De Sitter Vacua and Inflation in no-scale String Models

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Abstract

This thesis studies the question of how de Sitter vacua and slow-roll inflation may be realized in string-motivated models. More specifically, we consider 4d $\mathcal{N} = 1$ supergravity theories (without vector multiplets) with Kähler potentials which are 'no-scale' at leading order. Such theories frequently arise in the moduli sector of string compactifications. We discuss a condition on the scalar geometry (defined by the Kähler potential) and on the direction of supersymmetry breaking in the scalar manifold, which has to be met in order for the average of the masses of the sGoldstinos to be positive, and hence for metastable vacua to be possible. This condition also turns out to be necessary for the existence of trajectories admitting slow-roll inflation. Its implications for certain scalar manifolds which arise from Calabi-Yau string compactifications are discussed. In particular, for two-moduli models arising from compactifications of heterotic- and type IIB string theory, a simple criterion on the intersection numbers needs to be satisfied for possible de Sitter phases to exist. In addition, we show that subleading corrections breaking the no-scale property may allow the condition on the scalar geometry to be fulfilled, even when it is violated at leading order. Finally, we develop a procedure to construct superpotentials for a given viable Kähler potential, such that the scalar potential has a realistic local minimum. We propose two-moduli models, with superpotentials which could arise from flux backgrounds and non-perturbative effects, which have a viable vacuum without employing subleading corrections or an uplifting sector.

Zusammenfassung

Diese Arbeit behandelt die Frage wie de Sitter Vakua und 'slow-roll' Inflation in stringtheoretisch motivierten Modellen realisiert werden können. Genauer gesagt betrachten wir 4d $\mathcal{N} = 1$ Supergravitations-Theorien (ohne Vektor Multiplets) mit Kähler Potentialen, die in führender Ordnung eine 'no-scale' Eigenschaft haben. Derartige Theorien treten häufig im Moduli-Sektor von String-Kompaktifizierungen auf. Wir diskutieren eine Bedingung an die skalare Geometrie (die durch das Kähler Potential definiert ist) und an die Richtung der Supersymmetrie-Brechung in der skalaren Mannigfaltigkeit, die erfüllt sein muss, damit der Durchschnitt der Massen der sGoldstinos positiv ist, und folglich damit metastabile Vakua möglich sind. Es zeigt sich, dass diese Bedingung auch für die Existenz von Trajektorien, die 'slow-roll' Inflation erlauben, notwendig ist. Die Auswirkungen dieser Bedingung auf bestimmte skalare Mannigfaltigkeiten, die in Calabi-Yau Kompaktifizierungen auftreten, werden diskutiert. Insbesondere muss für zwei-Moduli Modelle, die in Kompaktifizierungen von heterotischer- und Typ IIB String Theorie auftreten, ein einfaches Kriterium an die 'intersection numbers' erfüllt sein, damit de Sitter Zustände möglich sind. Außerdem zeigen wir, dass nachrangige Korrekturen, welche die no-scale Eigenschaft verletzen, es ermöglichen können die Bedingung zu erfüllen, auch falls sie in führender Ordnung verletzt ist. Schließlich entwickeln wir ein Verfahren, um Superpotentiale für ein vorgegebenes brauchbares Kähler Potential zu konstruieren, derart, dass das skalare Potential ein realistisches lokales Minimum aufweist. Wir schlagen zwei-Moduli Modelle vor, mit Superpotentialen die aus Hintergründen mit Flüssen und aus nicht-störungstheoretischen Effekten resultieren könnten, welche eine brauchbares Vakuum haben, ohne dass nachrangige Korrekturen oder ein 'uplifting' Sektor benutzt wird.

Contents

1 Introduction

Cosmological observations over the past two decades have significantly changed our view of the universe. One of the most prominent and important results of these efforts is the unexpected discovery of the accelerated expansion of the universe, based on observations of type Ia supernovae [1, 2]. This has since been independently confirmed by, for instance, measurements of the Cosmic Microwave Background (CMB) [3–6] and of the large-scale structure of the universe [7–9]. It is arguably fair to say that the most straightforward reason for cosmic acceleration is the presence of some form of dark energy, i.e. a tiny positive energy density filling the entire space (see, for instance, [10–13] for a recent review).* The previous idea that the vacuum energy – for some unknown reason – vanishes has been ruled out. While it is easy to model dark energy in an effective field theory – for instance by means of a tiny positive cosmological constant –, it is unknown why this parameter should be so much smaller than other dimensionful parameters of the theory. This 'cosmological constant problem' [14] – which is one of the biggest puzzles at the interface of high energy physics and cosmology – typically reappears in other disguises in alternative models for cosmic acceleration.

Furthermore, recent cosmological observations are in excellent agreement [3–6, 8, 9] with the predictions of slow-roll inflation [15, 16], which is another phase of cosmic acceleration (much faster than today's), that may have taken place in the very early universe (see e.g. [17–19] for a recent review). Inflation was originally 'invented' to account for the observed absence of magnetic monopoles. In addition, it serves as an explanation for the spatial homogeneity, isotropy (both on very large scales) and flatness of the universe, as verified by measurements of the CMB and large-scale structure. Finally – and perhaps most importantly – slow-roll inflation can also provide the 'seeds' for the creation of the structure of the universe [20–24]. More precisely, it generates primordial density perturbations which are (for the simplest models) almost scale-invariant, Gaussian and adiabatic. This is consistent with cosmological precision measurements. We thus have good reasons to believe that some form of slow-roll inflation indeed has taken place in the very early universe.† While it is easy to model inflation at the level of an effective field theory, a convincing more fundamental explanation is not agreed on.

*A different explanation could be the modification of the laws of gravity on large scales (see also e.g. [10–13] for a review).

†There are however also alternative scenarios that aim to solve the aforementioned problems, for instance the 'ekpyrotic model of the universe' [25].

On the theoretical side, significant progress has been made in the development and understanding of string theory over the past decades. Let us recall the basic principles: String theory is a quantum theory of relativistic strings (see for instance the textbooks [26–28] for an introduction). Compared to quantum theories of relativistic particles (such as the Standard Model of particle physics) the usual concept of quantum mechanics is kept, but point-like particles are replaced by one-dimensional strings.* Analogously, worldlines become worldsheets and Feynmann diagrams become worldsheets with a certain topology. This rather simple assumption that the most fundamental objects are strings rather than particles has remarkable consequences. One of them – arguably the primary reason why string theory has received so much attention – is that it can reconcile quantum field theory (QFT) and General Relativity (GR), i.e. string theory provides a theory of quantum gravity.

There are various reasons why such a theory should exist (see e.g. [30] for an extensive discussion). One reason is mostly aesthetical: The concept of unification of forces has proven to be very useful in the history of physics. One may thus expect that gravity and the three other forces are described by a single theory – and consequently gravity is also expected to be a quantum theory. A second reason is that under certain extreme conditions, such as in the very early universe, the effects of both QFT and GR are expected to become important.

Unfortunately, it seems to be very hard to make falsifiable predictions from string theory, in order to gain confidence that it is the correct quantum theory of gravity and UV completion of the Standard Model.† Actually, it is not even certain yet that string theory is consistent with the Standard Models of particle physics and cosmology. It is certainly an important task to show that known physics can be described by string theory.

The aim of this work is to make some further progress regarding the question of how dark energy and slow-roll inflation could be realized in string theory. More specifically, we employ the framework of 4d $\mathcal{N} = 1$ supergravity,

*As a side remark, note that there is however a crucial conceptual difference between string theories and relativistic quantum field theories. The former are 'first-quantized' in that the canonical commutation relations are applied to the coordinates (or 'string embedding functions') $X^M(\sigma, \tau)$ and the corresponding conjugate momenta. The latter are 'second-quantized' in that the canonical commutation relations are applied to the fields and the corresponding conjugate momenta. This has the consequence that string theory can only be defined perturbatively, whereas in QFT the perturbation series can be *derived*. There are however also theories of second-quantized strings, which are called string field theories (see e.g. [29] for a review).

†A recent discussion about prospects for testing string theory can for instance be found in Section 4 of Ref. [31].

which may be a low energy approximation of string theory. We will discuss conditions which have to be met by such theories in order for them to possibly include dark energy and slow-roll inflation. Using these conditions, we will also find examples for string-motivated models which include dark energy.

In Sect. 1.1, we remind the reader of a few basic facts of (aspects of) this area of research which are relevant for this work, while in Sect. 1.2 we give a brief introduction to slow-roll inflation. Readers to whom these topics are familiar may want to skip Sects. 1.1 and 1.2 and to immediately proceed to Sect. 1.3, where we give a more specific introduction to the present work and an outline of the thesis.

1.1 String model building

String theory and M-theory

String theories with fermions (the latter are obviously necessary for string theory to be potentially realistic), called 'superstring-theories', can be consistently formulated in up to 10 dimensions. (From now on, when we say 'string theory' we mean 'superstring-theory'.) There are five different string theories with a flat 10-dimensional (10d) background. These are called type IIA, type IIB, type I, heterotic $SO(32)$ and heterotic $E_8 \times E_8$. All of them have space-time supersymmetry (SUSY) – the former two have 32 supercharges while the latter three have 16 supercharges.

Clearly, string theory would lose some of its attractiveness if there are five different consistent theories rather than one unique theory. However, all five theories are linked, together with 11d supergravity, in a 'web of dualities' which suggests that they are different aspects of a single unified theory (see e.g. the review [32] and the references therein). Let us briefly summarize this situation: Type IIA and IIB, respectively the two heterotic theories, are connected by what is called T-duality. This means that the type IIA theory compactified on a circle of radius R gives the same theory as the IIB theory compactified on a circle of radius α'/R, where α' is the Regge-slope parameter (and analogous for the heterotic theories).* In an analogous way, S-duality is a weak coupling – strong coupling duality which relates a string theory at string coupling g_s with a string theory at coupling $1/g_s$. The type I theory is S-dual to the heterotic $SO(32)$ theory and the type IIB theory is S-dual to itself. By making use of S-duality, one thus can learn how these

*T-duality can also be extended to other compactifications. For instance, type IIA and type IIB compactified on a pair of 'mirror-CY manifolds' yield the same theory. This 'mirror symmetry' may be understood as a particular case of T-duality [33].

three theories behave at strong coupling.* On the other hand, the strong-coupling regime of the other two theories, type IIA and heterotic $E_8 \times E_8$, is believed to be described by what is called M-theory. By compactifying M-theory on a circle respectively on an interval of length $g_s\sqrt{\alpha'}$ one obtains the type IIA respectively heterotic $E_8 \times E_8$ theories. The low-energy limit of M-theory, on the other hand, is 11d supergravity. Taking into account also T- and S-duality, it thus seems plausible that there exists a single theory unifying the five string theories as well as 11d supergravity.

String compactifications

The first step towards making contact with phenomenology is to approximate each of the five string theories – for length scales which are much larger than the string length – by a 10d supergravity field theory. This can be done by calculating scattering amplitudes for the massless modes of the string spectrum and then writing down a 10d field theory Lagrangian which at tree-level yields the same scattering amplitudes.

The second step is to assume a 10d space-time background†

$$M_{10} = M_4 \times X \,, \tag{1.1}$$

where '×' denotes either a direct product or a warped product, M_4 is 'our' 4d non-compact maximally symmetric space-time and X is a 6d space which is small enough that these extra-dimensions are unobservable by means of present-day experiments and observations. While the extra dimensions are not directly observable, the topology of X determines the particle content, forces and symmetries of the 4d theory. Usually, one also requires that the 4d theory of the Kaluza-Klein zero-modes is supersymmetric, i.e. that supersymmetry is broken below the compactification scale (which is determined by the size of X). This is for phenomenological reasons (one would like SUSY to solve the gauge hierarchy problem) and also for 'practical' reasons (the resulting effective theory is under much better control), but (as far as is known today) not for fundamental reasons coming from string theory. For part of the 10d SUSY to be preserved in the 4d theory, X must allow for a globally well-defined nowhere vanishing spinor. Such spaces, by definition, have an $SU(3)$ structure group. If in addition this spinor is covariantly constant with

*Let us also mention 'F-theory' [34]: The self-S-duality of the type IIB theory can be extended to a larger $SL(2, \mathbf{Z})$-symmetry. This symmetry can be given a geometric meaning by understanding it as the modular group of a torus which is attached to each point of the 10d space. This is the basic idea of F-theory.

†The procedure is analogous for 11d supergravity, of course, but we focus on 10d supergravities here.

respect to the Levi-Civita connection, then X further has $SU(3)$ holonomy and is a Calabi-Yau (CY) manifold (see e.g. Ref. [35] for a review). Even restricting to SUSY-preserving compactifications leaves an enormous freedom for the choice of X however. Moreover, an explanation for why 6 of the 10 dimensions should be small at all is currently lacking.

Despite of these (so far) unanswered questions, one may nevertheless take compactification to a 4d space-time with SUSY broken below the compactification scale as an assumption and explore the phenomenological consequences of the 4d supersymmetric effective field theories which are obtained in this way – this is a common approach for doing 'string-phenomenology'. One problem that immediately arises is that the 4d field theories generically have many fields which are massless at tree level. These 'moduli' include for instance the dilaton, the geometric moduli, which parametrize the size and shape of the compact space, as well as axions originating from the Ramond-Ramond (RR) sector of the string spectrum. All moduli need to be 'made' massive in order not to be in conflict with observations (more precisely, massless scalars would lead to unobserved long-range forces and/or time-varying coupling constants). Furthermore, the moduli need to be stabilized at a point in field space where – for phenomenological reasons – supersymmetry is broken and the potential has the observed tiny positive value, and – for consistency – where the volume of X is large compared to the string length scale and the string coupling g_s is small. This is the problem of 'moduli stabilization'. To stabilize the moduli in a de Sitter vacuum (i.e. with a positive cosmological constant) is a very important and (perhaps at first sight surprisingly) difficult problem, on which an enormous amount of research has been done, particularly in the past several years (see for instance the recent reviews [31, 36–39] and the references therein). Here, we can just touch a selection of the challenges and possible solutions which were suggested.

To obtain massive moduli in a region in field space where the 4d effective theory is reliable is a major difficulty. For instance, it is hard to stabilize the moduli only with perturbative quantum corrections, which lift the flatness of the potential (this is sometimes referred to as the 'Dine-Seiberg problem' (cf. [40])). The reason is the following: In order not to have a runaway of the modulus to zero or to infinity, but instead to have a minimum of the potential at a finite field value, one obviously needs to balance at least first order and second order quantum corrections. If a higher order correction is of the same order of magnitude as the first order correction, one is (by definition) outside the regime of weak coupling – unless the coefficient of the higher order quantum correction is unnaturally large – and the effective action can no longer be trusted. From this argument, it would seem not unlikely that 'our vacuum' is in a strongly coupled regime and hardly accessible to calculations [40].

Fluxes and the 'landscape'

Fortunately however, it was discovered in the mid 90's that one can also obtain scalar potentials at tree level by 'turning on fluxes' [41,42]. The principal idea is to allow the various field strengths which arise in the 10d supergravity actions to assume non-zero background values. (In a higher dimensional space, this can be done in a way that preserves 4d Lorentz symmetry.) Due to a Dirac quantization condition, these 'fluxes' are integrally quantized and cannot be changed in a continuous way [41]. The gauge potentials for the field strengths include the Neveu-Schwarz (NS) two-form potential B_2 and the $(p+1)$-form potentials C_{p+1} that in addition arise in type II theories. If one compactifies on a space X which has a nontrivial $(p+2)$-cycle γ, the flux $\int_\gamma F_{p+2}$ (where locally $F_{p+2} = dC_{p+1}$) can be non-vanishing. Fluxes generate a potential for (at least some of) the moduli, because the 'F^2 term' depends on the metric of the compact space. An intuitive picture for this is that it costs energy to deform the space, since fluxes thread cycles in the compact geometry. Of course, the fluxes cannot be chosen completely arbitrary: The background fields need to solve the equations of motion and fulfill the Bianchi identities.

It has been argued that due to the presence of fluxes there exists an enormous number of vacua in string theory, so that the parameters (like the cosmological constant) of the effective 4d theory corresponding to each of them form a 'discretuum' of closely adjacent values [43]. The (simplified) argument is as follows: Assume that X has K different $(p+2)$-cycles $\gamma_i, (i \in 1, \ldots, K)$ around which fluxes $N_i = \int_{\gamma_i} F_{p+2}$ can thread. Then, the vacuum energy generically is of the form

$$V = V_0 + \sum_i c_i N_i^2 \,, \tag{1.2}$$

where the c_i are not unnaturally small or large coefficients and V_0 is assumed to be a large and negative contribution coming e.g. from orientifold-planes (O-planes) and/or quantum corrections. Further, due to Bianchi identities the fluxes are constrained by a condition of the form $\sum_i N_i^2 \leq L^2$. For large enough L, the number of vacua can be approximated by the volume of a K-dimensional ball with radius L, which is $(\sqrt{\pi}L)^K/\Gamma(1+K/2)$. The important point now is that K is usually of order hundreds in Calabi-Yau compactifications, so that the number of vacua can be very large. In case there are at least $\sim 10^{120}$ of them, one can expect – by arguing that the values of the vacuum energy are roughly uniformly distributed among them – that at least in a few of the vacua the cosmological constant assumes a value $\sim 10^{-120}$ in Planck units. The set of metastable vacuum states of string theory has been termed 'landscape' [44]. Systematic analyses of parts of the landscape have been done, starting with Ref. [45].

If this picture of a landscape of string vacua is true, it has implications on whether one should consider the fine-tuning of parameters (such as the cosmological constant) a problem, because the occurrence of very small parameters in an effective theory can then be explained without the need for fine-tuned parameters in the fundamental theory (see e.g. [46] for a discussion).* One could indeed speculate that there is no fundamental reason that our universe has such particular properties, in the same way as for instance there is no fundamental reason that our planet has such particular properties. String theory (together with 'eternal inflation' [47–50], see e.g. [51] for a review) would then allow for an 'anthropic selection' of the cosmological constant (see e.g. [52]). While this may be true, the major drawback of this idea is of course that we can only observe 'our universe' and it hence seems impossible to test this idea.

Explicit models

After this rather philosophical parenthesis, let us mention an important no-go theorem which restricts the construction of de Sitter (dS) vacua using fluxes only [53–55]. Note that this theorem has nothing to do with supersymmetry. The conditions of the theorem are as follows: One starts with a D-dimensional action (with $D > 2$) describing Einstein gravity minimally coupled to massless fields (scalars, p-forms) with positive kinetic terms and with a non-positive potential. Further, one takes a background of the form (1.1) (but for D dimensions) where in this case X is assumed to be a smooth compact $(D-d)$-dimensional space and the metric is

$$ds_D^2 = \Omega^2(y)\left(g_{\mu\nu}dx^\mu dx^\nu + \tilde{g}_{mn}dy^m dy^n\right) , \tag{1.3}$$

where $\Omega(y)$ is a warp factor, $g_{\mu\nu}$ is the metric of d-dimensional anti de Sitter-, Minkowski- or de Sitter space and $\tilde{g}_{mn}$ is the metric on X. The external part of the D-dimensional Einstein equations implies

$$\frac{d}{(D-2)\Omega^{D-2}}\tilde{\nabla}^2\Omega^{D-2} = \mathcal{R}(g_{\mu\nu}) + \Omega^2\,\hat{T} , \tag{1.4}$$

*Note that if this argument is correct there may also be no gauge hierarchy problem.

where

$$\hat{T} \equiv -T_\mu{}^\mu + \frac{d}{D-2} T_L{}^L \,, \tag{1.5}$$

and $\mathcal{R}(g_{\mu\nu})$ is the Ricci scalar computed from $g_{\mu\nu}$. It can be shown that the contribution of a negative potential and of fluxes to $\hat{T}$ is positive (except for one-form flux, where it vanishes). After multiplying (1.4) by $(\Omega^{D-2})^2$ and integrating by parts over the internal space, the left-hand side of (1.4) is $-d/(D-2) \int d^{(D-d)}y \sqrt{\tilde{g}}\, (\tilde{\nabla}\Omega^{D-2})^2$, which is non-positive. Since $\hat{T}$ is non-negative, it immediately follows that compactifications to dS space, where $\mathcal{R}(g_{\mu\nu}) > 0$, are excluded, while compactifications to Minkowski space are allowed only in case that there is only one-form flux and for constant Ω.

This no-go theorem means that it is not so simple to construct de Sitter vacua using fluxes as it may have appeared at first sight. However, string theory allows for various ways to circumvent the assumptions of this theorem, for instance by including localized energy sources (this violates the assumption that the compact space is smooth) and/or by taking into account corrections in α' or the string coupling g_s which arise in the 10d effective action (violating the assumption that there are no higher curvature corrections to the Einstein-Hilbert term in the higher-dimensional theory). For instance, it was shown by Giddings, Kachru and Polchinski [56] that in type IIB string theory compactified on Calabi-Yau orientifolds with D-branes wrapping around cycles and nontrivial background fluxes it is possible to obtain *warped* compactifications to Minkowski space. At the same time, many of the moduli present in the 4d $\mathcal{N} = 1$ supergravity are stabilized in this setup. Including in addition also non-perturbative effects, all moduli can be stabilized, but generically in a supersymmetric ground state which is either anti de Sitter (AdS) or Minkowski [57–60], whereas a positive cosmological constant necessarily requires the breaking of supersymmetry. With a combination of non-perturbative effects and α' corrections, it is also possible to obtain *non*-supersymmetric AdS vacua [61] (see also [62,63]). For the 'uplifting' from an AdS- or Minkowski vacuum to a dS vacuum, a variety of mechanisms has been proposed and studied. For example, it was shown by Kachru, Kallosh, Linde and Trivedi (KKLT) [57] that an explicit supersymmetry-breaking term induced by anti-D3 branes (together with non-perturbative effects) can lead to dS vacua with realistic cosmological constant and all moduli stabilized. In the past several years, many more possibilities for realizing dS vacua in string theory have been explored. (A little more about this will be mentioned below in Sect. 1.3.)

4d $\mathcal{N} = 1$ supergravity

Before we come back to models with dS vacua in Sect. 1.3, let us briefly summarize some aspects of 4d $\mathcal{N} = 1$ supergravity which are relevant for this work. This also serves to introduce our notation. For extensive reviews, see for instance Refs. [64,65] (see the references therein for the original literature on supersymmetry and supergravity). Note that this is independent of string theory and thus somewhat apart from the rest of this section.

Supergravity is, by definition, a field theory which is invariant under local supersymmetry transformations. It turns out that such a theory necessarily includes gravity, which explains the name. Apart from the supergravity multiplet (which consists of the graviton and the gravitino), a 4d $\mathcal{N} = 1$ supergravity theory may also contain chiral multiplets and vector multiplets. Throughout this thesis we consider only theories *without* vector multiplets. The theory is in this case completely determined by one real function $G(\Phi, \bar{\Phi})$ of the n chiral multiplets $\Phi^i = (\phi^i, \chi^i, F^i)$, $(i = 1, \ldots, n)$. Here, ϕ^i is a scalar field, χ^i is a Weyl-fermion and F^i is an auxiliary field. The Kähler function G can be decomposed in terms of a real Kähler potential K and a holomorphic superpotential W in the following way:*

$$G(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi}) + \log |W(\Phi)|^2 \,. \tag{1.6}$$

This splitting is however ambiguous, because K and W are defined only up to Kähler transformations $K \to K + f + \bar{f}$ and $W \to We^{-f}$, where f is an arbitrary holomorphic function of the chiral multiplets.

The bosonic part of the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}\mathcal{R} - g_{i\bar{\jmath}}\,\partial\phi^i \partial\bar{\phi}^{\bar{\jmath}} - V(\phi, \bar{\phi}) \,. \tag{1.7}$$

The first term is the Einstein-Hilbert term which will not play an important role in this thesis. The second term is the kinetic energy of the scalar fields. It is determined by the sigma-model metric $g_{i\bar{\jmath}}$, which – due to supersymmetry – is given by the Kähler metric $G_{i\bar{\jmath}} \equiv \partial_i \partial_{\bar{\jmath}} G \; (= \partial_i \partial_{\bar{\jmath}} K)$, which defines a Kähler geometry for the manifold spanned by the scalar fields.† The Kähler metric is used to raise and lower indices and needs to be positive definite, so that the kinetic energy of the scalar fields is positive. The last term in (1.7), the potential energy density, is – again due to supersymmetry – given by

$$V = e^G \big(G^i G_i - 3\big) \,. \tag{1.8}$$

*We always use Planck units where $\hbar = 1, c = 1, M_P \equiv (8\pi G_N)^{-\frac{1}{2}} = 1$.

†Derivatives with respect to Φ^i and $\bar{\Phi}^{\bar{\jmath}}$ are denoted by lower indices i and $\bar{\jmath}$.

