Higher-Dimension Operators in Higher-Dimensional Field Theories

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Abstract

The present thesis investigates effects of higher-dimension operators in higher-dimensional field theories. 1-loop power corrections to the gauge couplings in 5d and 6d are studied. They become calculable in the case of a softly broken grand unified theory (GUT) combined with minimal symmetry assumptions. Using the analysis of the exact quantum effective action of supersymmetric gauge theories in 5d, these power corrections are demonstrated to be the effect of higher-dimension operators. They can be unambiguously determined within 5d low-energy effective field theory. Thus, large and quantitatively controlled power-law contributions to gauge couplings arise naturally and can, in the most extreme case, lead to calculable TeV-scale power law unification. A simple 5d SU(5) model with one massless **10** in the bulk is identified where the power-law effect is exactly MSSM-like.

In compactifications of 10d type IIB supergravity higher-dimension operators are an important ingredient for the stabilization of the moduli. Their cosmological impact in stabilized string vacua is analyzed and a model of inflation is constructed where these operators are essential. This model can accommodate the WMAP data of the cosmic microwave background (CMB) with a spectral index of the density fluctuations $n_s = 0.93$.

Zusammenfassung

Die vorliegende Arbeit untersucht Effekte von Operatoren höherer Massendimension in höherdimensionalen Feldtheorien. Zunächst werden 1-Loop-Potenzkorrekturen zu den Eichkopplungen in 5d und 6d analysiert. Diese werden im Falle weicher Brechung einer GUT und minimaler Symmetrieannahmen berechenbar. Die Potenzkorrekturen werden dann mittels Analyse der exakten quanten-effektiven Wirkung supersymmetrischer Eichtheorien in 5d auf die Effekte von Operatoren höherer Massendimension zurückgeführt. Diese Operatoren sind durch die Eigenschaften der 5d effektiven Feldtheorie eindeutig bestimmt. Damit ergibt sich, daß diese Quantenkorrekturen zu den Eichkopplungen berechenbare 'Power-Law'-Vereinigung an der TeV-Skala ermöglichen. Es wird ein einfaches 5d SU(5) GUT Modell mit einer Materie-**10** im 5d 'Bulk' beschrieben, in dem die Quantenkorrekturen exakt die Vereinigung der Eichkopplungen im MSSM imitieren.

Operatoren höherer Massendimension sind in Kompaktifizierungen der 10d Typ IIB Supergravitation ein wichtiger Faktor bei der Modulistabilisierung. Die kosmologischen Implikationen dieser Operatoren werden untersucht und es wird ein Inflationsmodell konstruiert, das essentiell auf ihrer Gegenwart beruht. Dieses Modell mit einem spektralen Index der Dichtefluktuationen von $n_s = 0.93$ ist mit den WMAP-Daten des kosmischen Mikrowellenhintergrunds verträglich.

Contents

Introduction

How can we describe this world? This is a question physics has tried to answer in terms of a set of laws as simple and elegant (and thus aesthetic in a sense) as possible since its very beginnings. Coupled with the fact that the laws of nature appear to be written in mathematical language - a highly nontrivial fact in itself - it is tempting to describe physics as being the quest for unification. On the long way to the goal of a theory of everything, some steps have already been taken successfully. Maxwell's electromagnetism merged the phenomena of electricity and magnetism, Einstein explained gravitation as a dynamical property of space-time. And after the advent of quantum mechanics, field and quantum theory were forged together to yield quantum field theory which, in turn, led to the unification of the electromagnetic and the weak interactions. Combined with QCD this structure forms the Standard Model (SM) of particle physics today.

The next step of this process is still speculative. There are, however, two facts which may point us in the right direction. Firstly, grand unified theories (GUTs), which are built from gauge symmetries described by the powerful structure of Lie groups, provide an elegant explanation of the fermion quantum numbers of the Standard Model (SM) [1] (also [2, 3]). Secondly, supersymmetry (SUSY) links together matter fields - fermions - and force fields - bosons - in a unified picture.

