega}) \ , \tag{E.62}$$

where Ω is a complex three-form satisfying $\big\langle \Omega, \bar{\Omega} \big\rangle \neq 0$. The imaginary part of Ω is unique up to ordering and we denote it by $\hat{\rho} = \mathrm{Im}(\Omega)$. The real three-forms $\hat{\rho}(\rho)$ can also be defined by using the map q introduced in eqn. (E.60). On forms $\rho \in U^3$ one defines the Hitchin function

$$\mathcal{H}(\rho) := \sqrt{-\tfrac{1}{3} q(\rho)} \quad \in \ \Lambda^6 TY \ , \tag{E.63}$$

which is well defined since $q(\chi) < 0$. The form $\hat{\rho}$ is then defined to be the Hamiltonian vector field on $TU^3 \cong \Lambda^3 T^*$ [3]

$$4 \big\langle \hat{\rho}, \alpha \big\rangle = -\mathcal{D}\mathcal{H}(\alpha) \ , \qquad \forall \alpha \ \in \ \Lambda^3 T^* \ , \tag{E.64}$$

[3] The factor 4 is not present in the corresponding expression in ref. [100]. It arises due to the fact that we have set $\rho = \mathrm{Re}(\Omega)$ and not $\rho_{\mathrm{Hitchin}} = 2\mathrm{Re}(\Omega)$ as in ref. [100]

where $\mathcal{D}$ is the (variational) differential on TU^3. Note that $H(\rho)$ can be rewritten as $\mathcal{H}(\rho) = i\langle \Omega, \bar{\Omega}\rangle$.

In this thesis we mostly use the integrated version of the Hitchin function $\mathcal{H}(\rho)$. Since $\mathcal{H}(\rho)$ is a volume form it is natural to define the Hitchin functional

$$H[\rho] = \int_{\mathcal{M}_6} \mathcal{H}(\rho) = i \int_{\mathcal{M}_6} \langle \Omega, \bar{\Omega} \rangle \ . \tag{E.65}$$

Its first (variational) derivative is precisely the form $\hat{\rho}$ such that

$$\partial_\rho H = -4\hat{\rho} \ , \qquad \qquad \partial_\rho H(\alpha) = -4 \int_{\mathcal{M}_6} \langle \hat{\rho}, \alpha \rangle \ . \tag{E.66}$$

Here we also displayed how $\partial_\rho H$ is evaluated on some real form $\alpha \in \Lambda^3 T^*$. The second derivative of $H[\rho]$ is given by

$$\partial_\rho \partial_\rho H = -4\mathcal{I} \ , \qquad \partial_\rho \partial_\rho H(\alpha, \beta) = -4 \int_{\mathcal{M}_6} \langle \alpha, \mathcal{I}\beta \rangle \ . \tag{E.67}$$

The map $\mathcal{I} : \Lambda^3 T^* \to \Lambda^3 T^*$ is shown to be an almost complex structure on U^3. It is used to prove that U^3 is actually a rigid special Kähler manifold [100]. The real form ρ can be also used to define an almost complex structure I_ρ on $\mathcal{M}_6$ itself by setting

$$(I_\rho)_n^m = \frac{1}{\mathcal{H}(\rho)} (e^m \wedge f_n \lrcorner \rho \wedge \rho) \ , \tag{E.68}$$

where $\mathcal{H}(\rho)$ is defined in eqn. (E.63). With respect to I_ρ one decomposes complex three-forms as

$$\Lambda^3 T^*_{\mathbb{C}} = \Lambda^{(3,0)} \oplus \Lambda^{(2,1)} \oplus \Lambda^{(1,2)} \oplus \Lambda^{(0,3)} \ . \tag{E.69}$$

Using this decomposition the complex structure $\mathcal{I}$ on U^3 is evaluated to be $\mathcal{I} = i$ on $\Lambda^{(3,0)} \oplus \Lambda^{(2,1)}$ and $\mathcal{I} = -i$ on $\Lambda^{(1,2)} \oplus \Lambda^{(0,3)}$. Furthermore, assuming that $\mathcal{M}_6$ possesses a metric which is hermitian with respect to I_ρ the six-dimensional Hodge-star obeys $*_6 = i$ on $\Lambda^{(0,3)} \oplus \Lambda^{(2,1)}$, while $*_6 = -i$ on $\Lambda^{(3,0)} \oplus \Lambda^{(1,2)}$. This implies the identifications

$$\mathcal{I} = *_6 \text{ on } \Lambda^{(2,1)} \oplus \Lambda^{(1,2)} \ , \qquad \mathcal{I} = - *_6 \text{ on } \Lambda^{(3,0)} \oplus \Lambda^{(0,3)} \ . \tag{E.70}$$

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