The auxiliary fields of the chiral multiplets are fixed by their equations of motion to be $F^i = e^{G/2}G^i$. Supersymmetry is spontaneously broken if and only if $< F^i > \neq 0$. The Goldstone fermion (which is called 'Goldstino') corresponding to the spontaneous symmetry breaking is $\eta = G_i \chi^i$.* It is 'absorbed' by the gravitino in the super-Higgs effect. The mass of the gravitino is given by $m_{3/2} =< e^{G/2} >$. For the sake of brevity, we will for the rest of this work omit the brackets '$< \ldots >$' for expressions which are evaluated at a vacuum. It should always be clear from the context what is meant.

1.2 Slow-roll inflation

History of the theory of inflation

We start with giving a brief sketch of the development of theories of inflation. 'Inflation', by definition, is a period of (almost) exponential expansion of the universe. Such a theory has first been realized by Starobinsky in 1979/80 [66]. This model did however not attempt to solve the cosmological problems of homogeneity and isotropy, but rather assumed homogeneous and isotropic initial conditions. The first inflationary model which could solve the above cosmological problems was constructed by Guth in 1981 [67]. The main idea of his model, which is often called 'old inflation', is that a scalar field – trapped in a supercooled 'false vacuum' – provides a constant energy density leading to exponential expansion. Inflation ends by quantum-mechanical tunneling to the 'true vacuum'. The energy of the bubbles, which are formed in these (first-order) phase transitions, is almost entirely located in their walls. If inflation lasts long enough to solve the cosmological problems, the bubbles never meet (because the background out of which they arise inflates too fast), leading to distinct bubbles of empty universes. Thus, this model suffers from the 'graceful exit problem' and does not work.

In 1982, Linde [15] and Albrecht and Steinhardt [16] introduced the idea that inflation could instead be caused by a scalar field which slowly rolls down a potential (rather than a field trapped in a false vacuum). In these models, which were named 'new inflation', there is no graceful exit problem. Still, these models had a problem: they assumed a state of thermal equilibrium as initial condition for inflation.† This was solved in 1983 by Linde with the

*As a side remark, the name 'Goldstino' strictly speaking does not comply with the otherwise common nomenclature in supersymmetry, because the Goldstino is *not* the superpartner of a Goldstone field.

†Besides the problem that it is hard to realize thermal equilibrium due to the usually very small inflaton coupling, this is a problem since a mechanism to explain the homogeneity of the universe which relies on homogeneous initial conditions is not satisfactory.

theory of 'chaotic inflation' [68]. The key observation was that – provided the potential has a flat-enough region somewhere – inflation also occurs if a scalar field initially varies from place to place in a random way, i.e. if it has chaotic initial conditions. Essentially, this is because there will be some patches of space where the scalar field attains a value at which the potential is flat enough for inflation to last long enough.

Linde also noticed that even monomial potentials can be flat enough, provided that the scalar field value is much bigger than the Planck mass [68] (such a large field value makes it hard to make contact with particle physics however). It should be stressed though that the principle of chaotic initial conditions applies to any flat-enough potential, such as to a 'hilltop potential' (similar to which was used in new inflation) or to a hybrid inflation potential [69] (where the potential energy responsible for inflation comes from a different field than the slowly rolling field).*

Slow-roll background

Let us now briefly review some basic equations of slow-roll inflation. We start with giving the most general spatially homogeneous and isotropic metric, the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] . \tag{1.9}$$

Here, $a(t)$ is the scale factor and $k/a^2(t)$ is the inverse squared radius of spatial curvature. One further assumes an energy-momentum tensor which has the perfect fluid form, i.e.

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)u^\mu u^\nu , \tag{1.10}$$

where u^μ is a normalized four-velocity, p is the pressure and ρ is the energy density. The Einstein equations, which are in this case called Friedmann equations, then read

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} \tag{1.11}$$

$$\frac{\ddot{a}}{a} = -\frac{\rho+3p}{6} , \tag{1.12}$$

*Let us give two remarks on the terminology: (1) Often, inflationary models with a hilltop potential are called 'new inflation', even though they assume chaotic initial conditions. (2) The term 'chaotic inflation' often more specifically refers only to models with monomial potentials.

while the equation for energy momentum conservation reads

$$\dot{\rho} = -3H(\rho + p)\,. \tag{1.13}$$

Here, we have defined the Hubble parameter $H = \dot{a}/a$.

We now assume that the universe is filled with a single spatially homogeneous scalar field $\phi(t)$. In this case one has

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \tag{1.14}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V\,. \tag{1.15}$$

Then, Eq. (1.13) reads*

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0\,. \tag{1.16}$$

Almost exponential expansion requires $|\dot{H}|/H^2 \ll 1$. Using (1.11)† and (1.16), one easily finds $\dot{H} = -\dot{\phi}^2/2$, so that $|\dot{H}|/H^2 \ll 1$ requires

$$\dot{\phi}^2 \ll V \tag{1.17}$$

(which means $H^2 \simeq V/3$). This is the first slow-roll condition. One also demands a second slow-roll condition,

$$|\ddot{\phi}| \ll |H\dot{\phi}| \quad \text{and} \quad |\ddot{\phi}| \ll |V'|\,, \tag{1.18}$$

so that $3H\dot{\phi} \simeq -V'$. Using this relation, one finds that

$$\frac{\dot{H}}{H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2 \equiv \epsilon\,. \tag{1.19}$$

A necessary condition for slow-roll is thus $\epsilon \ll 1$. It is also easy to verify that in order for (1.18) to be true one needs to have [70]

$$|\eta| \ll 1 \qquad \text{where} \qquad \eta \equiv \frac{V''}{V}\,. \tag{1.20}$$

It should be stressed that the smallness of the flatness parameters ϵ and $|\eta|$ is a condition on the form of the potential only (and not a condition on the solutions to the field equations). Hence, this is only necessary but not sufficient for slow-roll, because $\dot{\phi}$ could be arbitrarily large even where ϵ

*The prime ′ denotes a derivative with respect to a canonically normalized field.

†Because any spatial curvature which may have been present before inflation is almost completely 'diluted' during inflation, one can neglect the term k/a^2.

and $|\eta|$ are small. Still, generic initial conditions are usually attracted to an inflationary phase if $\epsilon, |\eta| \ll 1$.*

Density perturbations

We would also like to summarize a few basic equations regarding the density perturbations originating from quantum fluctuations of the inflaton field. These relation are crucial because they allow to constrain and test models of inflation.

The spectrum of the perturbations of the inflaton is given by [72, 73]

$$P_\phi = \left(\frac{H}{2\pi}\right)^2 . \tag{1.21}$$

The inflation field perturbations in turn induce curvature perturbations† [20–24]

$$\delta_H(k) = -\frac{H}{\dot\phi}\delta\phi . \tag{1.22}$$

Using the Friedmann equation (1.11) and the slow-roll approximation $3H\dot\phi \simeq -V'$, one immediately obtains the famous result [21–24]

$$\frac{4}{25}P_{\mathcal{R}}(k) \equiv \delta_H^2(k) \simeq \frac{1}{150\pi^2}\frac{V}{\epsilon} . \tag{1.23}$$

Here, V and ϵ have to be evaluated at the epoch of horizon exit ($k = aH$) of the scale k. The observed value for the spectrum of curvature perturbations at the scale $k \simeq 7.5H_0$, which is often called 'CMB-constraint', is given by [6, 9]

$$\begin{aligned} & \delta_H(k)|_{k\simeq 7.5H_0} \simeq 1.9\times 10^{-5} \\ \Leftrightarrow \quad & V/\epsilon\big|_{k\simeq 7.5H_0} \simeq (6.6\times 10^{16}\mathrm{GeV})^4 . \end{aligned} \tag{1.24}$$

This means that the scale of inflation must be well below the Planck scale.

*One can instead of ϵ, η define slow-roll parameters ϵ_H, δ_H in terms of the Hubble parameter [71]:

$$\epsilon_H \equiv -\frac{\dot H}{H^2} , \qquad \delta_H \equiv -\frac{\ddot\phi}{H\dot\phi} .$$

This has the advantage that no other condition needs to be satisfied for the slow-roll approximations to be valid.

†The letter k used in the following equations should not be confused with that used in (1.9).

Besides the amplitude of the spectrum of curvature perturbations, its scale-dependence can also be measured. The latter is defined by the spectral index

$$n_s \equiv 1 + \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k} = 1 + 2\eta - 6\epsilon + \cdots . \tag{1.25}$$

Another quantity which is useful to constrain models is the tensor fraction

$$r \equiv \frac{P_T}{P_{\mathcal{R}}} = 16\epsilon + \cdots \tag{1.26}$$

where P_T is the spectrum of tensor perturbations. In the above two equations, the last equalities apply in the slow-roll approximation. Observationally, one has $r \lesssim 0.3$. Note that this limits the size of ϵ and hence the scale of inflation. Regarding the spectral index, the observed value is $n_s \simeq 0.95$ (if one assumes $r \ll 0.1$) [6,9].

Finally, the scale-dependence of tensor perturbations is defined by

$$n_T \equiv \frac{d \ln P_T(k)}{d \ln k} = -2\epsilon + \cdots . \tag{1.27}$$

The last equality is valid for single-field inflation models. In case n_T could be measured in the future and a significant deviation of this relation would be detected, one could conclude that multi-field effects are important.

1.3 Introduction to this thesis and outline

We now come to a more specific introduction and preview of this thesis. In Sect. 1.1, we mentioned the KKLT model, where the moduli are stabilized in a supersymmetric AdS vacuum which is then uplifted to a dS vacuum due to the presence of anti-D3 branes. From the low-energy perspective, this is an explicit SUSY breaking (see also e.g. Refs. [74, 75]). However, it is desirable to have SUSY broken *spontaneously* at low energies in order to have better control over the theory. A multitude of such models have been proposed in string-theoretical frameworks. One possibility is to consider an extra SUSY breaking sector, i.e. to include extra light degrees of freedom for the purpose of breaking SUSY spontaneously by D- or F-terms and for providing the uplift [76–94]. Another possibility to achieve de Sitter vacua is to employ subleading corrections in α' and/or g_s to the tree-level Kähler potential [95–99].

None of these models is of the type, however, where supersymmetry is broken spontaneously only by the Kähler moduli (or alternatively by the complex structure moduli) – i.e. without an extra uplifting sector – and also without subleading corrections to the Kähler potential. This is a bit surprising, at first sight, since the no-go theorem [53–55] mentioned in Sect. 1.1 can easily be circumvented by including localized sources, while the superpotentials available in flux compactifications seem to be sufficiently generic that one could expect no obstacle towards this end. Nevertheless, it was shown in Ref. [100] that for $\mathcal{N} = 1$ supergravities describing string compactifications with a single volume modulus T and a no-scale (defined by $K^i K_i = 3$) Kähler potential

$$K = -3 \log(T + \bar{T}) \tag{1.28}$$

(as used by KKLT), stationary points with positive potential energy V generated only by F-terms are always characterized by the existence of at least one tachyonic direction, independently of the superpotential $W = W(T)$.

It turns out that this is only a special case of a much more general no-go theorem. Namely, one can show that – for an arbitrary 4d $\mathcal{N} = 1$ supergravity theory – in order for the scalar potential to have local minima with F-term SUSY breaking, with a vacuum energy V and a gravitino mass $m_{3/2}$ – a necessary condition is that, in the direction $f^i \equiv G^i/\sqrt{G^k G_k}$ of SUSY breaking, the 'holomorphic sectional curvature' $R(f^i) \equiv R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}$ of the manifold spanned by the scalar fields must fulfill* [102–105]

$$R(f^i) < \frac{2}{3}\frac{1}{1+\gamma} \qquad \text{where} \qquad \gamma \equiv \frac{V}{3m_{3/2}^2}\,. \tag{1.29}$$

(This is only a preview to Sect. 2 where this theorem will be derived and explained in detail.) Note that this condition is independent of the no-go theorem [53–55] mentioned in Sect. 1.1. It is an additional restriction which has to be met for successful models. The result found in Ref. [100] follows from the general condition (1.29) since the sectional curvature calculated from Eq. (1.28) is 2/3 so that (1.29) can never be fulfilled for $V \geq 0$.

The condition (1.29) was applied in Ref. [102] to compactifications for which the Kähler geometry spanned by the moduli is factorized into submanifolds of constant curvature. More precisely, it was shown that also for the no-scale Kähler potential

$$K = -\sum_i n_i \log(T^i + \bar{T}^i) \qquad \text{with} \qquad \sum_i n_i = 3\,, \tag{1.30}$$

stationary points with $V > 0$ have at least one tachyonic direction (which can become marginally flat when $V = 0$), independently of W. In light of this

*A similar strategy has been used in Ref. [101] to explore the statistics of supersymmetry breaking vacua in certain classes of string models.

fact, it is instructive to consider some known results about the existence or non-existence of de Sitter vacua. First, one immediately sees that in the case where the dilaton S (with Kähler potential $K = -\log(S + \bar{S})$) dominates supersymmetry breaking, it is not possible to have a metastable minimum (except if there are large corrections to K, see e.g. [107]), as was already concluded in Ref. [100, 108]. Second, for models with a single overall volume modulus with $K = -3\log(T + \bar{T})$, the condition (1.29) is (as already mentioned above) violated, but only marginally (for $\gamma \ll 1$), so that small corrections violating the no-scale structure of K may help. The situation is the same [103] for $K = -3\log(T + \bar{T} - 1/3\sum_i |\Phi^i|^2)$, which for instance was considered in Ref. [109]. Finally, for canonical Kähler potentials, $K = \sum_i |\Phi^i|^2$, the scalar field space is flat, so that the condition (1.29) is always fulfilled and it is possible to build models with de Sitter or Minkowski vacua and broken supersymmetry. A simple example realizing this (which is hard to motivate from string theory however) is, for instance, the 'Polonyi model' [110], where $K = |\Phi|^2$ and the superpotential is of the form $W = \mu^2(\Phi + \beta)$. Choosing $\beta = 2 - \sqrt{3}$ leads to a non-supersymmetric vacuum at $\phi = \sqrt{3} - 1$ with vanishing vacuum energy and gravitino mass $m_{3/2} = \mu^2 e^{(2-\sqrt{3})}$.

We now turn to inflation. While much progress has been made on the issue of realizing slow-roll inflation in supergravity and string theory, a completely satisfactory model has remained elusive, the main reason for this being the difficulty of ensuring the flatness of the inflaton potential [111]. Similar to the construction of models possessing dS vacua, due to the large amount of freedom for achieving potentials in string compactifications, one may expect no obstacles for obtaining scalar potentials with flat directions. However, in early attempts to achieve inflation, it was already understood that there are actually severe restrictions towards this possibility, particularly for the identification of the inflaton within the moduli sector [112–114]. Nevertheless, many interesting models of 'modular inflation' have recently been constructed (see e.g. [115–127]). These models typically have an extra uplifting sector beyond the moduli, and/or make use of subleading corrections.

It would certainly be interesting to have at hand models where inflation is realized in the moduli sector of string compactifications, without an uplifting sector and without relying on subleading corrections. Progress has been made very recently towards an understanding of the origin of the difficulties associated with this [128, 129]. As a matter of fact, the problem of finding viable models of slow-roll inflation turns out to be closely related to the problem of finding metastable vacua [105]. In Sect. 2, we will give, based on Ref. [105], a more general characterization for the possibility (or impossibility, depending on the point of view) to realize models of slow-roll inflation: the inequality (1.29) has to be fulfilled approximately. The reason that a condition arises, which is approximately the same as that for the possibility of realizing dS vacua, is actually easy to understand: an inflationary trajectory roughly corresponds to an 'almost metastable' de Sitter state. The word 'almost' is the reason why (1.29) has to be fulfilled only up to corrections determined by the flatness parameters – this will be rendered more precise in Sect. 2.

In summary, the inequality (1.29) poses an important necessary condition for the possibility to implement de Sitter vacua as well as slow-roll inflation in $\mathcal{N} = 1$ supergravity models. In fact, as we will argue in Sect. 2, this is also a sufficient condition provided one has the freedom to fine-tune the superpotential in an arbitrary way.

Above, we have already stated that for multi-field no-scale Kähler potentials of the form (1.30) (which occur for instance in orbifold compactifications of string theory), this condition cannot be met, independently of the superpotential. It is then natural to ask if this is the case for all no-scale Kähler potentials. The answer is no, as we will show in Sect. 3. More precisely, we will analyze, based on Refs. [104, 130], a class of no-scale Kähler potentials arising from Calabi-Yau compactifications of string theory, where the Kähler geometry spanned by the moduli is 'more complicated'. We will nevertheless identify a surprisingly simple criterion (for models with two fields) which allows us to immediately decide if a given such Kähler potential allows for building models with dS vacua (and slow-roll inflation) or not. For those no-scale models where it turns out that (1.29) cannot be fulfilled, it would further be interesting to know under what conditions this can be circumvented by including subleading corrections breaking the no-scale property of the Kähler potential. Moreover, one would like to know if, when the latter turns out to be possible, the condition (1.29) nevertheless implies important general restrictions (such as upper bounds on masses or on the scale of inflation) for constructing models. This will be explored and answered affirmatively in Sect. 3.3.1.

Once one has at hand a criterion to decide which Kähler potentials from string compactifications can be used to build viable models, one would like to go ahead and actually do the latter, i.e. to find a superpotential such that the scalar potential indeed has a metastable vacuum with realistic vacuum energy and masses.* This is the topic of Sect. 4, which is based on Ref. [130]. To find such superpotentials, one could in principle make an ansatz for it, find the stationary points of the resulting scalar potential and then choose

*Building models which also have an inflationary phase is left for future research.

suitable parameters. Since it is in practice however extremely difficult to find non-supersymmetric stationary points, we will follow a more feasible route, namely to first specify field values where we wish the scalar potential to have a minimum and then to construct a superpotential which at these field values has the desired properties. We will describe how this can be accomplished in Sect. 4.1. In a second step, this 'local superpotential' can then be matched to a string-motivated one. This leads to several explicit examples, which we present in Sect. 4.2. Note that the Kähler potentials which we use for these examples can be considered as string-*derived*. For the superpotentials, on the other hand, we content ourselves with a form that is string-*motivated*. This may be justified due to the huge amount of freedom (from fluxes and non-perturbative effects) for the choice of the parameters of the superpotential.

Finally, our conclusions and an outlook to future research will be presented in Sect. 5. Note that, for the convenience of the reader, we will provide at the end of each of the main sections 2, 3 and 4 a brief summary of that section.

2 A stability condition in 4d $\mathcal{N} = 1$ supergravity

2.1 Condition for metastable vacua

The aim of this section is to derive a necessary condition on the geometry of the scalar manifold and on the SUSY breaking direction in the scalar manifold for the existence of metastable vacua [102–105]. By definition, metastable vacua are stationary points of the potential, which are stable against small fluctuations of the fields.* The latter is the case if and only if all masses are positive (for Minkowski and de Sitter space), respectively if and only if all masses are larger than the negative Breitenlohner-Freedman bound [131] (for anti de Sitter space).

In this work we only discuss Minkowski and de Sitter vacua. This is for phenomenological reasons: the universe is in a de Sitter state today and also is likely to have undergone a period of slow-roll inflation (corresponding to an 'almost stationary' de Sitter state) at very early times. It is straightforward to extend the condition we will find to AdS space, however [132]. Furthermore, we are interested in vacua with spontaneously broken SUSY (the latter is necessary for de Sitter vacua, but not for Minkowski vacua). In summary, we consider local minima of the potential at which $F^i \neq 0$ and $V \geq 0$.

We consider 4d $\mathcal{N} = 1$ supergravity theories with n chiral multiplets and no vector multiplets. There are n complex stationarity conditions which are derived by computing V_i. Since V is a scalar with respect to the Kähler geometry, one has $V_i = \nabla_i V$ where ∇_i is a covariant derivative. The covariant derivative of some vector field v_j is given by $\nabla_i v_j = \partial_i v_j - \Gamma^k{}_{ij} v_k$ and $\nabla_{\bar{\imath}} v_j = \partial_{\bar{\imath}} v_j - \Gamma^k{}_{\bar{\imath}j} v_k$. Here, $\Gamma^k{}_{ij} = \Gamma^k{}_{ji} = g^{k\bar{l}} \partial_i g_{j\bar{l}}$ (and analogous for the complex conjugates) are the only non-vanishing connection coefficients of the Kähler geometry (i.e. $\Gamma^k{}_{\bar{\imath}j} = 0$). (See Ref. [65] for more details on Kähler geometry.) From (1.8) one then immediately finds

$$V_i = e^G \left(G_i + G^k \nabla_i G_k \right) + G_i V = 0 \,. \tag{2.1}$$

The $2n$-dimensional mass matrix for scalar fluctuations around a vacuum

*We use the term 'metastable vacua' (as opposed to 'stable vacua'), since states in a *local* minimum could decay via quantum tunneling to a state with lower energy. The lifetime depends on the details of the action. Strictly speaking, one should only call a state metastable once one has found that the lifetime is larger than the age of the universe. However, we do not discuss this issue here.

takes the form

$$V_{I\bar{J}} \equiv \begin{pmatrix} V_{i\bar{j}} & V_{ij} \\ V_{\bar{i}\bar{j}} & V_{\bar{i}j} \end{pmatrix} , \tag{2.2}$$

where $I = (i, \bar{\imath})$ and $\bar{J} = (\bar{\jmath}, j)$. The second derivatives of the potential, $V_{i\bar{j}}$ and V_{ij}, may also be computed using covariant derivatives, i.e. $V_{i\bar{j}} = \nabla_i \nabla_{\bar{j}} V$ (since $\Gamma^k{}_{\bar{i}j} = 0$) and $V_{ij} = \nabla_i \nabla_j V$. The latter uses the stationarity, since $\nabla_i \nabla_j V = \nabla_i V_j = V_{ij} - \Gamma^k{}_{ij} V_k$. One easily finds*

$$\begin{aligned} \nabla_i \nabla_{\bar{j}} V &= e^G \left(g_{i\bar{j}} + \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}m\bar{n}}\, G^m G^{\bar{n}} \right) \\ &\quad + \left(g_{i\bar{j}} - G_i G_{\bar{j}} \right) V + 2G_{(i} \nabla_{\bar{j})} V , \end{aligned} \tag{2.3}$$

$$\begin{aligned} \nabla_i \nabla_j V &= e^G \left(2\nabla_{(i} G_{j)} + G^k \nabla_{(i} \nabla_{j)} G_k \right) \\ &\quad + \left(\nabla_{(i} G_{j)} - G_i G_j \right) V + 2G_{(i} \nabla_{j)} V , \end{aligned} \tag{2.4}$$

where the last terms in (2.3) and (2.4) of course vanish at stationary points. In (2.3), $R_{i\bar{j}m\bar{n}}$ is the Riemann tensor of the scalar geometry (see Appendix A for our conventions). A necessary and sufficient condition for metastability is the requirement that the $2n$-dimensional mass matrix (2.2) should be positive definite.† It is known from elementary linear algebra that a matrix is positive definite if and only if all of its principal submatrices are positive definite (a $p \times p$ matrix P is a principal submatrix of a $q \times q$ matrix Q, if one obtains P by removing any $q - p$ rows of Q and the same $q - p$ columns of Q). Hence, a necessary condition for positive-definiteness of $V_{I\bar{J}}$ is the positive-definiteness of its upper-left block $V_{i\bar{j}}$. By definition, this means that $z^i V_{i\bar{j}} \bar{z}^{\bar{j}}$ must be positive for all non-vanishing complex vectors z^i.

In particular, $z^i V_{i\bar{j}} \bar{z}^{\bar{j}} > 0$ must hold for the special vector $z^i = f^i$, where

$$f^i \equiv \frac{G^i}{\sqrt{G^k G_k}} \tag{2.5}$$

is the normalized Goldstino vector, i.e. the direction of SUSY breaking in the space of chiral fermions. We then finally get the metastability condition

$$m^2 \equiv f^i V_{i\bar{j}} f^{\bar{j}} > 0 \, . \tag{2.6}$$

*The round brackets around tensor indices denote, as usual, symmetrization of indices:

$$T^{\cdots}{}_{(\mu_1 \ldots \mu_p) \ldots} = \frac{1}{p!} (T^{\cdots}{}_{\mu_1 \ldots \mu_p \ldots} + \text{sum over all permutations of indices } \mu_1 \ldots \mu_p) \, .$$

The prefactor $1/p!$ is chosen such that for symmetric tensors one has $T_{(\mu_1 \ldots \mu_p)} = T_{\mu_1 \ldots \mu_p}$.

†Note that the eigenvalues of $V_{I\bar{J}}$ are not the canonically normalized squared masses if the kinetic term is not canonical. In order to determine the canonical masses, one has to perform a field-redefinition $\phi^i \to \phi^i e_i{}^m$, where $e_i{}^m$ is (the vev of) the holomorphic n-bein of the scalar geometry, i.e. $g_{i\bar{j}} = e_i{}^m e_{\bar{j}}{}^{\bar{n}} \eta_{m\bar{n}}$. Because the kinetic term is positive definite, this field redefinition does not affect the metastability condition however.

One can find a more explicit expression for m^2. Using Eqs. (2.1) and (2.3), one obtains

$$m^2 = \left[2 - 3(1+\gamma)R(f^i)\right] m_{3/2}^2\,, \tag{2.7}$$

where we defined

$$R(f^i) \equiv R_{i\bar{\jmath}m\bar{n}} f^i f^{\bar{\jmath}} f^m f^{\bar{n}} \tag{2.8}$$

and

$$\gamma \equiv \frac{V}{3m_{3/2}^2}\,. \tag{2.9}$$

The quantity $R(f^i)$ actually has a geometrical interpretation: it is the holomorphic sectional curvature of the scalar manifold along the direction f^i. $R(f^i)$ is determined by the Riemann-tensor $R_{i\bar{\jmath}m\bar{n}}$ of the scalar manifold and by the SUSY breaking direction f^i. The condition $m^2 > 0$ implies then the constraint [102–105]

$$R(f^i) < \frac{2}{3}\frac{1}{1+\gamma}\,. \tag{2.10}$$

Note that the larger γ, the harder it is to fulfill this condition.

Note also that in the particular case of only a single chiral multiplet, this condition also follows from the supertrace

$$\mathrm{STr}\mathcal{M}^2 \equiv \sum_{spins\ J} (-1)^{2J}(2J+1)\mathrm{Tr}\,\mathcal{M}_J^2 \tag{2.11}$$

of the mass matrix, which (for n chiral multiplets) is found to be [133, 134]

$$\mathrm{STr}\mathcal{M}^2 = \left[2(n-1)(1+3\gamma) - 8\gamma^2 - 6(1+\gamma)R_{i\bar{\jmath}}f^i f^{\bar{\jmath}}\right] m_{3/2}^2\,. \tag{2.12}$$

To check that for $n = 1$ the condition $\mathrm{Tr}\,\mathcal{M}_{J=0}^2 > 0$ is equivalent to (2.10), one needs to use that the mass of the spin 1/2 particle in (2.11), the Goldstino, is $m_\eta = -2\gamma m_{3/2}$.*

Before closing this section, we would like to explain in more physical terms why the requirement that the projection of $V_{i\bar{\jmath}}$ along the direction f^i (and not some other direction) should be positive is a particularly interesting condition. First, we show how m^2 ($\equiv f^i V_{i\bar{\jmath}} f^{\bar{\jmath}}$) is related to the masses of the two real components of the complex scalar $\tilde{\eta} = G_i\phi^i$ (called 'sGoldstino') in the Goldstino multiplet. To this end, it is useful to define the $2n$-dimensional unit vector

$$f^I_{(\alpha)} \equiv (e^{i\alpha}f^i, e^{-i\alpha}f^{\bar{\imath}})/\sqrt{2}\,, \tag{2.13}$$

*Indeed, using the stationarity condition (2.1) one immediately finds that the mass matrix for the Weyl-fermions χ^i (see [135]) has an eigenvalue m_η with eigenvector proportional to G^i.

where α is an angle. Choosing the two values (say) $\alpha = 0$ and $\alpha = \pi/2$ in $f^I_{(\alpha)}$ defines two orthogonal real sGoldstino directions. The squared masses of these two sGoldstino fields are then given by $m^2_{(\alpha)} \equiv f^I_{(\alpha)} V_{I\bar{J}} \bar{f}^{\bar{J}}_{(\alpha)}$, for $\alpha = 0$ respectively $\alpha = \pi/2$. Now, using (2.2) and the definition of $f^I_{(\alpha)}$ one immediately sees that

$$m^2_{(\alpha)} = m^2 + \mathrm{Re}\{e^{2i\alpha} f^i V_{ij} f^j\} \,. \tag{2.14}$$

Since $(m^2_{(0)} + m^2_{(\pi/2)})/2 = m^2$, m^2 is the average (the arithmetic mean) of the two sGoldstino squared masses [132]. If $V_{ij} = 0$, then both sGoldstinos have a squared mass m^2, otherwise the masses split and m^2 is an upper bound for one of them.*

The crucial point now is that the Goldstino multiplet cannot receive a supersymmetric mass, since in the limit of rigid supersymmetry the fermionic component must be massless. This means that – if the squared masses which the sGoldstinos receive from supersymmetry breaking turn out negative – one cannot render them positive by simply adding supersymmetric mass terms. This is reflected in the expression (2.7) by the fact that the average sGoldstino squared mass m^2 depends on the superpotential only via the vector f^i (which defines the SUSY breaking direction) and the gravitino mass (which for Minkowski vacua is an order parameter of SUSY breaking).†

By contrast, the masses of the remaining $(2n-2)$ real scalar fields can be tuned positive (and in fact arbitrarily large) by a suitable choice of the superpotential. This can be seen from the mass matrix (2.2). Recall that G_i, $\nabla_i G_j$ and $\nabla_i \nabla_j G_k$ depend, respectively, on $(\log W)_i$, $(\log W)_{ij}$ and $(\log W)_{ijk}$ and hence – for a given fixed K – can be tuned to any desired values. For instance, one can first choose $\nabla_i \nabla_j G_k$ such that the elements of V_{ij} are zero. Then, one just has to make sure that the block $V_{i\bar{j}}$ is positive definite. This can be achieved by tuning $\nabla_i G_j$, which occurs in $V_{i\bar{j}}$ via the positive semidefinite part $e^G \nabla_i G_k \nabla_{\bar{j}} G^k$ of $V_{i\bar{j}}$. While $f^k \nabla_i G_k$ – and hence $f^i e^G \nabla_i G_k \nabla_{\bar{j}} G^k f^{\bar{j}}$ –

*Notice that even when $V_{ij} = 0$ and hence $m^2_{(\alpha)} = m^2$, the Goldstino direction f_j is in general not an eigenvalue of $V_i^{\ j}$. Interestingly, it turns out, however, that the 'optimal direction' f_{0j} which – for some given vev – extremizes m^2, is always an eigenvector of $V_i^{\ j}$ with eigenvalue m_0^2 [130]. To show this, one has to vary m^2 with respect to f_i, enforcing $f^i f_i = 1$ with a Lagrange multiplier. This gives an implicit equation for f_{0j}. Using this and the stationarity condition (2.1) one arrives at $V_i^{\ j} f_{0i} = m_0^2 f_{0i}$. The direction which maximizes m^2 will be determined in Sect. 3.2 for the particular scalar geometries discussed in that Section.

†As mentioned before, in supergravity, the (would-be) Goldstino, which is absorbed by the gravitino in the super Higgs effect, has a in general non-vanishing mass parameter $m_\eta = -2\gamma m_{3/2}$. The fact that the sGoldstinos cannot have supersymmetric mass contributions is however unaffected by this.

is constrained by the stationarity condition (2.1), the projection along directions perpendicular to f^i – and hence all masses but the two sGoldstino masses – can be tuned to any desired values by a suitable choice of the superpotential. How this can be done in practice will be discussed in some detail in Sect. 4, where we will also present explicit two-field models (with string-motivated superpotentials) where all moduli are stabilized.

2.2 Condition for slow-roll inflation

In this section we show that in order for viable models of slow-roll inflation to exist, the condition Eq. (2.10) also needs to be fulfilled, up to corrections (which will be made more precise below) which are suppressed by the flatness parameters of the potential: [105]

$$R(f^i) \lesssim \frac{2}{3}\frac{1}{1+\gamma}\,. \tag{2.15}$$

Notice the meaning of the parameter γ in terms of the Hubble parameter during inflation: Recalling from Sect. 1.2 that $H^2 \simeq V/3$, one has

$$\gamma \equiv \frac{V}{3m_{3/2}^2} \simeq \frac{H^2}{m_{3/2}^2}\,. \tag{2.16}$$

Here, $m_{3/2}$ is the gravitino mass during inflation, which in general differs from the gravitino mass today.

Even though (2.15) is of course valid for any positive value of the parameter γ, for generic and realistic slow-roll inflation models it is natural to have $\gamma \gg 1$. The reason is that H should be much larger than the weak scale, while $m_{3/2}$ is expected to be not much larger than the weak scale. Of course, the gravitino mass during inflation is not identical to that today (which should be not much higher than the weak scale in order for SUSY to solve the hierarchy problem), so that one could in principle also build models with a large $m_{3/2}$. However, it is a reasonable assumption that the gravitino masses during inflation and today are at least roughly of the same order of magnitude in generic models (i.e. without fine-tuning). Then, one indeed expects $H \gg m_{3/2}$ (i.e. $\gamma \gg 1$) for phenomenologically viable inflationary models. This means that the condition on the sectional curvature is more restrictive for achieving realistic inflationary models than it is for achieving realistic vacua (where $\gamma \ll 1$).

We now proceed with proving (2.15). In essence, the reason why (2.10) has to be approximately fulfilled in order for inflation to be possible is simply that an inflationary phase is similar to a metastable vacuum, the difference being that (i) an inflationary trajectory corresponds to 'almost' (rather than exactly) stationary points and that (ii) one of the squared masses may be slightly negative. We will now discuss this quantitatively and give a detailed derivation of (2.15).

For a successful inflationary model, the potential should be sufficiently flat in order for inflation to last long enough to produce our spatially flat, homogeneous and isotropic universe. For models with a single real scalar field, as discussed in Sect. 1.2, this necessitates the smallness of the flatness parameters $\epsilon = 1/2(V'/V)^2$ and $\eta = V''/V$. In case of several fields, the flatness parameters can be generalized to (cf. for instance [136, 137])*

$$\begin{aligned} \epsilon &= \frac{\nabla^i V \nabla_i V}{V^2} , && (2.17) \\ \eta &= \text{min eigenvalue}\,\{N\} . && (2.18) \end{aligned}$$

Here, the $(2n \times 2n)$-matrix N is defined as

$$N^I{}_J = \frac{L^{I\bar{P}} V_{\bar{P}J}}{V} = \frac{1}{V} \begin{pmatrix} \nabla^i \nabla_j V & \nabla^i \nabla_{\bar{\jmath}} V \\ \nabla^{\bar{\imath}} \nabla_j V & \nabla^{\bar{\imath}} \nabla_{\bar{\jmath}} V \end{pmatrix} , \tag{2.19}$$

where we used

$$L_{I\bar{J}} \equiv \begin{pmatrix} g_{i\bar{\jmath}} & 0 \\ 0 & g_{\bar{\imath}j} \end{pmatrix} , \tag{2.20}$$

respectively its inverse $L^{\bar{J}I}$, to raise indices.

Next, notice that for any unit vector u^I one has†

$$\eta \leq u_I N^I{}_J u^J . \tag{2.21}$$

*A more rigorous and very general characterization of slow-roll conditions for multi-field models with noncanonical kinetic terms can be found in Ref. [138]. In particular, a distinction between dynamical effects parallel and perpendicular to the inflaton's trajectory (which at leading order in the slow-roll approximation is $\propto \nabla_I V$) can be made. Instead of the single-field parameter η, one then has two parameters $\eta_{||}$ and $\eta_\perp$, where $\eta_{||}$ roughly corresponds to the projection of N along the direction $\nabla_I V$ of the inflationary trajectory and $\eta_\perp$ corresponds to those elements of N mixing the vector $\propto \nabla_I V$ with the normal vector relative to the inflaton's trajectory. Since $\eta_\perp$ vanishes in the single-field case, it is a measure of the multi-field effects.

In any case, the condition $|\eta| \ll 1$ (for η as defined in (2.18)) is a necessary condition, because it is not stronger than demanding $|\eta_{||}| \ll 1$ (due to $\eta \leq \eta_{||}$ which follows from (2.21)). Actually, it is also not much weaker than demanding $|\eta_{||}| \ll 1$: In order for isocurvature perturbations to be not too large (which is a phenomenological requirement), the projection of N along directions perpendicular to $\nabla_I V$ should be much larger than $\eta_{||}$. This means that $\eta \simeq \eta_{||}$, as contributions to η coming from projecting N along directions perpendicular to $\nabla_I V$ have to be suppressed.

†Indeed, one can always decompose u^I as $u^I = \sum_k c_{(k)} \omega^I_{(k)}$, where the $\omega^I_{(k)}$'s represent

In particular, one may choose the sGoldstino direction, i.e. $u^I = f^I_{(\alpha)}$ (see (2.13)). This yields, in analogy to (2.14),

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} + \mathrm{Re}\left\{ e^{2i\alpha} \frac{\nabla_i \nabla_j V}{V} f^i f^j \right\} . \qquad (2.22)$$

Furthermore, one can average over two orthogonal sGoldstino directions (e.g. $\alpha = 0$ and $\alpha = \pi/2$) which results in the weaker (but simpler) bound

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} . \qquad (2.23)$$

This inequality should be contrasted with (2.6). Using Eq. (2.3), one finds after some algebra that

$$\begin{aligned} \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} &= \left[2 - 3(1+\gamma) R(f^i) \right] \frac{1}{3\gamma} \\ &\quad + \frac{4}{\sqrt{3(1+\gamma)}} \mathrm{Re}\left\{ \frac{\nabla_i V}{V} f^i \right\} + \frac{\gamma}{1+\gamma} \frac{\nabla^i V \nabla_i V}{V^2} . \end{aligned} \qquad (2.24)$$

The terms in the second line, which do not occur in the corresponding Eq. (2.7), are due to the non-vanishing of the first derivatives of the potential. Also, note that the factor $1/(3\gamma)$, multiplying the rectangular bracket in the first line, comes just from the factor $1/V$ in the definition of η (compared to the definition of m^2). Since f^i is a unit vector, it is clear from the definition of ϵ that $|f^i \nabla_i V/V| \leq \sqrt{\epsilon}$. This inequality, together with (2.23) and Eq. (2.24), then implies*

$$R(f^i) \leq \frac{2}{3} \frac{1}{1+\gamma} + \frac{4}{\sqrt{3}} \frac{\gamma}{(1+\gamma)^{\frac{3}{2}}} \sqrt{\epsilon} + \left(\frac{\gamma}{1+\gamma} \right)^2 \epsilon - \frac{\gamma}{1+\gamma} \eta . \qquad (2.25)$$

We now see quantitatively what was meant with the '$\lesssim$'-symbol in (2.15), namely the disregard of the second, third and fourth terms of the right-hand side of (2.25). Neglecting these terms is certainly justified for slow-roll inflation where $\epsilon \ll 1$, $|\eta| \ll 1$. Note that for $\gamma \ll 1$, these three terms are not only small because of the smallness of the flatness parameters, but they are also suppressed by the smallness of γ. For the (more realistic) case

a basis of orthonormal eigenvectors of N with eigenvalues $\lambda_{(k)}$. Since the u^I's are unit vectors, the coefficients $c_{(k)}$ satisfy $\sum_k |c_{(k)}|^2 = 1$ and so it immediately follows that $u_I N^I{}_J u^J = \sum_k |c_{(k)}|^2 \lambda_{(k)} \geq \min\{\lambda_{(k)}\} = \eta$.

*As a check, one immediately sees that for $\epsilon = \eta = 0$ (and replacing $\leq \to <$ so that all masses are positive and not only nonnegative), one reobtains the condition (2.10) for the existence of metastable vacua.

$\gamma \gg 1$, on the other hand, the second term is in addition suppressed by a factor $1/\sqrt{\gamma}$. Denoting by R_{max} the right-hand side of the inequality (2.25), we have for different regimes of γ

$$R_{max} = \begin{cases} \frac{2}{3} + \mathcal{O}(\gamma) & \text{for} \quad \gamma \ll 1 \\ \frac{2}{3}\frac{1}{1+\gamma} + \mathcal{O}(\max\{\sqrt{\epsilon}, \eta\}) & \text{for} \quad \gamma \sim 1 \\ 0 + \mathcal{O}(\max\{\epsilon, \eta, 1/\gamma, \sqrt{\epsilon/\gamma}\}) & \text{for} \quad \gamma \gg 1 \end{cases} \tag{2.26}$$

Notice also that the corrections coming from the non-vanishing of the slow-roll parameters are positive (i.e. they make the condition slightly milder) as long as $\eta < 0$. Using for instance a canonical Kähler potential for which $R(f^i) = 0$, it is thus always possible to build viable models with $\eta < 0$.

Summary of Section 2:

The main results are the inequalities (2.10) and (2.25) (actually, (2.10) can be considered a special case of (2.25)). Both are conditions on $R(f^i)$, i.e. the sectional curvature of the manifold spanned by the scalar fields along the Goldstino direction; (2.10) is necessary for metastable vacua with non-negative energy density $V = 3\gamma m_{3/2}^2$ to exist, while (2.25) is necessary for inflationary trajectories with Hubble parameter $H^2 = \gamma m_{3/2}^2$ to exist. We have explained that these conditions, which are valid for 4d supergravity theories without vector multiplets, are equivalent to the requirement that the average of the squared masses of the sGoldstinos is positive (respectively only slightly negative in case of inflation). We have also argued that the squared masses of all remaining scalars can be made arbitrarily large by a tuning of the superpotential.

3 Implications for scalar manifolds from string theory

The objective of this Section is the computation of $R(f^i)$ for certain 'interesting' classes of scalar geometries. $R(f^i)$ depends on the point in the scalar manifold (i.e. on the values of the moduli) and on the direction f^i, which in turn, for a given K, depends on the parameters of the superpotential as well as on the values of the moduli. While $R(f^i)$ is of course not constant in general, it turns out that in some cases the range of $R(f^i)$ is restricted. More precisely, for certain scalar geometries one finds that $R(f^i) \geq 2/3$, so that both de Sitter vacua and inflation are excluded (and this cannot be remedied by any fine-tuning of the superpotential). We will show that this property holds for particular scalar manifolds which are obtained from string compactifications (in the large-volume- respectively large-complex-structure- and weak-coupling-limit). In Sect. 3.3, in turn, we will show that this may be circumvented by subleading corrections, which slightly modify the scalar geometry.

3.1 No-scale Kähler potentials

Before we come to specific scalar manifolds from string compactifications, let us first consider the general class of Kähler manifolds (of which the scalar manifolds from string compactifications which we will consider in Sect. 3.2 are a subclass) defined by no-scale Kähler potentials. The latter are characterized by the property [139]

$$K^i K_i = 3\,. \tag{3.1}$$

As will be reviewed within Sect. 3.2, such Kähler potentials arise for the geometric moduli of both heterotic and IIB string compactifications at leading order in the perturbative expansion. In this section, we will discuss some interesting properties of $R(f^i)$ which are valid for no-scale Kähler potentials *in general*. First, we consider completely arbitrary no-scale Kähler potentials. Then, in addition to the no-scale Kähler property, we will impose also a 'shift symmetry' (which also occurs in string-derived Kähler potentials, as we will see in Sect. 3.2). Finally, we will further specify to the particular case of models with only two fields. The results we will find here will be very useful later on.

For arbitrary no-scale Kähler potentials, the property (3.1) implies

$$R_{i\bar{\jmath}m\bar{n}} K^i K^{\bar{\jmath}} K^m K^{\bar{n}} = 6\,, \tag{3.2}$$

$$R_{i\bar{\jmath}m\bar{n}} K^{\bar{\jmath}} K^m K^{\bar{n}} = 2K_i\,, \tag{3.3}$$

as one finds by taking two derivatives $\partial_i \partial_{\bar{\jmath}}$ of Eq. (3.1) and contracting the result with $K^i K^{\bar{\jmath}}$ respectively $K^{\bar{\jmath}}$. From Eq. (3.2) we see that

$$R(k^i) = \frac{2}{3}\,, \tag{3.4}$$

where we have defined the normalized vector

$$k^i \equiv \frac{K^i}{\sqrt{K^j K_j}} = \frac{K^i}{\sqrt{3}}\,. \tag{3.5}$$

Recall that 2/3 is the critical value to violate the inequality (2.10) (respectively (2.15)) in the case $\gamma = 0$.

Furthermore, from (3.3) it follows that $R(f^i)$ is stationary at $f^i = k^i$, i.e.

$$\frac{\delta}{\delta f^i} R(f^i)|_{f^i = k^i} = 0\,. \tag{3.6}$$

To verify this (and also for later purposes), it is useful to decompose the vector f^i into a part which is parallel to k^i and a part which is orthogonal to k^i:

$$f^i = e^{i\varphi}\left(\sin\chi\, k^i + e^{i\phi}\cos\chi\, n^i\right)\,, \qquad f_i = e^{-i\varphi}\left(\sin\chi\, k_i + e^{-i\phi}\cos\chi\, n_i\right)\,,$$
$$f^{\bar{\imath}} = e^{-i\varphi}\left(\sin\chi\, k^i + e^{-i\phi}\cos\chi\, n^{\bar{\imath}}\right)\,, \quad f_{\bar{\imath}} = e^{i\varphi}\left(\sin\chi\, k_i + e^{i\phi}\cos\chi\, n_{\bar{\imath}}\right)\,. \tag{3.7}$$

Here, we have $n^i n_i = 1$ and $n^i k_i = 0$ by definition, and χ, φ, ϕ are angles.* Notice that the term in $R(f^i)$ which is linear in n^i is proportional to $R_{i\bar{\jmath}m\bar{n}} n^i k^{\bar{\jmath}} k^m k^{\bar{n}}$ and hence vanishes by Eq. (3.3) and $n^i k_i = 0$. This means that $R(f^i)$ is indeed stationary at $f^i = k^i$.

To decide if (2.10) respectively (2.15) can be satisfied for some f^i or not, we need to know if the stationary point $f^i = k^i$ is a minimum, a maximum or a saddle point (i.e. the convexity of $R(f^i)$ at $f^i = k^i$). De Sitter vacua and inflation are excluded if it is a minimum. Otherwise, they are possible, given a suitable superpotential. It turns out, however, that the convexity is *not* determined by the no-scale property. In the following paragraphs, we can at least show that $R(f^i)$ either has a saddle point or a minimum if one in addition has a shift-symmetry.

Let us then make the further assumption, besides the no-scale property, that K depends only on $\Phi^i + \bar{\Phi}^i$, but is independent of the imaginary part

*For the sake of brevity, we will set the overall phase φ to zero in most equations. We have actually already done this for (3.4) and (3.6), which are also true if there is an additional phase $k^i \to e^{i\varphi} k^i$.

of Φ^i. (Such a shift-symmetry is actually realized for the string-derived no-scale Kähler potentials which will be discussed in Sect. 3.2.) In this case, we can drop the distinction between holomorphic and antiholomorphic indices in quantities deduced only from K. However, for quantities which depend also on the superpotential (such as f_i, G_i or n_i), this distinction *cannot* simply be dropped, because such quantities are not real. In order to get rid of indices with bars also for these quantities, we introduce the following new notation by making the replacements

$$f_i \to f_i, \qquad f_{\bar{\imath}} \to \bar{f}_i, \qquad f^i \to \bar{f}^i, \qquad f^{\bar{\imath}} \to f^i\,, \tag{3.8}$$

and analogously for G_i, n_i etc.

For no-scale Kähler potentials with shift symmetry, there exists a special coordinate frame in which e^{-K} is a homogeneous function of degree 3 in $\Phi^i + \bar{\Phi}^i$. This means that $(\Phi^i + \bar{\Phi}^i)\frac{d}{d\Phi^i} e^{-K} = 3e^{-K}$ which immediately implies

$$-\big(\Phi^i + \bar{\Phi}^i\big) K_i = 3\,. \tag{3.9}$$

Taking a derivative of (3.9), it follows that

$$K^i = -\big(\Phi^i + \bar{\Phi}^i\big)\,. \tag{3.10}$$

Note that this equation, together with (3.9), implies the no-scale property (3.1). Finally, taking two more derivatives and contracting with K^i's, one finds that

$$R_{ijmn}K^m = K_{ijn}\,, \tag{3.11}$$

$$R_{ijmn}K^m K^n = R_{imjn}K^m K^n = 2\,g_{ij}\,. \tag{3.12}$$

As we now discuss, using the above expressions one finds that $R(f^i)$ either has a minimum or a saddle point at $f^i = k^i$, but not a maximum. For convenience (and in order to comply with the notation of Refs. [104,105,130]), we introduce the quantity

$$\hat{\sigma}(f^i) \equiv \frac{2}{3} - R(f^i)\,. \tag{3.13}$$

After some algebra and with the help of Eqs. (3.11) and (3.12), one finds that

$$\hat{\sigma}(f^i) = \hat{\omega} - 2\hat{s}^i\hat{s}_i\,, \tag{3.14}$$

where

$$\hat{\omega} = \left[\frac{2}{3}g_{ij}g_{mn} - R_{ijmn} + \frac{1}{2}K_{ijk}P^{kl}K_{lmn}\right] n^i\bar{n}^j n^m \bar{n}^n \cos^4\chi \tag{3.15}$$

$$\hat{s}^i = \left[\frac{2}{\sqrt{3}}\frac{e^{-i\phi}n^i + e^{i\phi}\bar{n}^i}{2}\tan\chi + \frac{1}{2}P^{ij}K_{jmn}n^m\bar{n}^n\right]\cos^2\chi\,. \tag{3.16}$$

Here, we have used the projector

$$P^{ij} \equiv g^{ij} - k^i k^j \tag{3.17}$$

on the subspace orthogonal to k^i. We now see that $\hat{\sigma}(f^i)$ does not have a minimum (corresponding to a maximum of $R(f^i)$) at $f^i = k^i$, because $\hat{\sigma}(f^i)$ can always be made negative by choosing a value for χ such that $\tan\chi$ is large enough that the negative term $-2\hat{s}^i\hat{s}_i$ dominates over the term $\hat{\omega}$ (whose sign we cannot tell just from the no-scale property and the shift symmetry). It seems that for no-scale Kähler potentials with shift symmetry, both situations ($R(f^i)$ having a saddle point or a minimum) can arise. (In Sect. 3.2 we will see that this expectation is correct.)

For the particular case of models with only two fields, Eqs. (3.15) and (3.16) can be further simplified. Since by definition $n^i k_i = 0$, we have $(n^1, n^2) \propto (k_2, -k_1)$ in this case. In particular, we may choose n^i to be real (i.e. $n^i = \bar{n}^i$). Including the correct normalization for a unit vector, one finds that

$$(n_1, n_2) = \sqrt{\det g}\,(k^2, -k^1)\,, \qquad (n^1, n^2) = \frac{1}{\sqrt{\det g}}(k_2, -k_1)\,. \tag{3.18}$$

Since the scalar manifold is two-dimensional, the vectors k^i and n^i span a basis. Therefore, the projector is simply

$$P^{ij} = n^i n^j\,. \tag{3.19}$$

It follows that (3.15) and (3.16) now simplify to

$$\hat{\omega} = \left[\frac{2}{3} - R_{ijmn}n^i n^j n^m n^n + \frac{1}{2}\left(K_{ijk}n^i n^j n^k\right)^2\right]\cos^4\chi\,, \tag{3.20}$$

$$\hat{s}^i = n^i\left[\frac{2}{\sqrt{3}}\cos\phi\tan\chi + \frac{1}{2}K_{jmn}n^j n^m n^n\right]\cos^2\chi\,. \tag{3.21}$$

In particular, (3.21) shows that one can always tune $\hat{s}^i = 0$ by choosing an appropriate value for χ and $\cos\phi \neq 0$ (note that χ and ϕ depend on the superpotential). This in turn means that in order to find out if $R(f^i)$ has a saddle point or a minimum at $f^i = k^i$, for the case of two-field models we just have to check whether the quantity $\hat{\omega}$ can be positive or not. In the next section, we will identify a (surprisingly simple) criterion on Kähler potentials from heterotic respectively IIB string compactification to decide this.

3.2 String compactifications

At low energies, i.e. below the mass scale set by the characteristic length scale of the strings, it is justified to approximate a string theory by a 10d supergravity field theory. Dimensional reduction of the latter on 6d compact spaces then leads to a 4d field theory. How much of the original supersymmetry of the 10d theory remains in 4d depends on the type of the compact 6d space. For instance, in case of Calabi-Yau manifolds, one quarter of the original SUSY is preserved.

Here, we are interested in the for contact to particle phenomenology most interesting case – where the 4d theory has $\mathcal{N} = 1$ SUSY. It turns out that – at tree-level in the string coupling – the Kähler potential for the Kähler moduli has a no-scale structure in the limit where the length scale of the compact space is much larger than the length scale of the strings (this is the 'large-volume limit'), while the Kähler potential for the complex structure moduli – which in general is *not* of the no-scale type even at large volume – has a no-scale structure in the 'large-complex-structure limit'.* One can distinguish between (at least) two types of such no-scale Kähler potentials arising from string compactifications. The first one, which we treat in Sect. 3.2.1, arises for instance – but not only – for the Kähler moduli of Calabi-Yau compactifications of the heterotic string. The second one (Sect. 3.2.2) occurs for the Kähler moduli of IIB string theories compactified on Calabi-Yau orientifolds with O3/O7 planes.

Note that we always assume that the dilaton and either the complex structure moduli (if we consider the Kähler potential for the Kähler moduli) or the Kähler moduli (if we consider the Kähler potential for the complex structure moduli) are stabilized in a way that they can be integrated out without affecting the low energy dynamics. Under what conditions this is justified is a subtle issue which we will not touch here (see e.g. [59, 141–144] for recent studies in this direction).

*The large-complex-structure limit can roughly be understood as follows: Mirror symmetry identifies the moduli space for the Kähler moduli of a manifold X with the moduli space for the complex structure moduli of a 'mirror manifold' $\tilde{X}$. Since the former moduli space obeys the no-scale property in the limit where X is large, the moduli space for the complex structure moduli of the corresponding mirror manifold $\tilde{X}$ should also be of the same no-scale form. This limit for the moduli space of the complex structure moduli of $\tilde{X}$ is called 'large-complex structure limit' (see e.g. Ref. [140]).

3.2.1 Heterotic moduli spaces

The moduli of Calabi-Yau compactifications of the heterotic string include the dilaton/axion and the deformations of the Calabi-Yau metric. The latter are divided into deformations of the Kähler class and deformations of the complex structure. Locally, the moduli space $\mathcal{M}$ is (at tree-level) the product manifold [145]

$$\mathcal{M} = \mathcal{M}^{\rm ks} \times \mathcal{M}^{\rm cs} \times \frac{SU(1,1)}{U(1)}\,, \qquad (3.22)$$

where $\mathcal{M}^{\rm ks}$ is the space spanned by the Kähler moduli, $\mathcal{M}^{\rm cs}$ is spanned by the complex structure moduli while the dilaton/axion are the coordinates of the last factor. One has

$$K = -\log Y\,, \qquad (3.23)$$

where in the large-volume limit $Y^{\rm cs/ks}$ are given by [145–147]

$$Y^{\rm cs} = i\int_X \Omega\wedge\bar{\Omega}\,, \qquad Y^{\rm ks} = \mathcal{V} \equiv \frac{4}{3}\int_X J\wedge J\wedge J\,. \qquad (3.24)$$

Here Ω and J are, respectively, the holomorphic $(3,0)$-form and the Kähler $(1,1)$-form of the Calabi-Yau threefold. $\mathcal{V}$ is the classical volume in that the equality $Y^{\rm ks} = \mathcal{V}$ only holds in the large-volume limit, and is modified by α'- and worldsheet-instanton corrections.

For concreteness, let us focus on the Kähler moduli sector in the large-volume limit and assume that it induces supersymmetry breaking. Of course we could equally well consider the complex structure moduli in the large-complex-structure limit which – due to mirror symmetry – would lead to an identical analysis.

Since J is harmonic, it can be expanded in a $h^{1,1}$-dimensional basis w_i, $i = 1,\ldots,h^{1,1}$ of the cohomology group $H^{1,1}$ via $J = v^i w_i$.* The NS two-form has a similar expansion $B_2 = b^i\omega_i$. The coefficients in these expansions v^i and b^i are scalar fields which combine into the complex coordinates $T^i = v^i + ib^i$. Inserting this into (3.24), one obtains

$$K = -\log\mathcal{V}\,, \qquad \text{with} \qquad \mathcal{V} = \frac{1}{6}\,d_{ijk}\,(T^i+\bar{T}^i)(T^j+\bar{T}^j)(T^k+\bar{T}^k)\,, \quad (3.25)$$

where $d_{ijk} = \int_X w_i\wedge w_j\wedge w_k$ are the Calabi-Yau intersection numbers.

Before we continue let us emphasize that such a Kähler potential also appears as a sub-sector of other string compactifications, for example, in Calabi-Yau compactifications of type IIB with $O5/O9$-orientifold planes [148].

*Recall that the cohomology group $H^{p,q}$ for a manifold consists of the equivalence classes of closed (p,q)-forms on that manifold.

Therefore the following analysis is not only valid for heterotic compactifications, but rather for any moduli-sector with a Kähler potential of the form given in Eq. (3.25).

In order to compute $\hat{\sigma}(f^i)$ let us first recall a few further properties of K (for more details on the following computations we refer the reader to Appendix A). Its first derivatives read

$$K_i = -\frac{\mathcal{V}_i}{\mathcal{V}}\,, \quad \text{where} \quad \mathcal{V}_i = \frac{1}{2}\, d_{ijk}(T^j + \bar{T}^j)(T^k + \bar{T}^k)\,. \tag{3.26}$$

The Kähler metric is then given by

$$g_{ij} \;=\; -\frac{\mathcal{V}_{ij}}{\mathcal{V}} + \frac{\mathcal{V}_i\mathcal{V}_j}{\mathcal{V}^2} \;=\; e^K d_{ijk}K^k + K_iK_j\,, \tag{3.27}$$

where the matrix $\mathcal{V}_{ij} = d_{ijk}(T^k + \bar{T}^k)$ has a signature $(1, h^{1,1} - 1)$ for all allowed values of $T^i + \bar{T}^i$, i.e. those values for which $\mathcal{V}$ is positive and the Kähler metric is positive-definite [145]. The inverse metric is conveniently expressed in terms of the matrix $\mathcal{V}^{ij}$ which is defined as the inverse of $\mathcal{V}_{ij}$, i.e. $\mathcal{V}^{ij}\mathcal{V}_{jk} = \delta^i_k$. Multiplication with $T^k + \bar{T}^k$ gives $2\mathcal{V}^{ij}\mathcal{V}_j = T^i + \bar{T}^i = -K^i$. Using this, one easily finds that*

$$g^{ij} = -\mathcal{V}\mathcal{V}^{ij} + \frac{1}{2}K^iK^j\,. \tag{3.28}$$

Using (3.26) – (3.28) one also easily computes the third derivatives of K and the Riemann tensor of the Kähler manifold:

$$K_{ijk} \;=\; -e^K d_{ijk} + g_{ij}K_k + g_{ik}K_j + g_{jk}K_i - K_iK_jK_k\,, \tag{3.29}$$
$$R_{ijmn} \;=\; g_{ij}g_{mn} + g_{in}g_{mj} - e^{2K} d_{imp}g^{pq}d_{qjn}\,. \tag{3.30}$$

The reason why R_{ijmn} has this specific simple form is that $\mathcal{M}^{\text{ks}}$ (and also $\mathcal{M}^{\text{cs}}$) is not only a Kähler manifold, but moreover a 'special Kähler' manifold, meaning that its Kähler potential can be expressed in terms of a holomorphic function $F = F(\Phi)$ (the 'prepotential') via $Y = -2(F + \bar{F}) + (F_k + \bar{F}_{\bar{k}})(\Phi^k + \bar{\Phi}^k)$ [149]. Indeed, K in Eq. (3.25) can be derived from the prepotential $F(T) = 1/6\, d_{ijk}T^iT^jT^k$. Actually, $\mathcal{M}^{\text{ks}}$ and $\mathcal{M}^{\text{cs}}$ are special Kähler manifolds not only in the large-volume large-complex-structure limit. The specific form of the Riemann tensor holds for any special Kähler manifold, with d_{ijk} replaced by the third derivative F_{ijk} of the prepotential [150].

*As a check, note that (3.26) and (3.28) indeed imply the no-scale condition (3.1) and also the homogeneity property (3.9).

We will now determine $R(f^i)$ (respectively $\hat{\sigma}(f^i)$) – specifying to models with two moduli. We do so by making use of the decomposition $\hat{\sigma} = \hat{\omega} - 2\hat{s}^i\hat{s}_i$ which we found in Sect. 3.1. Inserting (3.29) and (3.30) into (3.20) and (3.21), one finds (using in particular (3.27)) that

$$\hat{\omega} = \left[\frac{3}{2}\Big(e^K d_{pqr}n^p n^q n^r\Big)^2 - 1\right]\cos^4\chi\,, \tag{3.31}$$

$$\hat{s}^i = n^i\left[\frac{2}{\sqrt{3}}\tan\chi\,\cos\phi - \frac{1}{2}e^K d_{pqr}n^p n^q n^r\right]\cos^2\chi\,, \tag{3.32}$$

where $i,p,q,r \in \{1,2\}$, since we have two moduli. Let us now define the quantity

$$a_{\mathcal{H}} \equiv \frac{3}{2}\Big(e^K d_{pqr}n^p n^q n^r\Big)^2 - 1\,, \tag{3.33}$$

so that $\hat{\omega} = a_{\mathcal{H}}\cos^4\chi$. Using Eq. (3.18), one finds after some algebra that

$$a_{\mathcal{H}} = -\frac{\Delta}{24}\,\frac{e^{4K}}{(\det g)^3}\,, \tag{3.34}$$

where Δ is the discriminant of the cubic polynomial defined by $d_{ijk}v^iv^jv^k$ after scaling out one variable,* and reads

$$\Delta \equiv -27\left(d_{111}^2d_{222}^2 - 3\,d_{112}^2d_{122}^2 + 4\,d_{111}d_{122}^3 + 4\,d_{112}^3d_{222} - 6\,d_{111}d_{112}d_{122}d_{222}\right)\,. \tag{3.35}$$

This is the most important result of this section: Since $\det g > 0$, it means that the sign of $\hat{\omega}$ is the sign of $-\Delta$, which is independent of the value of the moduli. Hence, for Kähler potentials of the form (3.25) with two moduli and with $\Delta < 0$, de Sitter vacua and inflation are possible, whereas they are excluded if $\Delta > 0$.

Let us now proceed and determine, for two-field models with $\Delta < 0$ and for some fixed value of $a_{\mathcal{H}} \in (0,+\infty)$,† the maximal value of $\hat{\sigma}$ which may be obtained by tuning ϕ and χ (i.e. by tuning the Goldstino direction). Inserting the definition (3.33) into (3.32), one immediately finds

$$\hat{s}^i = n^i\frac{2}{\sqrt{3}}\left[\tan\chi\,\cos\phi - s_{\mathcal{H}}\sqrt{\frac{1+a_{\mathcal{H}}}{8}}\right]\cos^2\chi\,, \tag{3.36}$$

where $s_{\mathcal{H}} = \mathrm{sign}(d_{pqr}n^pn^qn^r)$. Then, one has

$$\hat{\sigma} = \left[a_{\mathcal{H}} - \frac{8}{3}\left(\tan\chi\cos\phi - s_{\mathcal{H}}\sqrt{\frac{1+a_{\mathcal{H}}}{8}}\right)^2\right]\cos^4\chi\,. \tag{3.37}$$

*Hereby, we mean the cubic polynomial $P(x^2) = d_{ijk}x^ix^jx^k$ with $x^1 \equiv 1$.

†That indeed $a_{\mathcal{H}}$ can be arbitrarily large can be seen in simple examples.

In the following, we provide some details on how to find the maximum of $\hat{\sigma}$ (which we denote by $\hat{\sigma}_0$). First, one finds that at stationary points of $\hat{\sigma}$ one has $\cos\phi \in \{0, \pm 1\}$. One can check however that at a maximum one needs to have $\cos\phi \in \{\pm 1\}$. Since $\hat{\sigma}$ is invariant under $(\tan\chi \to -\tan\chi,\ \cos\phi \to -\cos\phi)$, we can take $\cos\phi_0 = 1$ without loss of generality. Next, it is convenient to define the new variable ε by

$$\tan\chi \equiv s_{\mathcal{H}}\sqrt{\frac{1+a_{\mathcal{H}}}{8}}\,(1+\varepsilon)\,. \tag{3.38}$$

The condition $\partial\hat{\sigma}/\partial\varepsilon\ |_{\cos\phi=1} = 0$ then determines the stationary points. Solving this equation amounts to finding the roots of a cubic polynomial in ε. Luckily, this polynomial factorizes in a linear and a quadratic part, so that the expressions for the roots are relatively simple. The solution which turns out to correspond to a maximum of $\hat{\sigma}$ is given by

$$\varepsilon_0 = \frac{3}{2}\left(\sqrt{\frac{1+a_{\mathcal{H}}/9}{1+a_{\mathcal{H}}}} - 1\right). \tag{3.39}$$

Plugging (3.38), (3.39) and $\cos\phi_0 = 1$ back into (3.37) one finds that the maximum is given by

$$\hat{\sigma}_0 = \frac{128}{3}\frac{a_{\mathcal{H}} + 9\sqrt{(1+a_{\mathcal{H}})(1+a_{\mathcal{H}}/9)} - 9}{\Big(21 + a_{\mathcal{H}} - 3\sqrt{(1+a_{\mathcal{H}})(1+a_{\mathcal{H}}/9)}\Big)^2}\,. \tag{3.40}$$

From Eq. (3.40) one sees that $\hat{\sigma}_0$ grows asymptotically as $2a_{\mathcal{H}}/3$ for large values of $a_{\mathcal{H}}$ and can thus be made arbitrarily large and positive.* This means that for heterotic models (with $\Delta < 0 \Leftrightarrow a_{\mathcal{H}} > 0$), by an appropriate tuning of the superpotential (i) the average sGoldstino squared mass

$$m^2 = [3(1+\gamma)\hat{\sigma} - 2\gamma]m_{3/2}^2 \tag{3.41}$$

can be made arbitrarily large, and (ii) the condition for slow-roll inflation can always be fulfilled (see (2.15)).

In the following subsection, we will repeat the analysis of this subsection for a different class of Kähler potentials which is obtained in type IIB

*Note that for the simple choice of parameters $\cos\phi = 1$ and $\varepsilon = 0$ one has $\hat{\sigma} = 64\,a_{\mathcal{H}}/(9+a_{\mathcal{H}})^2$, which is always positive (but of course smaller than the maximum $\hat{\sigma}_0$ found in (3.40)). This choice was adopted in [105]. However, ε_0 (see (3.39)) is close to zero only for small $a_{\mathcal{H}}$ so that $\hat{\sigma}_0$ will depart significantly from the approximate expression $64\,a_{\mathcal{H}}/(9+a_{\mathcal{H}})^2$ for large values of $a_{\mathcal{H}}$.

orientifold compactifications. While the equations turn out to be more complicated, we can follow exactly the same logic. One important difference will be, however, that for orientifold models with two moduli the corresponding quantity $\hat{\sigma}_0$ cannot be arbitrarily large, so that there is an upper bound for the masses of the moduli in terms of the gravitino mass.

3.2.2 IIB orientifold moduli spaces

In contrast to the heterotic string, type IIB Calabi-Yau compactifications give theories with $\mathcal{N} = 2$ supersymmetry in 4 dimensions [151–156]. The RR forms which are present in 10d type II supergravities lead to additional massless 4d fields which, together with the geometric moduli, arrange into $\mathcal{N} = 2$ supermultiplets. The scalars in the vector multiplets span again a special Kähler manifold $\mathcal{M}^{\text{SK}}$ whereas the scalars in the hypermultiplet span a 'dual quaternionic' manifold $\mathcal{M}^{\text{Q}}$.

One way to obtain a theory with $\mathcal{N} = 1$ supersymmetry is to impose an orientifold projection. In type IIA, this involves $O6$-planes while in type IIB one has $O3/O7$ or $O5/O9$-planes [157, 158]. The moduli space in all of these three cases has the form [148, 159] (a review can be found in Ref. [140])

$$\tilde{\mathcal{M}} = \tilde{\mathcal{M}}^{\text{SK}} \times \tilde{\mathcal{M}}^{\text{Q}} \,, \tag{3.42}$$

where $\tilde{\mathcal{M}}^{\text{SK}}$ is a special Kähler submanifold of the 'parent' $\mathcal{N} = 2$ moduli space $\mathcal{M}^{\text{SK}}$ while $\tilde{\mathcal{M}}^{\text{Q}}$ is a Kähler submanifold of $\mathcal{M}^{\text{Q}}$. In the large-volume large-complex-structure limit, the $\tilde{\mathcal{M}}^{\text{SK}}$ factor satisfies the no-scale property and the Kähler potential does in fact coincide with the Kähler potential of Eq. (3.25). Therefore the analysis of Sect. 3.2.1 holds unmodified for the moduli of $\tilde{\mathcal{M}}^{\text{SK}}$. On the other hand the $\tilde{\mathcal{M}}^{\text{Q}}$ sector, which includes the dilaton, satisfies $K^i K_i = 4$, and if the dilaton is fixed, the latter sector is also no-scale [148]. However, the Kähler potential of $\tilde{\mathcal{M}}^{Q}$ is different for the three orientifold compactifications.

For concreteness let us focus on type IIB with O3/O7 planes, where the Kähler potential in the large-volume limit reads $K_Q = K - \log(S + \bar{S})$ with [148]

$$K = -2 \log \mathcal{V} \,, \qquad \text{where} \qquad \mathcal{V} = \frac{1}{48}\, d^{ijk} v_i v_j v_k \,. \tag{3.43}$$

$\mathcal{V}$ is the classical volume of the Calabi-Yau orientifold, S is the dilaton/axion and the v_i, $i = 1, \ldots, h_+^{1,1}$ are the Kähler moduli of the Calabi-Yau orientifold. In order to comply with the standard notation whereby chiral coordinates carry upper indices, we have interchanged the positions of the indices of the Kähler moduli v_i and the intersection numbers d^{ijk} in comparison to Sect. 3.2.1. We stress that they are exactly the same objects as in the heterotic case. However, the v_i do not appear as components of chiral multiplets in the low energy effective action here. Instead, they determine the real part of the Kähler coordinates $T^i = \rho^i + i\zeta^i$ via the quadratic relation*

$$\rho^i = \frac{1}{16}\, d^{ijk} v_j v_k\,. \tag{3.44}$$

Due to this relation the Kähler potential of Eq. (3.43) cannot explicitly be expressed in terms of the coordinates T^i in general, but is only implicitly defined through Eq. (3.44). As in the previous section we assume that the dilaton is fixed to a supersymmetric configuration and focus only on the Kähler moduli.

The metric can be conveniently expressed in terms of

$$d^{ij} \equiv \frac{\partial \rho^i}{\partial v_j} = \frac{1}{8}\, d^{ijk} v_k\,, \qquad d_{ij} \equiv \frac{\partial v_i}{\partial \rho^j}\,. \tag{3.45}$$

Using (3.43) – (3.45), one computes (where $K_i \equiv \partial K / \partial T^i$)

$$K_i = -\frac{1}{2}\, e^{K/2} v_i\;, \qquad d^{ij} = -\frac{1}{4}\, e^{-K/2} d^{ijk} K_k\,. \tag{3.46}$$

This in turn determines the Kähler metric and its inverse to be

$$g_{ij} = \frac{1}{2}\, K_i K_j - \frac{1}{4}\, e^{K/2} d_{ij}\,, \qquad g^{ij} = 4\,\rho^i \rho^j - 4\, e^{-K/2} d^{ij}\,. \tag{3.47}$$

One can now check that K satisfies the no-scale property $K^i K_i = 3$ as well as the special identity $K^i = -(T^i + \bar{T}^i)$, which again results from the fact that e^{-K} is a homogeneous function of degree 3 in $T^i + \bar{T}^i$. This can be used to slightly rewrite the inverse metric as

$$g^{ij} = e^{-K} d^{ijk} K_k + K^i K^j\,. \tag{3.48}$$

Notice that the inverse metric is, up to a factor, equal to the metric (3.27) of the heterotic case (see Appendix B for more details).† In this sense,

*The spaces $H^{p,q}$ each split into even and odd spaces under the orientifold projection, i.e. $H^{p,q} = H^{p,q}_+ \oplus H^{p,q}_-$. Here, we assume for simplicity that $h^{p,q}_- = dim(H^{p,q}_-) = 0$. Otherwise, there would be – in addition to the $h^{1,1}_+$ coordinates T^i – also $h^{1,1}_-$ coordinates G_α with couplings specified in [148].

†Let us clarify that this means that the inverse of the 'orientifold metric' and the 'heterotic metric' are (up to a factor) the same functions of the Kähler *moduli*, but they are of course different functions of the Kähler *coordinates* of the orientifold- respectively heterotic geometry.

the Kähler geometry discussed in this section is dual to the one which one obtains from heterotic compactifications [160]. This duality will be used below to infer the quantity $\hat{\omega}$ for IIB O3/O7 Kähler geometries from the expression (3.34) found in the last section for the heterotic case.

We also need the third derivatives of K and the Riemann tensor. Using the relations (3.45) – (3.47) one finds after some algebra that [161] (see Appendix A for more details)

$$
\begin{aligned}
K_{ijm} &= e^{-K}\hat{d}_{ijm} - 3\, g_{(ij}K_{m)} + K_i K_j K_k \,, && (3.49)\\
R_{ijmn} &= -g_{im}g_{jn} + 2\, e^{-2K}\hat{d}_{i(j|k}g^{kl}\hat{d}_{l|n)m} + 6\, g_{(ij}K_m K_{n)} \\
&\quad -3\, K_i K_j K_m K_n - 4\, e^{-K}\hat{d}_{(ijm}K_{n)} \,, && (3.50)
\end{aligned}
$$

where we abbreviated

$$\hat{d}_{ijk} \equiv g_{ip}g_{jq}g_{kl}d^{pql} \,. \qquad (3.51)$$

As in Sect. 3.2.1, we now determine $R(f^i)$ (respectively $\hat{\sigma}(f^i)$) – specifying to models with two moduli – by making use of the decomposition $\hat{\sigma} = \hat{\omega} - 2\hat{s}^i\hat{s}_i$. Inserting (3.49) and (3.50) into (3.20) and (3.21), one finds (using in particular (3.48)) that

$$
\begin{aligned}
\hat{\omega} &= \left[1 - \frac{3}{2}\Big(e^{-K}d^{pqr}n_p n_q n_r\Big)^2\right]\cos^4\chi \,, && (3.52)\\
\hat{s}^i &= n^i\left[\frac{2}{\sqrt{3}}\tan\chi\,\cos\phi - \frac{1}{2}e^{-K}d^{pqr}n_p n_q n_r\right]\cos^2\chi \,. && (3.53)
\end{aligned}
$$

Let us also, in analogy to Eq. (3.33), define the quantity

$$a_{\mathcal{O}} \equiv 1 - \frac{3}{2}\left(e^{-K}d^{pqr}n_p n_q n_r\right)^2 \,. \qquad (3.54)$$

Notice the similarity of the expression for $a_{\mathcal{O}}$ and the expression for $a_{\mathcal{H}}$ (Eq. (3.33)). In fact, one can show that $a_{\mathcal{O}} = -a_{\mathcal{H}}$ (this is done in Appendix B). This is a consequence of the duality between the heterotic- and orientifold scalar geometries, which was mentioned above. Also, one can show (see Appendix B) that under this duality the right-hand side of Eq. (3.34) is 'mapped' to $-\frac{\Delta}{24}\frac{(\det g)^3}{e^{4K}}$, where Δ is defined as in (3.35), but with intersection numbers with upper indices.* Hence, Eq. (3.34) implies that†

$$a_{\mathcal{O}} = \frac{\Delta}{24}\,\frac{(\det g)^3}{e^{4K}} \,. \qquad (3.55)$$

*Recall that raising the indices of the intersection numbers is only a convention.

†This can also be determined by a 'brute force' calculation from the Kähler potential (3.43). This is very cumbersome however, because performing the change of variables (3.44) involves finding the roots of a quartic polynomial.

We conclude that for orientifold models the situation is exactly the opposite as that for heterotic models: Here, the sign of $\hat{\omega}$ is the sign of Δ, so that de Sitter vacua and inflation are possible for $\Delta > 0$, but excluded for $\Delta < 0$.

We now determine, for a fixed value of $a_{\mathcal{O}} \in (0,1]$ (note that from (3.54) it follows that $a_{\mathcal{O}} \leq 1$), the maximal value of $\hat{\sigma}$ which may be obtained by tuning ϕ and χ. We proceed as in Sect. 3.2.1. Using the definition (3.54) and introducing the sign $s_{\mathcal{O}} = \mathrm{sign}(d^{pqr} n_p n_q n_r)$, one finds

$$\hat{\sigma} = \left[a_{\mathcal{O}} - \frac{8}{3}\left(\tan\chi\cos\phi - s_{\mathcal{O}}\sqrt{\frac{1-a_{\mathcal{O}}}{8}}\right)^2\right]\cos^4\chi\,. \tag{3.56}$$

As for (3.37) in Sect. 3.2.1, the maximum is at $\cos\phi_0 = 1$. We also define

$$\tan\chi = s_{\mathcal{O}}\sqrt{\frac{1-a_{\mathcal{O}}}{8}}\,(1+\varepsilon)\,. \tag{3.57}$$

One then has

$$\hat{\sigma}|_{\cos\phi=1} = 64\frac{a_{\mathcal{O}} - (1-a_{\mathcal{O}})\,\varepsilon^2/3}{[8+(1-a_{\mathcal{O}})(1+\varepsilon)^2]^2}\,. \tag{3.58}$$

One sees that one gets a lower bound $64\,a_{\mathcal{O}}/(9-a_{\mathcal{O}})^2$ on $\hat{\sigma}_0$ by setting $\varepsilon = 0$. This lower bound grows as $a_{\mathcal{O}}$ is increased until the point $a_{\mathcal{O}} = 1$, where it reaches its maximal value 1. The exact maximum of $\hat{\sigma}$ for a given $a_{\mathcal{O}}$ occurs for a in general non-vanishing value ε_0 (and hence is larger in general), determined by $\partial\hat{\sigma}/\partial\varepsilon|_{\cos\phi=0} = 0$. Solving this equation again amounts to solving a cubic polynomial, which – unlike in Sect. 3.2.1 – does not factorize, so that the expression for the value of ε_0 is somewhat complicated. One finds

$$\varepsilon_0 = \frac{\sqrt{1+5\,a_{\mathcal{O}}/9}}{\sqrt{1-a_{\mathcal{O}}}}\big(3\sin\theta - \sqrt{3}\cos\theta\big)\,, \tag{3.59}$$

where

$$\theta \equiv \frac{1}{3}\arccos\left(\frac{a_{\mathcal{O}}}{\sqrt{3}}\frac{\sqrt{1-a_{\mathcal{O}}}}{(1+5\,a_{\mathcal{O}}/9)^{3/2}}\right)\,. \tag{3.60}$$

Plugging this into (3.58), one finds that the exact maximal value $\hat{\sigma}_0$ is given by a relatively complicated expression, which we do not report here. Fortunately, one can however check that the quantity ε_0 given by (3.59) is always quite small for any value of $a_{\mathcal{O}} \in (0,1]$. In particular, one easily verifies that also the exact $\hat{\sigma}_0$ increases monotonically as a function of $a_{\mathcal{O}}$, and that for $a_{\mathcal{O}} = 1$ one obtains $\hat{\sigma}_0 = 1$. In practice one can then approximate the maximal value of $\hat{\sigma}$ with the one associated with $\varepsilon = 0$, namely

$$\hat{\sigma}_0 \simeq \frac{64\,a_{\mathcal{O}}}{(9-a_{\mathcal{O}})^2}\,. \tag{3.61}$$

Notice finally that the fact that $\hat{\sigma}(f^i)$ can be at most 1 implies the following upper bound for the sGoldstino mass scale m (the fact that $\hat{\sigma}(f^i) \leq 1$ is however not relevant for the possibility of building inflationary models, since even for $\gamma \gg 1$ it is always possible to fulfill the condition (2.15)):

$$m^2 \leq (3+\gamma) m_{3/2}^2 \,. \tag{3.62}$$

It means that there is at least one scalar field whose mass is not larger than $\sqrt{3+\gamma}$ times the gravitino mass – independently of the superpotential. This is an interesting result concerning the phenomenology of IIB O3/O7 orientifold compactifications, which may point towards a large gravitino mass to ease the cosmological moduli problem* (unless there is a phase of 'Thermal Inflation' [162]).

3.3 Subleading corrections

So far we have analyzed models that respect the no-scale property $K_i K^i = 3$. This property is however violated when α'-, worldsheet-instanton- or string-loop-corrections to the Kähler potential are taken into account, although they are suppressed in the large-volume and weak-coupling limit. It is therefore interesting to study how $R(f^i)$ (or equivalently $\hat{\sigma}(f^i)$) is modified by these effects, particularly for those models for which $\hat{\sigma}(f^i) \leq 0$ at leading order, which therefore have neither de Sitter vacua nor inflation at lowest order in the perturbative expansion. For concreteness we here consider only α' corrections, but the effect of other corrections can be studied in a similar way. Note that the analysis of this section holds for an arbitrary number of moduli.

3.3.1 Heterotic moduli spaces

If one takes α' corrections into account, the Kähler potential for the Kähler moduli of Calabi-Yau compactifications of the heterotic string is $K = -\log Y$ where [169]

$$Y = \mathcal{V} + 4\xi \,. \tag{3.63}$$

The quantity $\xi = -\zeta(3)\chi/2$ is a real constant determined by the Euler characteristic of the Calabi-Yau manifold, which is given by $\chi = 2(h^{1,1} - h^{2,1})$. The geometry is still of the special Kähler type, with prepotential

*Recall that the cosmological moduli problem [163–167] is the problem that light scalar fields may either overclose the universe (if they are stable and have a mass $\gtrsim 10^{-26}$ eV) or that they may spoil nucleosynthesis (if they are unstable, gravitationally coupled and lighter than ~ 100 TeV). A brief review of this can for instance be found within [168].

$F(T) = 1/6\, d_{ijk}T^iT^jT^k - \xi$. However, as mentioned above, α' corrections break the no-scale property (3.1), which is seen from Eqs. (A.2) and (A.3) of Appendix A with $n = 1$ and $\theta = (3/2)\mathcal{V}/(\mathcal{V} + 4\xi)$.

The natural small dimensionless parameter controlling the effect of α' corrections relative to the leading-order Kähler potential (3.25) is given by

$$\delta_{\mathcal{H}} = \frac{4\xi}{\mathcal{V}}\,. \tag{3.64}$$

In the following, we work at leading order in this parameter (this is signaled by the use of the symbol '$\simeq$'), which is small when the volume is large. Using Eqs. (A.1) and (A.3) with $\theta \simeq 3/2(1 - \delta_{\mathcal{H}})$, one then finds that

$$K_iK^i \simeq 3 + 6\,\delta_{\mathcal{H}}\,. \tag{3.65}$$

The Riemann tensor is given by Eq. (A.7). The quantities F_{ijk} in that equation are as before given by the intersection numbers, whereas the metric g_{ij} and its inverse g^{ij} are affected by the corrections and can be computed from (A.1).

In order to understand how α' corrections modify the sectional curvature along the Goldstino direction, let us compute the function $\hat{\sigma}(f^i)$ up to second order in the n^i's. To this end one needs the contractions

$$\begin{aligned} R_{ijmn}K^jK^n &\simeq 2g_{im} + 18\delta_{\mathcal{H}}(g_{im} - K_iK_m) \\ R_{ijmn}K^mK^n &\simeq 2g_{ij} - 12\delta_{\mathcal{H}}K_iK_j \\ R_{ijmn}K^jK^mK^n &\simeq 2(1 - 18\delta_{\mathcal{H}})K_i \\ R_{ijmn}K^iK^jK^mK^n &\simeq 6(1 - 16\delta_{\mathcal{H}})\,, \end{aligned} \tag{3.66}$$

which are computed using the formulae found in Appendix A. As a check, one immediately verifies that for $\delta_{\mathcal{H}} = 0$ one reobtains Eqs. (3.2), (3.3) and (3.12). Using furthermore the definition (3.7) of f^i, the definition $k^i = K^i/\sqrt{K^jK_j}$ and Eq. (3.65), one finds

$$\begin{aligned} \hat{\sigma}(f^i) \simeq\;& \frac{40}{3}\,\delta_{\mathcal{H}}\sin^4\chi - \frac{2}{3}\sin^2\chi\cos^2\chi \\ & \big\{(1 - 4\,\delta_{\mathcal{H}})\,2\,g_{ij}n^i\bar{n}^j + (1 + 7\,\delta_{\mathcal{H}})(e^{2i\phi}g_{ij}\bar{n}^i\bar{n}^j + c.c.)\big\} + \mathcal{O}\left((n^i)^3\right)\,. \end{aligned} \tag{3.67}$$

One observes that, also in the presence of subleading corrections breaking the no-scale property, $\hat{\sigma}(f^i)$ is stationary at $n^i = 0$ (since there is no term linear in n^i), but its value at the stationary point now is

$$\hat{\sigma}(k^i) \simeq \frac{40}{3}\,\delta_{\mathcal{H}} \tag{3.68}$$

rather than zero. If one has $\chi < 0$ (i.e. $h^{2,1} > h^{1,1}$) this is positive.

This is particularly important for models where $\hat{\sigma}(f^i) \leq 0$ at leading order. For such models, the maximal value which the sGoldstino mass scale (3.41) can attain (by choosing $f^i = k^i$) is given by

$$m^2 \simeq [40(1+\gamma)\,\delta_{\mathcal{H}} - 2\gamma]\, m_{3/2}^2\,. \tag{3.69}$$

For a realistic vacuum we need $\gamma \ll 1$, so that the metastability condition can be fulfilled as long as $\delta_{\mathcal{H}} \gtrsim \gamma/20$. This gives a criterion on how large α' corrections have to be for a given γ in order to admit vacua with all moduli stabilized.* Notice however that m^2 is suppressed with respect to $m_{3/2}^2$ by $\delta_{\mathcal{H}}$. In order not to have too light particles to be in conflict with cosmological lower bounds on moduli masses, $\delta_{\mathcal{H}}$ would need to be correspondingly larger. For instance, requiring $m \gtrsim 100\,\mathrm{TeV}$ (cf. the comment on the cosmological moduli problem in the footnote $*$ on page 46) means that one would need $\delta_{\mathcal{H}} \gtrsim \gamma/20 + (100\,\mathrm{TeV}/m_{3/2})^2/40 \simeq (100\,\mathrm{TeV}/m_{3/2})^2/40$.

Regarding the realization of inflationary models (for Kähler potentials for which $\hat{\sigma}(f^i) \leq 0$ at leading order), for a given value of $\delta_{\mathcal{H}}$ the bound (2.15) is equivalent to a bound on the ratio of the Hubble scale H to the gravitino mass $m_{3/2}$ (recall $\gamma \simeq H^2/m_{3/2}^2$):

$$\frac{\gamma}{1+\gamma} \lesssim \frac{3}{2}\hat{\sigma}(k^i) \simeq 20\ \delta_{\mathcal{H}}\,. \tag{3.70}$$

Since the left-hand side is always less than one (even for $\gamma \gg 1$), models with an arbitrarily large Hubble scale can be realized as long as $\delta_{\mathcal{H}} \gtrsim 1/20$, which may still be considered subleading. However, for values of $\delta_{\mathcal{H}}$ which are much smaller than this (which rather should be considered the generic situation for a subleading correction), the Hubble scale is bounded by $\gamma \lesssim 20\,\delta_{\mathcal{H}}$, i.e.

$$H \lesssim \sqrt{20\ \delta_{\mathcal{H}}}\ m_{3/2}\,. \tag{3.71}$$

3.3.2 IIB orientifold moduli spaces

We now include α' corrections in orientifold compactifications. When these corrections are taken into account, the Kähler potential (see Eq. (3.43)) is modified to $K_Q = -2\log Y - \log(S+\bar{S})$, where [62]

$$Y = \mathcal{V} + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}. \tag{3.72}$$

*One should bear in mind, however, that other subleading corrections to the Kähler potential could compete against α' corrections and modify this result.

One difficulty arises from the fact that these corrections depend on the dilaton which, strictly speaking, now should be considered a dynamical quantity.* This is due to the fact that in the presence of α' corrections the Kähler potential is not anymore a sum of a part which depends only on the Kähler moduli and a part which depends only on the dilaton (as one sees from Eq. (3.72)) and hence the metric is not block-diagonal with one block for the Kähler coordinates and one block for the dilaton. For simplicity, we nevertheless assume that the dilaton is fixed to a constant value in Eq. (3.72), and define the new constant $\tilde{\xi} = (\xi/2)[(S+\bar{S})/2]^{3/2}$. We expect that the results of this Section would not be qualitatively different in the full computation with a dynamical dilaton, assuming that S is fixed to a supersymmetric configuration $G_S = 0$.

As before, α' corrections break the no-scale property (3.1), which can be seen from Eqs. (A.2) and (A.3) of the Appendix with $n = 2$ and $\theta = 3\mathcal{V}/(\mathcal{V}+\tilde{\xi})$. The small dimensionless parameter controlling the relative effect of the α' corrections is in this case given by

$$\delta_{\mathcal{O}} = \frac{\tilde{\xi}}{\mathcal{V}}\,. \tag{3.73}$$

We will work at leading order in this parameter. Using the results of Appendix A with $\theta \simeq 3(1-\delta_{\mathcal{O}})$, one then finds that

$$K_i K^i \simeq 3 + 3/2\,\delta_{\mathcal{O}}\,. \tag{3.74}$$

As was done in Sect. 3.3.1 for the case of heterotic compactifications, one may compute $\hat{\sigma}(f^i)$ up to second order in n^i (the computation is more tedious now, however). The Riemann tensor, given by Eq. (A.5), can be evaluated by using $Y_{ij} = 1/8\,d_{ij}$, $Y_{ijm} = -1/128\,d_{ir}d_{js}d_{mt}d^{rst}$ and $Y_{ijmn} = 24\,Y_{ijs}d^{sr}Y_{rmn}$. In order to compute the contractions of R_{ijmn} with two, three and four K^i's, we need the contractions $Y_{ij}K^iK^j = 3\,(\theta-1)^{-2}\mathcal{V}$, $Y_{ijm}K^iK^j = Y/2\,(\theta-1)^{-2}K_m$ and $Y_{ijmn}K^iK^jK^mK^n = 9\,(\theta-1)^{-4}\mathcal{V}$. This leads to

$$\begin{aligned}
R_{ijmn}K^jK^n &\simeq 2g_{im} + \frac{27}{8}\delta_{\mathcal{O}}\,(2g_{im} - K_iK_m) \\
R_{ijmn}K^mK^n &\simeq 2g_{ij} - \frac{3}{8}\delta_{\mathcal{O}}\,(6g_{ij} - 5K_iK_j) \\
R_{ijmn}K^jK^mK^n &\simeq \left(2 - \frac{27}{8}\delta_{\mathcal{O}}\right)K_i \\
R_{ijmn}K^iK^jK^mK^n &\simeq 6 - \frac{41}{8}\delta_{\mathcal{O}}\,.
\end{aligned} \tag{3.75}$$

*Here we mean that one should first take derivatives of K with respect to the dilaton and then evaluate these at the vev of the dilaton.

In the same way as in Sect. 3.3.1, one then finds

$$\hat{\sigma}(f^i) \simeq \frac{35}{24}\,\delta_{\mathcal{O}}\,\sin^4\chi - \frac{2}{3}\sin^2\chi\cos^2\chi \tag{3.76}$$
$$\left\{\left(1+\frac{5}{4}\delta_{\mathcal{O}}\right)2\,g_{ij}n^i\bar{n}^j + \left(1+\frac{23}{8}\delta_{\mathcal{O}}\right)\left(e^{2i\phi}g_{ij}\bar{n}^i\bar{n}^j + c.c.\right)\right\} + \mathcal{O}\left((n^i)^3\right)\,.$$

Again, $\hat{\sigma}(f^i)$ is stationary at $n^i = 0$ with a value

$$\hat{\sigma}(k^i) \simeq \frac{35}{24}\,\delta_{\mathcal{O}}\,. \tag{3.77}$$

The only difference in comparison to the result (3.68) found for heterotic models is the numerical factor in front of $\delta_{\mathcal{O}}$, which is of order unity now rather than $\mathcal{O}(10)$ in (3.68). The implications for the realization of de Sitter vacua and slow-roll inflation (for Kähler potentials with $\hat{\sigma}(f^i) \leq 0$ at leading order) are thus similar to those for heterotic models: (i) The maximal sGoldstino mass scale (3.41) is given by

$$m^2 \simeq \left[\frac{35}{8}(1+\gamma)\,\delta_{\mathcal{O}} - 2\gamma\right] m_{3/2}^2\,, \tag{3.78}$$

which is suppressed by $\delta_{\mathcal{O}}$. (ii) Inflationary models with a parameter $\gamma \simeq H^2/m_{3/2}^2$ are possible as long as

$$\frac{\gamma}{1+\gamma} \lesssim \frac{3}{2}\hat{\sigma}(k^i) \simeq \frac{35}{16}\,\delta_{\mathcal{O}}\,. \tag{3.79}$$

Since the numerical factor in front of $\delta_{\mathcal{O}}$ is smaller here (compared to (3.70)), one cannot have inflationary models with $\gamma \sim 1$ or larger, as that would necessitate $\delta_{\mathcal{O}} \sim 1$, which is a contradiction to $\delta_{\mathcal{O}}$ being a subleading correction. We thus have $\gamma \ll 1$, and more precisely – for a (positive) subleading correction $\delta_{\mathcal{O}}$ – we obtain the bound

$$H \lesssim m_{3/2}\sqrt{35\delta_{\mathcal{O}}}/4\,. \tag{3.80}$$

Summary of Section 3:

In this Section we have discussed the implications of the metastability bound for certain scalar geometries which arise from string compactifications. In a first step, we discussed what can be said about the sectional curvature for general no-scale models. We have shown that $R(f^i)$ is stationary for $f^i = k^i$, with critical value $2/3$. We have further shown that, assuming in addition a shift symmetry, $\hat{\sigma}(f^i) \equiv 2/3 - R(f^i)$ can be written in the form $\hat{\sigma}(f^i) = \hat{\omega} - 2\hat{s}^i\hat{s}_i$. For two-field models, the negative semi-definite part $-2\hat{s}^i\hat{s}_i$ can always be set to zero independently of $\hat{\omega}$, so that for such models, in order to know if de Sitter vacua and inflation are possible, it is sufficient to find out if $\hat{\omega}$ can be positive.

We have done this in Sect. 3.2 for two classes of scalar manifolds which arise, on the one hand, (for instance) from heterotic Calabi-Yau compactifications and, on the other hand, from IIB Calabi-Yau orientifold compactictifications with O3/O7 planes. For the former, we could show that the sign of $\hat{\omega}$ is minus the sign of Δ (the discriminant of the cubic function defining K), while for the latter the situation is the other way around (this is a consequence of a duality between both geometries). We have also demonstrated that – if Δ has the right sign – in the heterotic case all moduli can be made arbitrarily heavy by choosing a suitable superpotential, whereas for orientifold geometries at least one of the scalar's masses is not larger than $\sqrt{3+\gamma}\, m_{3/2}$.

We have finally analyzed in Sect. 3.3 what happens to $R(f^i)$ if the no-scale property of the Kähler potentials of Sect. 3.2 is sligthly violated by α' corrections. It turned out that the effect of the latter is to move the critical value of $R(f^i)$ a little bit away from the value 2/3 obtained for no-scale models – in which direction depends on the sign of the α' correction. This is similar for heterotic and orientifold geometries, the difference being just the numerical factors.

4 Constructing string-inspired models

4.1 General procedure

We now come to the question of how, for a given two-field Kähler potential which satisfies the necessary condition for metastability on the sign of Δ (as discussed in Sect. 3.2), one may find (preferably string-motivated) superpotentials such that the resulting scalar potential has a local minimum with a tiny positive cosmological constant. In principle, one could achieve this by making an ansatz for W with enough free parameters, then determine the stationary points as a function of the parameters, and finally choose parameters such that V and all masses have the desired positive value. However, it is very hard to find *non-supersymmetric* stationary points as soon as there are $n > 1$ complex fields and several parameters. The reason is that finding the stationary points amounts to solving a set of $2n$ coupled nonlinear equations.*

Our strategy will instead be to assume some reference values (T_0^1, T_0^2) for the fields and then to 'construct' a superpotential such that, at these reference values, the scalar potential has a local minimum with the desired vacuum energy.† The four masses of scalar fluctuations around the vacuum depend on the Taylor expansion (around the reference values) of W up to third order, i.e. we need to consider

$$W(T^1, T^2) = W_0 + W_i(T - T_0)^i + \frac{1}{2} W_{ij}(T - T_0)^i (T - T_0)^j + \frac{1}{6} W_{ijk}(T - T_0)^i (T - T_0)^j (T - T_0)^k + \cdots . \qquad (4.1)$$

The goal is to determine suitable coefficients W_0, W_i, W_{ij} and W_{ijk}. Since we are demanding stabilization at field values $(T^1, T^2) = (T_0^1, T_0^2)$, these coefficients depend on (T_0^1, T_0^2) via K and its derivatives evaluated at these field values. More precisely, they depend only on $\mathrm{Re}\, T_0^{1,2}$, because of the shift symmetry of K. Hence, the vevs of the axions $\mathrm{Im}\, T^{1,2}$ do not affect the coefficients in Eq. (4.1). This means that – once one has found suitable coefficients W_0, W_i, W_{ij} and W_{ijk} – one can insert arbitrary values for $\mathrm{Im}\, T_0^{1,2}$ in (4.1) and the resulting scalar potential has a stationary point with

*For this purpose, the algebraic method for finding stationary points (and the corresponding computer program) developed in Ref. [170] can be helpful. Still, in case of several free parameters, we could not find solutions in a reasonable time even using that powerful computer program.

†Of course, we do not solve the cosmological constant problem, but we can fine-tune the vacuum energy to a realistic value.

all four masses positive at that given vev. After having constructed the Taylor coefficients, one can then, in a final step, propose a string-motivated superpotential which has the required local behavior at $(T^1, T^2) = (T_0^1, T_0^2)$.

Let us now describe a systematic procedure for constructing the coefficients W_0, W_i, W_{ij} and W_{ijk}. Of course, we have in principle already done this at the end of Sect. 2.1, but we here outline the procedure in much more detail for the case of two-field models. Before starting, notice that the freedom in choosing the two vevs $T_0^{1,2}$ can be used to achieve any desired value for the volume $\mathcal{V}$, and a suitable positive value for the parameter $a_{\mathcal{H}}$ respectively $a_{\mathcal{O}}$. More precisely, the value of $a_{\mathcal{H}}$ respectively $a_{\mathcal{O}}$ fixes the ratio of T_0^1 and T_0^2, whereas the value of the volume $\mathcal{V}$ fixes their overall size. Note also from Eq. (4.1) that rescaling the vevs of the fields $T_0^{1,2}$ can be compensated by rescaling the coefficients appropriately, after factorizing out the overall superpotential scale W_0.

Tuning W_0

The coefficient W_0 is fixed, up to an irrelevant phase which we set to zero, by the gravitino mass and the volume. The relation $m_{3/2} = e^{G/2}$ means that

$$|W_0| = m_{3/2}\, e^{-K/2} \,. \tag{4.2}$$

Due to the different dependence of the Kähler potentials of heterotic and orientifold models on the volume, this equation translates into different relations between $m_{3/2}$ and $\mathcal{V}$ in heterotic and orientifold models. In the two cases one finds respectively

$$|W_0| = m_{3/2}\sqrt{\mathcal{V}_{\mathcal{H}}}\,, \qquad |W_0| = m_{3/2}\mathcal{V}_{\mathcal{O}}\,, \tag{4.3}$$

In any case, the value of $|W_0|$ fixes the overall scale of the potential.

Tuning W_i

The two coefficients W_i are fixed by the parameter γ and the direction of supersymmetry breaking that one desires to achieve. Indeed, one has by definition $G_i = K_i + W_i/W_0$, and G_i can be parametrized in terms of γ and f_i as $G_i = \sqrt{3(1+\gamma)}f_i$. Recalling also the definition $K_i = \sqrt{3}\,k_i$, it follows then that

$$\frac{W_i}{W_0} = \sqrt{3}\left(\sqrt{1+\gamma}\,f_i - k_i\right) \,. \tag{4.4}$$

This fixes W_i/W_0 in terms of γ and f_i. The direction f_i which we have parametrized by χ, ϕ and φ in Eq. (3.7) must be chosen such that $m^2 > 0$.

Tuning W_{ij}

The three coefficients W_{ij} are fixed by demanding stationarity of the potential at the reference field values, $\nabla_i V = 0$, and positivity of the two-dimensional diagonal blocks $V_{i\bar{\jmath}}$ of the mass matrix, which is necessary for positivity of the full mass matrix. It is convenient to first implement the stationarity conditions (2.1). This implies the following two relations, which allow to fix two of the three parameters W_{ij} in terms of the last one (understanding now G_i as fixed):

$$\frac{W_{ij}}{W_0} G^j = -(1+3\gamma) G_i - G_{\bar{\imath}} + \Gamma^k_{ij} G_k G^j + \frac{W_i W_j}{W_0^2} G^j \,. \tag{4.5}$$

The remaining parameter among the W_{ij} which is still free is then fixed by demanding positive-definiteness of the two-dimensional matrix $V_{i\bar{\jmath}}$. We have already ensured that the projection $m^2 = V_{i\bar{\jmath}} f^i f^{\bar{\jmath}}$ is positive in the last step. In order to see how $V_{i\bar{\jmath}}$ may be tuned to be positive definite, one calculates the projection of $V_{i\bar{\jmath}}$ along the direction u^i orthogonal to f^i, defined (up to an overall phase) by

$$\begin{aligned} u^i &= \cos\chi\, k^i - e^{i\phi} \sin\chi\, n^i \,, \qquad u_i = \cos\chi\, k_i - e^{-i\phi} \sin\chi\, n_i \,, \\ u^{\bar{\imath}} &= \cos\chi\, k^i - e^{-i\phi} \sin\chi\, n^i \,, \qquad u_{\bar{\imath}} = \cos\chi\, k_i - e^{i\phi} \sin\chi\, n_i \,. \end{aligned} \tag{4.6}$$

Using the fact that $\nabla_i G_j u^i f^j = 0$ by the stationarity condition, one finds

$$V_{i\bar{\jmath}} u^i u^{\bar{\jmath}} = \Big[1 + 3\gamma - 3(1+\gamma) R_{i\bar{\jmath}m\bar{n}} u^i u^{\bar{\jmath}} f^m f^{\bar{n}} + |\nabla_i G_j u^i u^j|^2\Big] m^2_{3/2} \tag{4.7}$$

$$V_{i\bar{\jmath}} u^i f^{\bar{\jmath}} = -3(1+\gamma)\, R_{i\bar{\jmath}m\bar{n}} u^i f^{\bar{\jmath}} f^m f^{\bar{n}}\, m^2_{3/2} \,. \tag{4.8}$$

From Eq. (4.7) we see that it is always possible to tune the quantity $\nabla_i G_j$ in order to make the last positive term arbitrary large and achieve $V_{i\bar{\jmath}} u^i u^{\bar{\jmath}} > 0$. This is compatible with the two stationarity conditions that also involve $\nabla_i G_j$, since there are three parameters W_{ij}. Notice also that the off-diagonal elements (4.8) are independent of $\nabla_i G_j$, so that it is always possible to make $V_{i\bar{\jmath}} u^i u^{\bar{\jmath}}$ large enough that both eigenvalues of $V_{i\bar{\jmath}}$ are positive.

Tuning W_{ijk}

Finally, the four coefficients W_{ijk} need to be chosen in such a way that all of the four eigenvalues of the full mass matrix $V_{I\bar{J}}$ are positive when taking into account the effect of the off-diagonal block V_{ij}. Solving the expression for V_{ij} in terms of the W_{ijk}, one deduces the following three relations (where now both G_i and $\nabla_i G_j$ are understood as fixed):

$$\begin{aligned}\frac{W_{ijk}}{W_0}G^k &= \Bigg[R_{ijkm}G^{\bar{m}} + \Gamma^m_{ij}\nabla_m G_k + \Gamma^m_{(ik}\nabla_m G_{j)} - 2\frac{W_iW_jW_k}{W_0^3} \\ &\quad +2\frac{W_{(i}W_{j)k}}{W_0^2} + \frac{W_kW_{ij}}{W_0^2} + \Gamma^m_{(ik}\left(\frac{W_{mj)}}{W_0} - \frac{W_mW_{j)}}{W_0^2}\right)\Bigg]G^k \\ &\quad -(2+3\gamma)\nabla_{(i}G_{j)} + 3\gamma\, G_iG_j + \frac{V_{ij}}{m^2_{3/2}}\,. \end{aligned} \tag{4.9}$$

Recall that for $V_{ij} = 0$, the mass spectrum is degenerate, with two states for each of the two eigenvalues of $V_{i\bar{j}}$, which have already been adjusted to be positive in the previous step. When instead $V_{ij} \neq 0$, the spectrum splits and one has to make sure that no eigenvalue becomes negative. This represents three constraints on the four parameters W_{ijk}. If for simplicity one requires $V_{ij} = 0$, then these become three relations, which allow to express three of the four parameters W_{ijk} in terms of the last one. More generally, however, one can leave V_{ij} arbitrary and compute the four eigenvalues as functions of the W_{ijk}'s. In generic situations it is hard to do this by hand, but it can easily be done with computer assistance. One can then scan the multi-parameter space of the W_{ijk}'s for regions where all masses are positive.

The next step is to match these 'local superpotentials' with the expansion of some string-motivated superpotential around the given vevs. To this end we will consider in the next section superpotentials with enough parameters and determine these parameters in such a way that the Taylor expansion around the reference field values matches the cubic superpotential constructed as we just outlined.

4.2 Explicit examples

Let us now apply the procedure described in the last subsection to construct some illustrative examples of string models with a sector of two volume moduli admitting a metastable dS vacuum. The value of the vacuum energy required to explain the observed accelerated expansion of the universe is of the order $V \sim 10^{-120}$ in Planck units. This is so small that we can set $\gamma = 0$ for the numerical examples we will discuss. Also, we will assume that the superpotentials are separable:

$$W(T^1, T^2) = W^{(1)}(T^1) + W^{(2)}(T^2)\,. \tag{4.10}$$

This choice implies further restrictions on the coefficients of the Taylor expansion of the superpotential about the vacuum, namely

$W_{12} = W_{112} = W_{221} = 0$. The existence of a solution with these characteristics is no longer guaranteed from the beginning. For instance, in the third step ('tuning W_{ij}'), one has only two free parameters in this case. It turns out however that it is nevertheless possible to find simple examples of this type.

4.2.1 IIB orientifold models

Let us start with type IIB orientifold models. For these models, a way in which the dilaton and the complex structure moduli may be stabilized is well understood [56], and restricting to the sector of volume moduli may be justified because the other moduli are generically stabilized at a higher mass scale than the Kähler moduli. In this case, the necessary condition for metastability is that the discriminant Δ (see (3.35)) should be positive. As a prototype example, let us take a CY manifold with intersection numbers given by $d^{111} = -1$, $d^{112} = 0$, $d^{122} = 1$ and $d^{222} = 0$, for which $\Delta = 108 > 0$. Using (3.43) and (3.44), the Kähler potential is then found to be

$$K = -\log\left[\frac{8}{9}\Big((T^1+\bar{T}^1) + \sqrt{(T^1+\bar{T}^1)^2 + (T^2+\bar{T}^2)^2}\Big)\left(\frac{(T^2+\bar{T}^2)^2 + (T^1+\bar{T}^1)^2 - (T^1+\bar{T}^1)\sqrt{(T^1+\bar{T}^1)^2+(T^2+\bar{T}^2)^2}}{T^2+\bar{T}^2}\right)^2\right]. \tag{4.11}$$

For definiteness we require that $a_{\mathcal{O}} = 1$ at the stationary point. As seen in Sect. 3.2.2, this choice allows to maximize the sGoldstino mass and corresponds to setting $\hat{s}^i = 0$. Using the definitions of $a_{\mathcal{O}}$ and $\mathcal{V}_{\mathcal{O}}$, one finds that the condition $a_{\mathcal{O}} = 1$ fixes the vevs of the real parts of the two fields to the following values, in units of $\mathcal{V}_{\mathcal{O}}^{2/3}$:

$\mathrm{Re}\, T_0^1$	0.412741
$\mathrm{Re}\, T_0^2$	0.714888

(4.12)

We then apply the procedure described in the previous section: $|W_0|$ is fixed by Eq. (4.3), W_1 and W_2 are fixed by the choice of the Goldstino direction, W_{11} and W_{22} are determined by the stationarity condition, while W_{111} and W_{222} are determined by computing the four masses as a function of W_{111} and W_{222} and searching for a region of parameters where all masses are positive. In this way we find that the local behavior that the superpotential needs to have is specified by, for instance, the following Taylor coefficients, which are given in units of $m_{3/2}\mathcal{V}_{\mathcal{O}}$ for W_0, $m_{3/2}\mathcal{V}_{\mathcal{O}}^{1/3}$ for W_i, $m_{3/2}\mathcal{V}_{\mathcal{O}}^{-1/3}$ for W_{ii} and

$m_{3/2}\mathcal{V}_{\mathcal{O}}^{-1}$ for W_{iii}:*

$$\begin{array}{|l|r|}\hline W_0 & 1.000000 \\ W_1 & 2.021311 \\ W_2 & 0.931223 \\ W_{11} & 0.999657 \\ W_{22} & -0.797685 \\ W_{111} & -0.827204 \\ W_{222} & 3.308820 \\ \hline \end{array} \tag{4.13}$$

For this example, the four physical square-mass eigenvalues m_i^2 at the minimum, obtained after canonically normalizing the fields, are given by 2.77, 2.95, 3.86, 5.14 in units of $m_{3/2}^2$.

Notice that the coefficients (4.13) scale in the following way with the size $T_0 \sim \mathcal{V}_{\mathcal{O}}^{2/3}$ of the field vevs:

$$W_0 : W_i : W_{ii} : W_{iii} \sim 1 : T_0^{-1} : T_0^{-2} : T_0^{-3} \,. \tag{4.14}$$

This scaling can be understood as naturally following from the structure of Eqs. (4.4), (4.5) and (4.9) (to see this, note that $K_i \sim T_0^{-1}$, $g_{ij} \sim T_0^{-2}$ etc). It is conceivable however that this scaling could be avoided with some additional fine-tuning of the parameters of the theory. This relation calls nevertheless for superpotentials with derivatives satisfying $(T)^n((\frac{\partial}{\partial T})^n W)/W \sim 1$.

Let us now try to match the coefficients (4.13) of the local expansion with an explicit superpotential of a form that may plausibly arise in type IIB orientifold models. The simplest possibility is to try with an exponential effective superpotential that typically arises from gaugino condensation. This has the simple form $W = Ae^{-aT}$ (where A and a are parameters), provided that $aT \gg 1$, corresponding to a weakly coupled 4d low-energy effective theory. For this type of superpotential, however, one gets $(T)^n((\frac{\partial}{\partial T})^n W)/W \sim (aT)^n$, which is much larger than 1 as soon as $aT \gg 1$. It is then not possible to reproduce the scaling (4.14). This problem might however be cured by adding a constant term $W = \Lambda$, or possibly also a linear term $W = FT$, which may arise from fluxes.† Notice also that one needs a superpotential with at least

*It should be clear that there is much freedom in this procedure and the parameters we provide are far from unique. We also stress that the fact that we choose to present the parameters with an accuracy of 6 digits does not mean that viable models involve a fine-tuning with a 6-digit-accuracy.

†The linear terms, which can result from metric fluxes and/or non-geometric fluxes [171], do not arise in Calabi-Yau compactifications (see [172] for a recent review). Note that the Kähler potential is at tree level unaffected by the presence of these fluxes. Such fluxes have also been used to construct supersymmetric vacua. See for instance Refs. [173, 174].

7 free parameters in order to be able to match all the local coefficients.

As a simple and 'symmetric' possibility to try out, one could then consider a superpotential with a constant term plus two exponential terms for each field:

$$W = \Lambda + A_1 e^{-a_1 T^1} + A_2 e^{-a_2 T^2} + B_1 e^{-b_1 T^1} + B_2 e^{-b_2 T^2} \,. \tag{4.15}$$

Such a combination of exponentials could arise for instance from gaugino condensation on two sets of D7-branes wrapping cycles controlled by the moduli T^1 and T^2, each giving rise to a gauge group consisting of two semisimple factors. This W has 9 coefficients which have to satisfy 7 equations. This allows to express 7 of them in terms of the other 2, say b_1 and b_2, and of the coefficients of the local superpotential. Among other relations, one finds that

$$a_i = -\frac{b_i W_{ii} + W_{iii}}{b_i W_i + W_{ii}} \,. \tag{4.16}$$

One can then choose the values of b_i in such a way that $b_i T_0^i \gg 1$, but by Eq. (4.14) one will then get $a_i T_0^i \sim 1$. This means that the constant term allows to make only some of the exponents in the exponential terms large, and some of them remain of order one, so that higher-order corrections may become relevant. We nevertheless present a numerical example of this type, given by the following values of the parameters, in units of $m_{3/2}\mathcal{V}_{\mathcal{O}}$ for Λ, A_i, B_i and $\mathcal{V}_{\mathcal{O}}^{-2/3}$ for a_i,b_i:

Λ	2.63036×10^{1}
A_1	7.37726×10^{1}
B_1	-9.77287×10^{1}
A_2	-1.50213×10^{0}
B_2	-2.80545×10^{0}

a_1	3.49830×10^{-1}
b_1	2.79764×10^{-1}
a_2	7.30908×10^{0}
b_2	4.19646×10^{-1}

(4.17)

These numbers are obtained by expanding the superpotential defined in Eq. (4.15) about the vev and then determining the Taylor coefficients of this expansion such that they match the values found in (4.13). Of course, this process is not unique, since there are 9 parameters for 7 equations.

A more satisfactory but slightly more complicated model may be obtained by adding linear terms. Let us consider for example the following form of the superpotential:

$$W = \Lambda + F_1 T^1 + F_2 T^2 + A_1 e^{-a_1 T^1} + A_2 e^{-a_2 T^2} + B_1 e^{-b_1 T^1} + B_2 e^{-b_2 T^2} \,. \tag{4.18}$$

While one still has $W_{iii}/W_{ii} = -a_i$, as this condition is unaffected by the addition of a linear term, the relation between the coefficients a_i, b_i and

W_{iii}/W_{ii} gets now more complicated and less constraining. This allows to find parameters such that all the exponents in the exponential terms are large. A working example of this type is obtained with the choice of parameters (4.19), in units of $m_{3/2}\mathcal{V}_{\mathcal{O}}$ for Λ, A_i, B_i, $m_{3/2}\mathcal{V}_{\mathcal{O}}^{1/3}$ for F_i and $\mathcal{V}_{\mathcal{O}}^{-2/3}$ for a_i, b_i.

Λ	-4.83093×10^{-1}
A_1	5.14986×10^{9}
B_1	-1.55366×10^{10}
A_2	-4.16798×10^{8}
B_2	2.38480×10^{10}

a_1	6.69463×10^{1}
b_1	6.99410×10^{1}
a_2	3.55839×10^{1}
b_2	4.19646×10^{1}

F_1	2.05036×10^{0}
F_2	8.92014×10^{-1}

(4.19)

Note that in order to achieve large values of the exponents $a_i T_0^i$, $b_i T_0^i$ at the minimum in this kind of models, one necessarily needs a hierarchy between the coefficients A_i, B_i of the gaugino condensation terms and the coefficients Λ and (if present) F_i. Indeed, in order for all the terms in W to be of comparable size at the minimum, the ratio of these two kinds of coefficients must be of order $e^{a_i T_0^i}$, $e^{b_i T_0^i}$. In (4.17) such a hierarchy is absent, because the exponents are of order one, whereas in (4.19) it is large, because the exponents are large.

The particular numbers chosen in the second example serve as an illustration but can correspond to realistic values for physical parameters. For a weak scale gravitino mass $m_{3/2} \sim 10^{-16} M_{\rm Pl} \sim 100\,\text{GeV}$ and a reasonably large volume in Planck units $\mathcal{V}_{\mathcal{O}} \sim 10^3$, one has $A_i^{1/3}, B_i^{1/3} \sim 10^{-1} M_{\rm Pl} \sim 10^{17}$ GeV, which is a not unrealistic gaugino condensation scale, and $\Lambda^{1/3} \sim 10^{-4} M_{\rm Pl} \sim 10^{14}\,\text{GeV}$, which could also be reasonable.

4.2.2 Heterotic models

Let us now consider heterotic models. In this case, the way in which the dilaton and the complex structure moduli may be stabilized (in particular at a high scale and at supersymmetric points) is less understood, but we will nevertheless assume that these do not play any role and focus on two volume moduli. As an explicit example satisfying the necessary condition $\Delta < 0$, let us consider a CY manifold with intersection numbers $d_{111} = 1$, $d_{112} = 0$, $d_{122} = 1$ and $d_{222} = 0$, for which $\Delta = -108 < 0$. Using Eq. (3.23), the corresponding Kähler potential is found to be

$$K = -\log\left[\frac{1}{6}(T^1 + \bar{T}^1)^3 + \frac{1}{2}(T^1 + \bar{T}^1)(T^2 + \bar{T}^2)^2\right]. \tag{4.20}$$

We choose in this case the vevs in such a way that $a_{\mathcal{H}} = 9$. The motivation for this choice is that for heterotic models we decide to fix the sGoldstino direction such that $\hat{s}^i = 0$, and in that case the maximal value of $\hat{\sigma}$ is reached for $a_{\mathcal{H}} = 9$, as one easily checks. This choice $\hat{s}^i = 0$ does not correspond to the largest possible sGoldstino mass (which is achieved for $\hat{s}^i \neq 0$), but it has the virtue of maintaining some similarity with the orientifold examples. The condition $a_{\mathcal{H}} = 9$ leads then to the following values of the vevs, in units of $\mathcal{V}_{\mathcal{H}}^{1/3}$:

$\mathrm{Re}\, T_0^1$	0.405666
$\mathrm{Re}\, T_0^2$	0.749277

(4.21)

Applying the same procedure as for orientifold models, one finds the following set of local parameters, in units of $m_{3/2}\mathcal{V}_{\mathcal{H}}^{1/2}$ for W_0, $m_{3/2}\mathcal{V}_{\mathcal{H}}^{1/6}$ for W_i, $m_{3/2}\mathcal{V}_{\mathcal{H}}^{-1/6}$ for W_{ii} and $m_{3/2}\mathcal{V}_{\mathcal{H}}^{-1/2}$ for W_{iii}:

W_0	1.00000
W_1	1.64415
W_2	2.60392
W_{11}	−17.4400
W_{22}	3.82418
W_{111}	616.732
W_{222}	2.31275

(4.22)

In this model, the four physical square-mass eigenvalues m_i^2 at the minimum are given by 4.43, 5.95, 203.88 and 311.92 in units of $m_{3/2}^2$.

We may now proceed as for orientifold models and fit these coefficients with a superpotential involving exponential, constant or linear terms. In this case, however, the possible origin of such terms is less clear as for orientifolds. For instance, gaugino condensation produces exponential contributions, but with an exponent involving in the first approximation only the dilaton. It is however common that the effective gauge coupling receives perturbative threshold corrections depending on the volume moduli as well. Assuming then that the dilaton does not play any role and the volume moduli are large, one can be left with an exponent linear in T. Notice moreover that, taking this perspective, there is no reason to require any longer that the exponent should be large and positive (see for example [175, 176]). As a toy illustrative example with enough parameters, we can thus again consider a superpotential of the form (4.15). In the same way as for orientifold models, one can then, for example, reproduce the local coefficients (4.22) with the following values of parameters, in units of $m_{3/2}\mathcal{V}_{\mathcal{H}}^{1/2}$ for Λ, A_i, B_i and $\mathcal{V}_{\mathcal{H}}^{-1/3}$

for a_i, b_i:

Λ	-5.97604×10^{-1}
A_1	-3.62358×10^{5}
B_1	-1.46692×10^{0}
A_2	7.98841×10^{-1}
B_2	7.49672×10^{-1}

a_1	4.36876×10^{1}
b_1	2.66924×10^{0}
a_2	-1.28225×10^{0}
b_2	5.33848×10^{0}

(4.23)

As before, the hierarchy arising between some of the coefficients A_i, B_i and Λ is related to the fact that some of the exponents $a_i T_0^i$, $b_i T_0^i$ are large at the minimum. In this case, for $m_{3/2} \sim 10^{-16}$ and $\mathcal{V}_{\mathcal{H}} \sim 10^3$ in Planck units, the particular numbers chosen in the example yield $A_i^{1/3}, B_i^{1/3} \sim 10^{13} - 10^{15}$ GeV and $\Lambda^{1/3} \sim 10^{13}$GeV.

Summary of Section 4:

The aim of this Section has been to build viable models, starting with a string-derived two-field no-scale Kähler potential for which models with metastable dS vacua are possible (i.e. Δ has the right sign, as discussed in Sect. 3.2). In other words, the task has been to find a corresponding string-motivated superpotential, such that the resulting scalar potential has a local minimum with realistic vacuum energy at a point in the moduli space where the effective 4d supergravity approximation can be trusted. To this end one could in principle have made an ansatz for the superpotential, identified the stationary points of the resulting scalar potential and chosen parameters such that all masses are positive. Since it is in practice however extremely difficult to find non-supersymmetric stationary points, we followed a more feasible route: We first specified field values where we wish the scalar potential to have a minimum and then constructed a superpotential which at these field values has the desired properties (but is not yet string-motivated). We outlined how this can be achieved in Sect. 4.1. In a second step, this 'local superpotential' could then be matched to a string-motivated one. The scalar potential which results from such a string-motivated W (and the corresponding K) then has a minimum at the field values which were specified in the beginning.

We presented explicit examples in Sect. 4.2, focussing on separable superpotentials. For IIB compactifications, where integrating out the complex structure moduli and the dilaton can be justified, we found a working example which employs terms arising both from fluxes and from nonperturbative effects (Eq. (4.18)). The coefficients we presented are given in units of the gravitino mass and the volume, so that we in fact exhibited a whole family of examples. For illustration, we also presented heterotic examples (even though in this case it is not so clear that neglecting the effects of the complex structure moduli and the dilaton can be justified).

5 Conclusions and outlook

The main objective of this thesis has been to make progress on the question of how dark energy and slow-roll inflation could be realized in string theory.

After motivating this area of research and briefly reviewing some background material in Sect. 1, we derived in Sect. 2, following Refs. [102–105], a necessary condition (which we called ‘metastability condition’) which has to be met in order for local minima with energy $V = 3\gamma\, m_{3/2}^2 M_{pl}^2$ in the scalar potential of arbitrary 4d $\mathcal{N} = 1$ supergravity theories (without vector multiples however) with supersymmetry broken spontaneously by F-terms to exist: Along the sGoldstino direction, the holomorphic sectional curvature $R(f^i)$ of the Kähler manifold spanned by the scalar fields must be smaller than $2/[3(1+\gamma)]$. (This is a consequence of the structure of supergravity theories and is independent of whether the supergravity theory is derived from string theory or not.)

Furthermore, we have proven in Sect. 2, following Ref. [105], that – up to small corrections determined by the flatness parameters – the same condition has to be satisfied in order for models of slow-roll inflation with a Hubble parameter $H = \sqrt{\gamma}\, m_{3/2}$ to be possible. In practice, the main difference between both situations is then that in realistic models of inflation the Hubble parameter is usually much larger than the gravitino mass so that one needs $R(f^i) \lesssim 0$, whereas for a realistic vacuum one has $\gamma \simeq 0$ so that only $R(f^i) \lesssim 2/3$ is required.

The sectional curvature $R(f^i)$ at a certain point in the moduli space depends both on the Riemann curvature tensor of the scalar manifold, which is defined by the Kähler potential of the supergravity theory, and on the direction f^i of supersymmetry breaking, which depends also on the superpotential. However, in some cases the range of $R(f^i)$ is restricted, independently of the superpotential, so that – if it turns out that $R(f^i)$ cannot be smaller than $2/[3(1+\gamma)]$ – one obtains a no-go theorem which, for the corresponding Kähler potentials, excludes dS vacua respectively inflation (with a scale determined by γ) independently of the form of the superpotential. On the other hand, for those cases where a given Kähler potential does allow to fulfill the metastability condition, we could show that it is always possible to stabilize *all* moduli – given the freedom to choose an arbitrary superpotential. In this sense, the condition that $R(f^i)$ must be smaller than $2/[3(1+\gamma)]$ may be considered not only a necessary, but also a sufficient condition for building viable models. Of course, if one insists that the superpotential should come from string theory, the choice for it is restricted. Nevertheless, there is still a very large amount of freedom (such as from fluxes and nonperturbative effects) to obtain superpotentials in string theory.

The topic of Sect. 3 has been, following Ref. [104] and also [130], to analyze $R(f^i)$ for certain classes of scalar geometries which occur in 4d low-energy effective theories derived from heterotic CY compactifications and type IIB CY orientifold compactifications. In particular, our interest was in the Kähler potentials for the geometric moduli of such string compactifications. In the limit of large volume (respectively large complex structure), these exhibit a no-scale structure. Motivated by this, we first studied in Sect. 3.1 the properties of $R(f^i)$ for arbitrary no-scale Kähler potentials. This lead to the interesting result that $R(f^i)$ is stationary for $f^i = k^i$ with critical value $2/3$, implying that in order to decide if de Sitter vacua are possible or not one needs to know the convexity of $R(f^i)$ at that point. In Sect. 3.2, we were able to show that, in case of two-moduli models, the latter is determined by the discriminant Δ of the cubic function which specifies the Kähler potential.

The final point of Sect. 3 has been the investigation of the effects which subleading corrections to the no-scale property of the scalar geometry in α' have on the sectional curvature. We have demonstrated that this effect consists in moving the critical value slightly away from the value $2/3$, thus allowing to circumvent the no-go theorem (this should be qualitatively the same for other corrections, such as string loop corrections). It is then feasible to also construct practicable models with Kähler potentials which are excluded at leading order. The restriction is however that the squared mass of the lightest scalar, respectively the square of the Hubble parameter, can in this case be at most of the order of $m_{3/2}^2$ times the small parameter specifying the corrections.

Section 4, which is based on Ref. [130], was devoted to building concrete models, i.e. to finding suitable string-motivated superpotentials which together with an eligible Kähler potential (which was found in Sect. 3) yield a scalar potential with a realistic metastable vacuum. This was not as simple as it may have appeared at first sight due to the difficulty of identifying non-supersymmetric stationary points in multi-field models. For this reason, we developed a systematic procedure to 'construct' viable models by imposing on them the desired properties only locally at the vacuum. In this way, we were able to find classes of string-motivated models admitting viable metastable vacua, both for type IIB CY orientifold compactifications and for heterotic CY compactifications. The superpotentials in these examples were of a form which can emerge from fluxes and gaugino condensation effects. The fact that these models need to have more than one dynamical field and at least seven independent parameters in the superpotential to allow for the construction is probably the reason why such models have not been noticed earlier. It is still an open question to study more realistic, more generic and more minimal models, but there now exist working examples for dS vacua arising from simple F-term supersymmetry breaking in both type IIB and heterotic compactifications.

We believe that these results emphasize in a clear way that it is actually possible to achieve genuine metastable dS vacua even in models satisfying the no-scale property, provided that the scalar geometry is sufficiently generic. This is the case for the volume moduli sector in the large-volume limit of smooth CY compactifications when at least two moduli arise. But of course, the examples we provide should be considered as toy models. In order to construct fully realistic models, there are several other issues to be addressed. One of them is the detailed mechanism stabilizing the other moduli and the impact of their dynamics on the dS vacuum admitted by the volume moduli sector (see e.g. [59, 141–144]). Another is the life-time of the metastable dS vacuum against decay to other supersymmetric AdS vacua that generically arise at different values of the fields (see e.g. [177–180]).

Also, we would like to stress that the presence of vector multiplets giving D-term contributions to supersymmetry breaking (which we have assumed to play no role in our analysis) can potentially further improve the situation [106]. More precisely, increasing the ratio between the D-term and F-term contributions (for a fixed value of V) has the net effect of making the left-hand side of (2.10) smaller and therefore making that constraint milder [106].

Let us finally give an outlook to future directions of research.

It would certainly be interesting to extent the analysis of Sect. 4 in such a way as to find examples which not only have a realistic vacuum, but also include a trajectory which is suitable for slow-roll inflation. To do this in a systematic way will be more challenging than finding models which only have a realistic vacuum, because in this case – in order to obtain a model leading to a viable cosmological evolution – it is not enough to impose local properties at one point in field space, but a whole trajectory has to be specified in principle. Nevertheless, we believe that this should be manageable.

Furthermore, it would be very useful to investigate how the results which were obtained in Sect. 3.2 are modified for related, but more general scalar geometries. For instance, one may wonder if a simple condition also arises in case that the coordinates of the moduli space for type IIB orientifolds with O3/O7 planes are not given by the relation (3.44), (which applies when $dim(H^{p,q}_-) = 0$), but by a more complicated relation, as specified in Ref. [148] (which applies when $dim(H^{p,q}_-) \neq 0$). Besides that, one would like to know what can be said for scalar geometries of the type discussed in Sect. 3.2, but with more than two moduli. At least for situations where supersymmetry breaking is dominated by two moduli, one may expect that an analysis similar to that done in Sect. 3.2 can be performed.

A Details of Kähler geometries

In this Appendix, we collect some useful formulae concerning the geometry of Kähler manifolds, which are needed in some derivations in the main text.

A.1 Logarithmic Kähler potentials

Let us consider a Kähler potential of the form $K = -n \log Y$, where Y is some real function of the scalar fields ϕ^i and n is a real number. Denoting by $Y^{i\bar{\jmath}}$ the inverse of $Y_{i\bar{\jmath}}$, one easily finds

$$
\begin{aligned}
K_i &= -n\frac{Y_i}{Y}\,, \\
g_{i\bar{\jmath}} &= -n\frac{Y_{i\bar{\jmath}}}{Y} + n\frac{Y_i Y_{\bar{\jmath}}}{Y^2} = -n\frac{Y_{i\bar{\jmath}}}{Y} + \frac{1}{n}K_i K_{\bar{\jmath}}\,, \\
g^{i\bar{\jmath}} &= -\frac{Y Y^{i\bar{\jmath}}}{n} + \frac{1}{n}\frac{1}{\theta - 1}Y^{i\bar{r}}Y_{\bar{r}}Y^{\bar{\jmath}s}Y_s = -\frac{Y Y^{i\bar{\jmath}}}{n} + \frac{\theta - 1}{n}K^i K^{\bar{\jmath}}\,, \\
K^i &= -\frac{1}{\theta - 1}Y^{i\bar{r}}Y_{\bar{r}}\,.
\end{aligned} \tag{A.1}
$$

The quantity θ is defined as

$$
\theta \equiv \frac{Y_i Y^{i\bar{\jmath}} Y_{\bar{\jmath}}}{Y}\,, \tag{A.2}
$$

and controls the value of the contraction defining the no-scale property:

$$
K^i K_i = n\frac{\theta}{\theta - 1}\,. \tag{A.3}
$$

The third derivatives of K are

$$
\begin{aligned}
K_{i\bar{\jmath}m} &= -\frac{n}{Y}Y_{i\bar{\jmath}m} + \frac{n}{Y^2}(Y_i Y_{\bar{\jmath}m} + Y_m Y_{\bar{\jmath}i} + Y_{\bar{\jmath}}Y_{im}) - \frac{2n}{Y^3}Y_i Y_{\bar{\jmath}} Y_m\,, \\
K_{i\bar{\jmath}\bar{n}} &= -\frac{n}{Y}Y_{i\bar{\jmath}\bar{n}} + \frac{n}{Y^2}(Y_{\bar{\jmath}} Y_{i\bar{n}} + Y_{\bar{n}} Y_{i\bar{\jmath}} + Y_i Y_{\bar{\jmath}\bar{n}}) - \frac{2n}{Y^3}Y_i Y_{\bar{\jmath}} Y_{\bar{n}}\,.
\end{aligned} \tag{A.4}
$$

Finally, the Riemann tensor for the Kähler manifold is

$$
\begin{aligned}
R_{i\bar{\jmath}m\bar{n}} &= K_{i\bar{\jmath}m\bar{n}} - K_{im\bar{r}}g^{\bar{r}s}K_{s\bar{\jmath}\bar{n}} \\
&= \frac{1}{n}\big(g_{i\bar{\jmath}}\,g_{m\bar{n}} + g_{i\bar{n}}\,g_{m\bar{\jmath}}\big) - \frac{n}{Y}Y_{i\bar{\jmath}m\bar{n}} - \frac{n}{Y^2}(nY_{im\bar{s}}g^{\bar{s}r}Y_{r\bar{\jmath}\bar{n}} + \frac{1}{\theta - 1}Y_{im}Y_{\bar{\jmath}\bar{n}}) \\
&\quad + \frac{n^2}{Y^3}(Y_{im}Y_{\bar{\jmath}\bar{n}r}g^{r\bar{s}}Y_{\bar{s}} + Y_{\bar{\jmath}\bar{n}}Y_{im\bar{s}}g^{\bar{s}r}Y_r)\,.
\end{aligned} \tag{A.5}
$$

A.2 Special Kähler geometries

We now consider the case of special Kähler geometries, for which the Kähler potential $K = -\log Y$ itself admits a holomorphic prepotential F, in terms of which [149]

$$Y = -2(F + \bar{F}) + (F_k + \bar{F}_{\bar{k}})(\phi^k + \bar{\phi}^k)\,. \tag{A.6}$$

The Riemann tensor simplifies substantially in this case. Indeed, one easily computes $Y_i + Y_{\bar{\imath}} = N_{ij}\big(\phi^j + \bar{\phi}^{\bar{\jmath}}\big)$ and $Y_{i\bar{\jmath}} = N_{ij}$, where $N_{ij} = F_{ij} + \bar{F}_{\bar{\imath}\bar{\jmath}}$. Combining these two expressions, one gets then $Y^{i\bar{\jmath}}(Y_j + Y_{\bar{\jmath}}) = (\phi^i + \bar{\phi}^{\bar{\imath}})$. Finally, combining this result with $Y_{ij} = F_{ijk}\big(\phi^k + \bar{\phi}^{\bar{k}}\big)$ and $Y_{ij\bar{k}} = F_{ijk}$, one obtains the relation $Y_{ij\bar{s}}Y^{\bar{s}r}\big(Y_r + Y_{\bar{r}}\big) = Y_{ij}$. Using these relations, one finally finds [150]

$$R_{i\bar{\jmath}m\bar{n}} = g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}}\, g_{m\bar{\jmath}} - \frac{1}{Y^2} F_{imr} g^{r\bar{s}} \bar{F}_{\bar{s}\bar{\jmath}\bar{n}}\,. \tag{A.7}$$

A.3 Kähler geometries from IIB orientifolds

Here we provide some details on how the Riemann tensor for the particular geometry which is discussed in Sect. 3.2.2 (defined by the Kähler potential (3.43)) can be calculated. To this end, it is convenient to first compute derivatives of

$$g^{ij} = e^{-K} d^{ijk} K_k + K^i K^j\,. \tag{A.8}$$

Using the relations (3.45) – (3.47) one finds

$$\begin{aligned} \big[g^{ij}\big]_k &= e^{-K} d^{ijm} g_{mk} - (g^{ij} - K^i K^j) K_k - \delta^i_k K^j - \delta^j_k K^i\,, && \text{(A.9)}\\ \big[g^{ij}\big]_{mn} &= -e^{-2K} d^{ijp} g_{pq} d^{qrs} g_{rm} g_{sn} + \delta^i_m \delta^j_n + \delta^i_n \delta^j_m\,. && \text{(A.10)} \end{aligned}$$

The third derivatives of K and the Riemann tensor are expressed in terms of these derivatives as

$$\begin{aligned} K_{ijm} &= -g_{ip}[g^{pq}]_j g_{qm}\,, && \text{(A.11)}\\ R_{ijmn} &= -g_{ip} g_{qj} [g^{pq}]_{mn} + g_{ir}[g^{rp}]_m g_{pq} [g^{qs}]_n g_{sj}\,. && \text{(A.12)} \end{aligned}$$

This then leads to

$$\begin{aligned} K_{ijm} &= e^{-K} \hat{d}_{ijm} - 3\, g_{(ij} K_{m)} + K_i K_j K_k\,, && \text{(A.13)}\\ R_{ijmn} &= -g_{im} g_{jn} + 2\, e^{-2K} \hat{d}_{i(j|k} g^{kl} \hat{d}_{l|n)m} + 6\, g_{(ij} K_m K_{n)} \\ &\quad -3\, K_i K_j K_m K_n - 4\, e^{-K} \hat{d}_{(ijm} K_{n)}\,, && \text{(A.14)} \end{aligned}$$

where we abbreviated

$$\hat{d}_{ijk} \equiv g_{ip} g_{jq} g_{kl} d^{pql}\,. \tag{A.15}$$

B Dual Kähler geometries

Here, we would like to prove that Eq. (3.34) implies Eq. (3.55). To this end, we make use of a 'duality' between, on the one hand, the scalar geometry (a) which was discussed in Sect. 3.2.1 and is defined by the Kähler potential

$$K = -\log \mathcal{V} \, , \qquad \text{with} \qquad \mathcal{V} = \frac{4}{3}\, d_{ijk}\, v^i v^j v^k \, , \tag{B.1}$$

with coordinates

$$T^i = v^i + ib^i \, , \tag{B.2}$$

and, on the other hand, the scalar geometry (b), which was discussed in Sect. 3.2.2 and is defined by the Kähler potential*

$$\hat{K} = 2(K + \log 64) \tag{B.3}$$

with coordinates $\hat{T}_i = \rho_i + i\zeta_i$, where

$$\rho_i = \frac{1}{16} d_{ijk} v^j v^k \, . \tag{B.4}$$

Note that in this Appendix we label the Kähler potential, the coordinates and derived quantities of the scalar geometry (b) with a 'hat' in order to distinguish them from those of the 'heterotic geometry' (a). Also, we interchanged upper and lower indices in comparison to the notation of Sect. 3.2.2, which is more convenient for the purposes of this Appendix.

One can then rewrite the metric of the heterotic geometry (3.27) and the inverse of the metric of the orientifold geometry (3.47) as

$$g_{ij} = \frac{1}{4\mathcal{V}^2}\left(\frac{\partial \mathcal{V}}{\partial v^i}\frac{\partial \mathcal{V}}{\partial v^j} - \mathcal{V}\frac{\partial^2 \mathcal{V}}{\partial v^i \partial v^j}\right) \tag{B.5}$$

$$\hat{g}_{ij} = \frac{4}{64^2}\left(\frac{\partial \mathcal{V}}{\partial v^i}\frac{\partial \mathcal{V}}{\partial v^j} - \mathcal{V}\frac{\partial^2 \mathcal{V}}{\partial v^i \partial v^j}\right) . \tag{B.6}$$

We stress that in this notation $\hat{g}_{ij}$ is the inverse metric for (b) while g_{ij} is the metric for (a), as explained above. The two geometries (a) and (b) are thus dual, in the sense that [160]

$$\hat{g}_{ij} = \frac{\mathcal{V}^2}{2^8} g_{ij} \, . \tag{B.7}$$

*The constant '$2\log 64$' is introduced in order to have the same 'normalization' for the Kähler potential as in Sect. 3.2.2. Note that in this Appendix, $\mathcal{V}$ is always defined by $\mathcal{V} = \frac{4}{3}\, d_{ijk}\, v^i v^j v^k$.

In particular, this implies

$$(\det g)^{-1} = \det \hat{g}(2^4 e^{-\hat{K}})^n\,, \tag{B.8}$$

for n-dimensional Kähler manifolds. Using this and (B.3), one immediately finds for the case $n = 2$ that

$$\frac{e^{4K}}{(\det g)^3} = \frac{(\det \hat{g})^3}{e^{4\hat{K}}}\,. \tag{B.9}$$

One also easily finds that $K_i = (16/\mathcal{V})\hat{K}_i$. Using this, (B.3), (B.8) and the definition (3.18) of the unit vector n^i orthogonal to K^i for two-field models, one finds that

$$(n^1, n^2) = 4e^{-\hat{K}/2}(\hat{n}^1, \hat{n}^2)\,. \tag{B.10}$$

Using this and (B.3), one finds that

$$e^K d_{ijk} n^i n^j n^k = e^{-\hat{K}} d_{ijk} \hat{n}^i \hat{n}^i \hat{n}^k\,, \tag{B.11}$$

which shows that $a_{\mathcal{O}} = -a_{\mathcal{H}}$ (see the definitions (3.33) and (3.54)). Then, using (B.9) one sees that indeed Eq. (3.34) implies Eq. (3.55).

Bibliography

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," Astron. J. **116** (1998) 1009 [arXiv:astro-ph/9805201].

[2] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," Astrophys. J. **517** (1999) 565 [arXiv:astro-ph/9812133].

[3] A. H. Jaffe *et al.* [Boomerang Collaboration], "Cosmology from Maxima-1, Boomerang and COBE/DMR CMB Observations," Phys. Rev. Lett. **86** (2001) 3475 [arXiv:astro-ph/0007333].

[4] C. Pryke, N. W. Halverson, E. M. Leitch, J. Kovac, J. E. Carlstrom, W. L. Holzapfel and M. Dragovan, "Cosmological Parameter Extraction from the First Season of Observations with DASI," Astrophys. J. **568** (2002) 46 [arXiv:astro-ph/0104490].

[5] D. N. Spergel *et al.* [WMAP Collaboration], "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters," Astrophys. J. Suppl. **148** (2003) 175 [arXiv:astro-ph/0302209].

[6] D. N. Spergel *et al.* [WMAP Collaboration], "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology," Astrophys. J. Suppl. **170** (2007) 377 [arXiv:astro-ph/0603449].

[7] D. J. Eisenstein *et al.* [SDSS Collaboration], "Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies," Astrophys. J. **633** (2005) 560 [arXiv:astro-ph/0501171].

[8] A. G. Sanchez *et al.*, "Cosmological parameters from CMB measurements and the final 2dFGRS power spectrum," Mon. Not. Roy. Astron. Soc. **366** (2006) 189 [arXiv:astro-ph/0507583].

[9] M. Tegmark *et al.* [SDSS Collaboration], "Cosmological Constraints from the SDSS Luminous Red Galaxies," Phys. Rev. D **74** (2006) 123507 [arXiv:astro-ph/0608632].

[10] E. J. Copeland, M. Sami and S. Tsujikawa, "Dynamics of dark energy," Int. J. Mod. Phys. D **15** (2006) 1753 [arXiv:hep-th/0603057].

[11] J. Frieman, M. Turner and D. Huterer, "Dark Energy and the Accelerating Universe," Ann. Rev. Astron. Astrophys. **46** (2008) 385 [arXiv:0803.0982 [astro-ph]].

[12] R. R. Caldwell and M. Kamionkowski, "The Physics of Cosmic Acceleration," arXiv:0903.0866 [astro-ph.CO].

[13] M. Sami, "A primer on problems and prospects of dark energy," arXiv:0904.3445 [hep-th].

[14] S. Weinberg, "The cosmological constant problem," Rev. Mod. Phys. **61** (1989) 1.

[15] A. D. Linde, "A New Inflationary Universe Scenario: A Possible Solution Of The Horizon, Flatness, Homogeneity, Isotropy And Primordial Monopole Problems," Phys. Lett. B **108** (1982) 389.

[16] A. Albrecht and P. J. Steinhardt, "Cosmology For Grand Unified Theories With Radiatively Induced Symmetry Breaking," Phys. Rev. Lett. **48** (1982) 1220.

[17] D. H. Lyth and A. Riotto, "Particle physics models of inflation and the cosmological density perturbation," Phys. Rept. **314** (1999) 1 [arXiv:hep-ph/9807278].

[18] A. Riotto, "Inflation and the theory of cosmological perturbations," arXiv:hep-ph/0210162.

[19] A. Linde, "Inflationary Cosmology," Lect. Notes Phys. **738** (2008) 1 [arXiv:0705.0164 [hep-th]].

[20] V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuation And Nonsingular Universe. (In Russian)," JETP Lett. **33** (1981) 532 [Pisma Zh. Eksp. Teor. Fiz. **33** (1981) 549].

[21] S. W. Hawking, "The Development Of Irregularities In A Single Bubble Inflationary Universe," Phys. Lett. B **115** (1982) 295.

[22] A. A. Starobinsky, "Dynamics Of Phase Transition In The New Inflationary Universe Scenario And Generation Of Perturbations," Phys. Lett. B **117** (1982) 175.

[23] A. H. Guth and S. Y. Pi, "Fluctuations In The New Inflationary Universe," Phys. Rev. Lett. **49** (1982) 1110.

[24] J. M. Bardeen, P. J. Steinhardt and M. S. Turner, "Spontaneous Creation Of Almost Scale - Free Density Perturbations In An Inflationary Universe," Phys. Rev. D **28** (1983) 679.

[25] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," Phys. Rev. D **64** (2001) 123522 [arXiv:hep-th/0103239].

[26] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol 1: Introduction." *Cambridge, Uk: Univ. Pr. (1987) 469 P. (Cambridge Monographs On Mathematical Physics)* and
"Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies And Phenomenology," *Cambridge, Uk: Univ. Pr. (1987) 596 P. (Cambridge Monographs On Mathematical Physics)*

[27] J. Polchinski, "String theory. Vol. 1: An introduction to the bosonic string," *Cambridge, UK: Univ. Pr. (1998) 402 p* and
"String theory. Vol. 2: Superstring theory and beyond," *Cambridge, UK: Univ. Pr. (1998) 531 p*

[28] K. Becker, M. Becker and J. H. Schwarz, "String theory and M-theory: A modern introduction," *Cambridge, UK: Cambridge Univ. Pr. (2007) 739 p*

[29] W. Siegel, "Introduction to string field theory," arXiv:hep-th/0107094.

[30] C. Kiefer, "Quantum gravity," Int. Ser. Monogr. Phys. **124** (2004) 1.

[31] F. Denef, M. R. Douglas and S. Kachru, "Physics of string flux compactifications," Ann. Rev. Nucl. Part. Sci. **57** (2007) 119 [arXiv:hep-th/0701050].

[32] J. H. Schwarz, "Lectures on superstring and M theory dualities," Nucl. Phys. Proc. Suppl. **55B** (1997) 1 [arXiv:hep-th/9607201].

[33] A. Strominger, S. T. Yau and E. Zaslow, "Mirror symmetry is T-duality," Nucl. Phys. B **479** (1996) 243 [arXiv:hep-th/9606040].

[34] C. Vafa, "Evidence for F-Theory," Nucl. Phys. B **469** (1996) 403 [arXiv:hep-th/9602022].

[35] B. R. Greene, "String theory on Calabi-Yau manifolds," arXiv:hep-th/9702155.

[36] M. Graña, "Flux compactifications in string theory: A comprehensive review," Phys. Rept. **423** (2006) 91 [arXiv:hep-th/0509003].

[37] M. R. Douglas and S. Kachru, "Flux compactification," Rev. Mod. Phys. **79** (2007) 733 [arXiv:hep-th/0610102].

[38] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, "Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes," Phys. Rept. **445** (2007) 1 [arXiv:hep-th/0610327].

[39] F. Denef, "Les Houches Lectures on Constructing String Vacua," arXiv:0803.1194 [hep-th].

[40] M. Dine and N. Seiberg, "Is The Superstring Weakly Coupled?," Phys. Lett. B **162** (1985) 299.

[41] J. Polchinski and A. Strominger, "New Vacua for Type II String Theory," Phys. Lett. B **388** (1996) 736 [arXiv:hep-th/9510227].

[42] J. Michelson, "Compactifications of type IIB strings to four dimensions with non-trivial classical potential," Nucl. Phys. B **495** (1997) 127 [arXiv:hep-th/9610151].

[43] R. Bousso and J. Polchinski, "Quantization of four-form fluxes and dynamical neutralization of the cosmological constant," JHEP **0006** (2000) 006 [arXiv:hep-th/0004134].

[44] L. Susskind, "The anthropic landscape of string theory," arXiv:hep-th/0302219.

[45] M. R. Douglas, "The statistics of string/M theory vacua," JHEP **0305** (2003) 046 [arXiv:hep-th/0303194].

[46] J. Polchinski, "The cosmological constant and the string landscape," arXiv:hep-th/0603249.

[47] A. Vilenkin, "The Birth Of Inflationary Universes," Phys. Rev. D **27** (1983) 2848.

[48] A. A. Starobinsky, "Stochastic de Sitter (inflationary) stage in the early universe," *In 'De Vega, H.j. (Ed.), Sanchez, N. (Ed.): Field Theory, Quantum Gravity and Strings', 107-126*

[49] A. D. Linde, "Eternally Existing Selfreproducing Chaotic Inflationary Universe," Phys. Lett. B **175** (1986) 395.

[50] A. S. Goncharov, A. D. Linde and V. F. Mukhanov, “The Global Structure Of The Inflationary Universe,” Int. J. Mod. Phys. A **2** (1987) 561.

[51] A. H. Guth, “Eternal Inflation,” arXiv:astro-ph/0101507.

[52] S. Weinberg, “Anthropic Bound on the Cosmological Constant,” Phys. Rev. Lett. **59** (1987) 2607.

[53] G.W. Gibbons, ”Aspects of Supergravity Theories”, in Supersymmetry, Supergravity and Related Topics, eds. F. del Aguila, J.A. de Azcarraga and L.E. Ibanez (World Scientific 1985) pp. 346-35.

[54] B. de Wit, D. J. Smit and N. D. Hari Dass, “Residual Supersymmetry of Compactified D=10 Supergravity,” Nucl. Phys. B **283** (1987) 165.

[55] J. M. Maldacena and C. Nunez, “Supergravity description of field theories on curved manifolds and a no go theorem,” Int. J. Mod. Phys. A **16** (2001) 822 [arXiv:hep-th/0007018].

[56] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” Phys. Rev. D **66** (2002) 106006 [arXiv:hep-th/0105097].

[57] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].

[58] F. Denef and M. R. Douglas, “Distributions of flux vacua,” JHEP **0405** (2004) 072 [arXiv:hep-th/0404116].

[59] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, “Stability of flux compactifications and the pattern of supersymmetry breaking,” JHEP **0411** (2004) 076 [arXiv:hep-th/0411066].

[60] D. Lust, S. Reffert, W. Schulgin and S. Stieberger, “Moduli stabilization in type IIB orientifolds. I: Orbifold limits,” Nucl. Phys. B **766** (2007) 68 [arXiv:hep-th/0506090] and
D. Lust, S. Reffert, E. Scheidegger, W. Schulgin and S. Stieberger, “Moduli stabilization in type IIB orientifolds. II,” Nucl. Phys. B **766** (2007) 178 [arXiv:hep-th/0609013].

[61] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, “Systematics of moduli stabilisation in Calabi-Yau flux compactifications,” JHEP **0503** (2005) 007 [arXiv:hep-th/0502058].

[62] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha'-corrections to flux induced potentials," JHEP **0206**, 060 (2002) [arXiv:hep-th/0204254].

[63] G. von Gersdorff and A. Hebecker, "Kaehler corrections for the volume modulus of flux compactifications," Phys. Lett. B **624** (2005) 270 [arXiv:hep-th/0507131].

[64] H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110** (1984) 1.

[65] J. Wess and J. Bagger, "Supersymmetry and supergravity," *Princeton, USA: Univ. Pr. (1992) 259 p*

[66] A. A. Starobinsky, "Spectrum of relict gravitational radiation and the early state of the universe," JETP Lett. **30** (1979) 682 [Pisma Zh. Eksp. Teor. Fiz. **30** (1979) 719] and
"A new type of isotropic cosmological models without singularity," Phys. Lett. B **91** (1980) 99.

[67] A. H. Guth, "The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems," Phys. Rev. D **23** (1981) 347.

[68] A. D. Linde, "Chaotic Inflation," Phys. Lett. B **129** (1983) 177.

[69] A. D. Linde, "Hybrid inflation,"
Phys. Rev. D **49** (1994) 748 [arXiv:astro-ph/9307002].

[70] E. W. Kolb and M. S. Turner, "The Early universe," Front. Phys. **69** (1990) 1.

[71] A. R. Liddle, P. Parsons and J. D. Barrow, "Formalizing the slow roll approximation in inflation," Phys. Rev. D **50** (1994) 7222 [arXiv:astro-ph/9408015].

[72] A. Vilenkin and L. H. Ford, "Gravitational Effects Upon Cosmological Phase Transitions," Phys. Rev. D **26** (1982) 1231.

[73] A. D. Linde, "Scalar Field Fluctuations In Expanding Universe And The New Inflationary Universe Scenario," Phys. Lett. B **116** (1982) 335.

[74] E. Silverstein, "Simple de Sitter Solutions," Phys. Rev. D **77**, 106006 (2008) [arXiv:0712.1196 [hep-th]].

[75] S. S. Haque, G. Shiu, B. Underwood and T. Van Riet, "Minimal simple de Sitter solutions," Phys. Rev. D **79** (2009) 086005 [arXiv:0810.5328 [hep-th]].

[76] C. P. Burgess, R. Kallosh and F. Quevedo, "de Sitter string vacua from supersymmetric D-terms," JHEP **0310** (2003) 056 [arXiv:hep-th/0309187].

[77] M. Becker, G. Curio and A. Krause, "De Sitter vacua from heterotic M-theory," Nucl. Phys. B **693** (2004) 223 [arXiv:hep-th/0403027].

[78] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, "Soft supersymmetry breaking in KKLT flux compactification," Nucl. Phys. B **718** (2005) 113 [hep-th/0503216];

[79] F. Saueressig, U. Theis and S. Vandoren, "On de Sitter vacua in type IIA orientifold compactifications," Phys. Lett. B **633** (2006) 125 [arXiv:hep-th/0506181].

[80] G. Villadoro and F. Zwirner, "de Sitter vacua via consistent D-terms," Phys. Rev. Lett. **95** (2005) 231602 [arXiv:hep-th/0508167] and "D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications," JHEP **0603** (2006) 087 [arXiv:hep-th/0602120].

[81] A. Achucarro, B. de Carlos, J. A. Casas and L. Doplicher, "de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings," JHEP **0606** (2006) 014 [arXiv:hep-th/0601190];

[82] O. Lebedev, H. P. Nilles and M. Ratz, "de Sitter vacua from matter superpotentials," Phys. Lett. B **636** (2006) 126 [arXiv:hep-th/0603047].

[83] K. Choi and K. S. Jeong, "Supersymmetry breaking and moduli stabilization with anomalous U(1) gauge symmetry," JHEP **0608** (2006) 007 [arXiv:hep-th/0605108].

[84] E. Dudas and Y. Mambrini, "Moduli stabilization with positive vacuum energy," JHEP **0610** (2006) 044 [arXiv:hep-th/0607077].

[85] E. Dudas, C. Papineau and S. Pokorski, "Moduli stabilization and uplifting with dynamically generated F-terms," JHEP **0702** (2007) 028 [arXiv:hep-th/0610297].

[86] C. P. Burgess, J. M. Cline, K. Dasgupta and H. Firouzjahi, "Uplifting and inflation with D3 branes," JHEP **0703** (2007) 027 [arXiv:hep-th/0610320].

[87] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, "Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms," Phys. Rev. D **75** (2007) 025019 [arXiv:hep-th/0611024].

[88] R. Kallosh and A. Linde, "O'KKLT," JHEP **0702** (2007) 002 [arXiv:hep-th/0611183].

[89] O. Lebedev, V. Lowen, Y. Mambrini, H. P. Nilles and M. Ratz, "Metastable vacua in flux compactifications and their phenomenology," JHEP **0702** (2007) 063 [arXiv:hep-ph/0612035].

[90] D. Cremades, M. P. Garcia del Moral, F. Quevedo and K. Suruliz, "Moduli stabilisation and de Sitter string vacua from magnetised D7 branes," JHEP **0705** (2007) 100 [arXiv:hep-th/0701154].

[91] H. Abe, T. Higaki and T. Kobayashi, "More about F-term uplifting," Phys. Rev. D **76** (2007) 105003 [arXiv:0707.2671 [hep-th]].

[92] P. Brax, A. C. Davis, S. C. Davis, R. Jeannerot and M. Postma, "Warping and F-term uplifting," JHEP **0709** (2007) 125 [arXiv:0707.4583 [hep-th]].

[93] E. Dudas, Y. Mambrini, S. Pokorski and A. Romagnoni, "Moduli stabilization with Fayet-Iliopoulos uplift," JHEP **0804** (2008) 015 [arXiv:0711.4934 [hep-th]].

[94] A. Achucarro and K. Sousa, "F-term uplifting and moduli stabilization consistent with Kahler invariance", arXiv:0712.3460 [hep-th].

[95] V. Balasubramanian and P. Berglund, "Stringy corrections to Kaehler potentials, SUSY breaking, and the cosmological constant problem," JHEP **0411** (2004) 085 [arXiv:hep-th/0408054].

[96] S. L. Parameswaran and A. Westphal, "de Sitter string vacua from perturbative Kaehler corrections and consistent D-terms," JHEP **0610** (2006) 079 [arXiv:hep-th/0602253].

[97] A. Westphal, "de Sitter String Vacua from Kahler Uplifting," JHEP **0703** (2007) 102 [arXiv:hep-th/0611332].

[98] M. Berg, M. Haack and E. Pajer, "Jumping through loops: on soft terms from large volume compactifications," JHEP **0709** (2007) 031 [arXiv:0704.0737 [hep-th]].

[99] E. Palti, G. Tasinato and J. Ward, "Weakly-coupled IIA Flux Compactifications." JHEP **0806** (2008) 084 [arXiv:0804.1248 [hep-th]].

[100] R. Brustein and S. P. de Alwis, "Moduli potentials in string compactifications with fluxes: Mapping the discretuum," Phys. Rev. D **69** (2004) 126006 [arXiv:hep-th/0402088].

[101] F. Denef and M. R. Douglas, "Distributions of nonsupersymmetric flux vacua," JHEP **0503** (2005) 061 [arXiv:hep-th/0411183].

[102] M. Gomez-Reino and C. A. Scrucca, "Locally stable non-supersymmetric Minkowski vacua in supergravity," JHEP **0605** (2006) 015 [arXiv:hep-th/0602246].

[103] M. Gomez-Reino and C. A. Scrucca, "Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity," JHEP **0609** (2006) 008 [arXiv:hep-th/0606273].

[104] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma and C. A. Scrucca, "de Sitter vacua in no-scale supergravities and Calabi-Yau string models," JHEP **0806** (2008) 057 [arXiv:0804.1073 [hep-th]].

[105] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma and C. A. Scrucca, "Constraints on modular inflation in supergravity and string theory," JHEP **0808** (2008) 055 [arXiv:0805.3290 [hep-th]].

[106] M. Gomez-Reino and C. A. Scrucca, "Metastable supergravity vacua with F and D supersymmetry breaking," JHEP **0708** (2007) 091 [arXiv:0706.2785 [hep-th]].

[107] P. Binetruy, M. K. Gaillard and Y. Y. Wu, "Modular invariant formulation of multi-gaugino and matter condensation," Nucl. Phys. B **493** (1997) 27 [arXiv:hep-th/9611149].

[108] J. A. Casas, "The generalized dilaton supersymmetry breaking scenario," Phys. Lett. B **384** (1996) 103 [hep-th/9605180].

[109] M. A. Luty and R. Sundrum, "Radius stabilization and anomaly-mediated supersymmetry breaking," Phys. Rev. D **62** (2000) 035008 [arXiv:hep-th/9910202].

[110] J. Polonyi, "Generalization of the Massive Scalar Multiplet Coupling to the Supergravity," *Hungary Central Inst Res - KFKI-77-93 (77,REC.JUL 78) 5p*

[111] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, "False vacuum inflation with Einstein gravity," Phys. Rev. D **49** (1994) 6410 [arXiv:astro-ph/9401011].

[112] P. Binetruy and M. K. Gaillard, "Candidates For The Inflaton Field In Superstring Models," Phys. Rev. D **34** (1986) 3069.

[113] R. Brustein and P. J. Steinhardt, "Challenges for superstring cosmology," Phys. Lett. B **302** (1993) 196 [arXiv:hep-th/9212049].

[114] T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore and P. J. Steinhardt, "Modular Cosmology," Phys. Rev. D **52** (1995) 3548 [arXiv:hep-th/9503114].

[115] J.J. Blanco-Pillado, C.P. Burgess, J.M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. Linde, F. Quevedo, "Racetrack inflation," JHEP **0411** (2004) 063 [arXiv:hep-th/0406230].

[116] Z. Lalak, G. G. Ross and S. Sarkar, "Racetrack inflation and assisted moduli stabilisation," Nucl. Phys. B **766** (2007) 1 [arXiv:hep-th/0503178].

[117] A. Westphal, "Eternal inflation with alpha' corrections," JCAP **0511** (2005) 003 [arXiv:hep-th/0507079].

[118] J. P. Conlon and F. Quevedo, "Kaehler moduli inflation," JHEP **0601** (2006) 146 [arXiv:hep-th/0509012].

[119] J.J. Blanco-Pillado, C.P. Burgess, J.M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. Linde, F. Quevedo, "Inflating in a better racetrack," JHEP **0609** (2006) 002 [arXiv:hep-th/0603129].

[120] J. Simon, R. Jimenez, L. Verde, P. Berglund and V. Balasubramanian, "Using cosmology to constrain the topology of hidden dimensions," arXiv:astro-ph/0605371 and
"Topology from Cosmology," JHEP **0806** (2008) 025 [arXiv:0712.1815 [hep-th]].

[121] J. R. Ellis, Z. Lalak, S. Pokorski and K. Turzynski, "The price of WMAP inflation in supergravity," JCAP **0610** (2006) 005 [arXiv:hep-th/0606133].

[122] J. R. Bond, L. Kofman, S. Prokushkin and P. M. Vaudrevange, "Roulette inflation with Kaehler moduli and their axions," Phys. Rev. D **75** (2007) 123511 [arXiv:hep-th/0612197].

[123] B. de Carlos, J. A. Casas, A. Guarino, J. M. Moreno and O. Seto, "Inflation in uplifted supergravities," JCAP **0705** (2007) 002 [arXiv:hep-th/0702103].

[124] Z. Lalak, D. Langlois, S. Pokorski and K. Turzynski, "Curvature and isocurvature perturbations in two-field inflation," JCAP **0707** (2007) 014 [arXiv:0704.0212 [hep-th]].

[125] T. W. Grimm, "Axion Inflation in Type II String Theory," Phys. Rev. D **77** (2008) 126007 [arXiv:0710.3883 [hep-th]].

[126] P. Brax, A. C. Davis, S. C. Davis, R. Jeannerot and M. Postma, "D-term Uplifted Racetrack Inflation," JCAP **0801** (2008) 008 [arXiv:0710.4876 [hep-th]].

[127] A. Linde and A. Westphal, "Accidental Inflation in String Theory," JCAP **0803** (2008) 005 [arXiv:0712.1610 [hep-th]].

[128] M. Badziak and M. Olechowski, "Volume modulus inflation and a low scale of SUSY breaking," JCAP **0807** (2008) 021 [arXiv:0802.1014 [hep-th]].

[129] I. Ben-Dayan, R. Brustein and S. P. de Alwis, "Models of Modular Inflation and Their Phenomenological Consequences," JCAP **0807** (2008) 011 [arXiv:0802.3160 [hep-th]].

[130] L. Covi, M. Gomez-Reino, C. Gross, G. A. Palma and C. A. Scrucca, "Constructing de Sitter vacua in no-scale string models without uplifting," JHEP **0903** (2009) 146 [arXiv:0812.3864 [hep-th]].

[131] P. Breitenlohner and D. Z. Freedman, "Stability In Gauged Extended Supergravity," Annals Phys. **144** (1982) 249.

[132] M. Gomez-Reino, J. Louis and C. A. Scrucca, "No metastable de Sitter vacua in N=2 supergravity with only hypermultiplets," JHEP **0902** (2009) 003 [arXiv:0812.0884 [hep-th]].

[133] M. T. Grisaru, M. Rocek and A. Karlhede, "The Superhiggs Effect In Superspace," Phys. Lett. B **120** (1983) 110.

[134] H. P. Nilles, "Supergravity Generates Hierarchies," Nucl. Phys. B **217** (1983) 366.

[135] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, "Spontaneous Symmetry Breaking And Higgs Effect In Supergravity Without Cosmological Constant," Nucl. Phys. B **147** (1979) 105.

[136] M. Sasaki and E. D. Stewart, "A General Analytic Formula For The Spectral Index Of The Density Perturbations Produced During Inflation," Prog. Theor. Phys. **95** (1996) 71 [arXiv:astro-ph/9507001].

[137] C. P. Burgess, J. M. Cline, H. Stoica and F. Quevedo, "Inflation in realistic D-brane models," JHEP **0409** (2004) 033 [arXiv:hep-th/0403119].

[138] S. Groot Nibbelink and B. J. W. van Tent, "Density perturbations arising from multiple field slow-roll inflation," [arXiv:hep-ph/0011325] and
"Scalar perturbations during multiple field slow-roll inflation," Class. Quant. Grav. **19** (2002) 613 [arXiv:hep-ph/0107272].

[139] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, "Naturally Vanishing Cosmological Constant In N=1 Supergravity," Phys. Lett. B **133** (1983) 61.

[140] T. W. Grimm, "The effective action of type II Calabi-Yau orientifolds," Fortsch. Phys. **53**, 1179 (2005) [arXiv:hep-th/0507153].

[141] S. P. de Alwis, "Effective potentials for light moduli," Phys. Lett. B **626**, 223 (2005) [arXiv:hep-th/0506266].

[142] A. Achucarro, S. Hardeman and K. Sousa, "F-term uplifting and the supersymmetric integration of heavy moduli," JHEP **0811** (2008) 003 [arXiv:0809.1441 [hep-th]] and
"Consistent Decoupling of Heavy Scalars and Moduli in N=1 Supergravity", Phys. Rev. D **78** (2008) 101901 [arXiv:0806.4364 [hep-th]].

[143] D. Gallego and M. Serone, "An Effective Description of the Landscape - I," JHEP **0901** (2009) 056 [arXiv:0812.0369 [hep-th]] and
"An Effective Description of the Landscape - II," arXiv:0904.2537 [hep-th].

[144] L. Brizi, M. Gomez-Reino and C. A. Scrucca, "Globally and locally supersymmetric effective theories for light fields," Nucl. Phys. B **820** (2009) 193 [arXiv:0904.0370 [hep-th]].

[145] P. Candelas and X. de la Ossa, "Moduli space of Calabi-Yau manifolds," Nucl. Phys. B **355**, 455 (1991).

[146] A. Strominger, "Yukawa Couplings In Superstring Compactification," Phys. Rev. Lett. **55** (1985) 2547.

[147] L. J. Dixon, V. Kaplunovsky and J. Louis, "On Effective Field Theories Describing (2,2) Vacua of the Heterotic String," Nucl. Phys. B **329**, 27 (1990).

[148] T. W. Grimm and J. Louis, "The effective action of N = 1 Calabi-Yau orientifolds," Nucl. Phys. B **699**, 387 (2004) [arXiv:hep-th/0403067].

[149] B. de Wit and A. Van Proeyen, "Potentials And Symmetries Of General Gauged N=2 Supergravity: Yang-Mills Models," Nucl. Phys. B **245**, 89 (1984).

[150] E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, "Vector Multiplets Coupled To N=2 Supergravity: Superhiggs Effect, Flat Potentials And Geometric Structure," Nucl. Phys. B **250**, 385 (1985).

[151] M. Bodner and A. C. Cadavid, "Dimensional Reduction of Type IIB Supergravity and Exceptional Quaternionic Manifolds," Class. Quant. Grav. **7** (1990) 829.

[152] R. Bohm, H. Gunther, C. Herrmann and J. Louis, "Compactification of type IIB string theory on Calabi-Yau threefolds," Nucl. Phys. B **569** (2000) 229 [arXiv:hep-th/9908007].

[153] M. Bodner, A. C. Cadavid and S. Ferrara, "(2,2) vacuum configurations for type IIA superstrings: N=2 supergravity Lagrangians and algebraic geometry," Class. Quant. Grav. **8** (1991) 789.

[154] S. Ferrara and S. Sabharwal, "Quaternionic Manifolds for Type II Superstring Vacua of Calabi-Yau Spaces," Nucl. Phys. B **332** (1990) 317.

[155] S. Ferrara and S. Sabharwal, "Dimensional Reduction of Type II Superstrings," Class. Quant. Grav. **6** (1989) L77.

[156] S. Cecotti, S. Ferrara and L. Girardello, "Geometry of Type II Superstrings and the Moduli of Superconformal Field Theories," Int. J. Mod. Phys. A **4** (1989) 2475.

[157] B. S. Acharya, M. Aganagic, K. Hori and C. Vafa, "Orientifolds, mirror symmetry and superpotentials," arXiv:hep-th/0202208.

[158] I. Brunner and K. Hori, "Orientifolds and mirror symmetry," JHEP **0411** (2004) 005 [arXiv:hep-th/0303135].

[159] T. W. Grimm and J. Louis, "The effective action of type IIA Calabi-Yau orientifolds," Nucl. Phys. B **718** (2005) 153 [arXiv:hep-th/0412277].

[160] R. D'Auria, S. Ferrara and M. Trigiante, "c-map,very special quaternionic geometry and dual Kaehler spaces," Phys. Lett. B **587** (2004) 138 [arXiv:hep-th/0401161].

[161] R. D'Auria, S. Ferrara and M. Trigiante, "Homogeneous special manifolds, orientifolds and solvable coordinates," Nucl. Phys. B **693** (2004) 261 [arXiv:hep-th/0403204].

[162] D. H. Lyth and E. D. Stewart, "Thermal Inflation And The Moduli Problem," Phys. Rev. D **53** (1996) 1784 [arXiv:hep-ph/9510204].

[163] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, "Cosmological problems for the Polonyi potential," Phys. Lett. B **131** (1983) 59.

[164] J. R. Ellis, D. V. Nanopoulos and M. Quiros, "On the Axion, Dilaton, Polonyi, Gravitino and Shadow Matter Problems in Supergravity and Superstring Models," Phys. Lett. B **174** (1986) 176.

[165] M. Y. Khlopov and A. D. Linde, "Is It Easy To Save The Gravitino?," Phys. Lett. B **138** (1984) 265.

[166] T. Banks, D. B. Kaplan and A. E. Nelson, "Cosmological Implications of Dynamical Supersymmetry Breaking," Phys. Rev. D **49** (1994) 779 [arXiv:hep-ph/9308292].

[167] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, "Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings," Phys. Lett. B **318** (1993) 447 [arXiv:hep-ph/9308325].

[168] J. P. Conlon and F. Quevedo, "Astrophysical and Cosmological Implications of Large Volume String Compactifications," JCAP **0708** (2007) 019 [arXiv:0705.3460 [hep-ph]].

[169] P. Candelas, X. C. De La Ossa, P. S. Green and L. Parkes, "A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory," Nucl. Phys. B **359**, 21 (1991).

[170] J. Gray, Y. H. He and A. Lukas, "Algorithmic algebraic geometry and flux vacua," JHEP **0609** (2006) 031 [hep-th/0606122] and
J. Gray, Y. H. He, A. Ilderton and A. Lukas, "A new method for finding vacua in string phenomenology," JHEP **0707** (2007) 023 [arXiv:hep-th/0703249] and
J. Gray, Y. H. He, A. Ilderton and A. Lukas, "STRINGVACUA: A Mathematica Package for Studying Vacuum Configurations in String Phenomenology," Comput. Phys. Commun. **180** (2009) 107 [arXiv:0801.1508 [hep-th]].

[171] I. Benmachiche and T. W. Grimm, "Generalized N = 1 orientifold compactifications and the Hitchin functionals," Nucl. Phys. B **748** (2006) 200 [arXiv:hep-th/0602241].

[172] B. Wecht, "Lectures on Nongeometric Flux Compactifications," Class. Quant. Grav. **24** (2007) S773 [arXiv:0708.3984 [hep-th]].

[173] A. Micu, E. Palti and G. Tasinato, "Towards Minkowski vacua in type II string compactifications," JHEP **0703**, 104 (2007) [arXiv:hep-th/0701173].

[174] E. Palti, "Low energy supersymmetry from non-geometry," JHEP **0710**, 011 (2007) [arXiv:0707.1595 [hep-th]].

[175] M. Badziak and M. Olechowski, "Volume modulus inflection point inflation and the gravitino mass problem," JCAP **0902** (2009) 010 [arXiv:0810.4251 [hep-th]].

[176] M. Serone and A. Westphal, "Moduli stabilization in meta-stable heterotic supergravity vacua" JHEP **0708** (2007) 080 [arXiv:0707.0497 [hep-th]].

[177] S. R. Coleman and F. De Luccia, "Gravitational effects on and of vacuum decay," Phys. Rev. D **21** (1980) 3305.

[178] S. Weinberg, "Does gravitation resolve the ambiguity among supersymmetry vacua?," Phys. Rev. Lett. **48** (1982) 1776.

[179] T. Banks, "Heretics of the false vacuum: Gravitational effects on and of vacuum decay. II," arXiv:hep-th/0211160.

[180] R. Kallosh and A. Linde, "Landscape, the scale of SUSY breaking, and inflation," JHEP **0412** (2004) 004 [arXiv:hep-th/0411011].

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