Despite the beauty of GUTs in providing symmetries which merge the different force and matter sectors, they suffer from the fact that the unified symmetry is, in a sense, too large. Symmetry has to be broken to be able to generate the world as we see it, and this is where the conventional GUTs become ugly. The additional matter and force fields of a GUT have to be super heavy ($\sim 10^{16}$ GeV) in order to accommodate the absence of proton decay in present experiments. However, in view of the sensitivity of the scalar sector of conventional GUTs to quadratically divergent radiative corrections, this leads to the hierarchy problem: the doublet-triplet splitting of the light $SU(2)$-Higgs doublet from its colored partners that necessarily exist in a 4d GUT has to be protected despite getting contributions from quadratically divergent loop corrections.

It has been possible to alleviate this problem to some extent with supersymmetry. This symmetry, which pairs each fermion with a boson of equal mass, can protect the Higgs from the quadratic corrections and thus stabilize its mass. Furthermore, the minimal supersymmetric extension of the SM, the MSSM [4], led to the phenomenological success of gauge coupling unification [5]. This has established high-scale SUSY GUTs as the standard framework for the discussion of physics above the electroweak scale. Still, the problems concerning symmetry breaking are not fully solved by SUSY. The gauge hierarchy between the EW and the GUT scale is stabilized by SUSY, however, problems such as the origin of the doublet-triplet splitting remain. In addition, low-energy phenomena are not supersymmetric, so SUSY has to be broken, too, above the EW scale.

Another direction of unification which has been around since Einstein is the geometrization of interactions. The Kaluza-Klein (KK) mechanism of compactifying field theories in a higher-dimensional space-time ($d > 4$) down to $\mathcal{M}_4 \times C$, where C denotes the compact internal manifold, allows for deriving gauge invariance geometrically as consisting of invariance under coordinate transformations along the internal directions. Furthermore, new geometrical ways of symmetry breaking arise by KK-compactification on topologically nontrivial internal manifolds.

The next big leap came when people started considering extended one-dimensional objects instead of point particles: string theory presently appears to be the only candidate for a finite theory of quantum gravity. Simultaneously, it contains an internal structure rich enough to encompass chiral fermions and non-Abelian gauge interactions. The requirement of internal consistency then led to the discovery that string theory demands both a 10d space-time and the presence of supersymmetry. (Supersymmetry was discovered this way first.) Superstring theory is therefore the only known candidate for a fully unified theory of all interactions. This provided additional strong motivation for studying higher-dimensional effective field theories since the superstring contains as its low energy effective theory 10d supergravity.

During the last few years the above 4d SUSY GUT paradigm has been challenged by various scenarios with extra dimensions compactified at scales below $M_{\rm GUT} \sim 10^{16}$ GeV. In particular, Dienes, Dudas and Gherghetta [6] have proposed low-scale gauge unification as a possible consequence of power-like loop corrections to gauge coupling constants [7, 8]. However, objections to this proposal were raised on the basis that the relevant loop corrections are completely UV dominated and that, as a result, no precise statement about the ratio of low-energy gauge couplings can be made without a UV completion of the higher-dimensional SM-like theory (see, e.g., [9, 10]). The issue of gauge coupling unification in higher dimensions (sometimes called 'power law running') was also discussed in connection with 'deconstruction' and warped 5d models (see, e.g., [11] and [12–14]). The geometric nature of gravity forces the geometric properties of higher-dimensional field theories to become dynamical if gravity is taken into account. A very interesting area in this context is concerned with the existence and structure of non-trivial string vacua where all moduli fields are stabilized. Moduli are 4d scalar fields which describe the geometric properties of the compactification from 10d to 4d. Recent studies have shown the possibility to construct such fully stabilized string vacua [15] using fluxes [16–31] and non-perturbative effects such as gaugino condensation. Another interesting topic deals with the cosmological aspects of the so-called string vacuum 'landscape' since it became known that the number of possible string vacua is enormous (a number in the range of 10^{500} [32]).

The present investigation concerns itself with the effects of higher-dimension operators in higher-dimensional field theories which can appear in the effective action at tree level or through radiative loop corrections.

The first part of this thesis focuses on loop corrections to the gauge couplings of higher-dimensional field theories. Their power-like behavior has sometimes been called 'power law running' [6] and the discussion will thus concentrate on this term. The power-like loop corrections derived in this thesis are shown to be numerically important, calculable, and of a universal nature in very general situations, if the GUT group is softly broken in the weak-coupling regime of the higher-dimensional theory [33]. Firstly, the general regime of validity for such power corrections to the gauge couplings is discussed. Then the structure of higher-dimensional SUSY is analyzed in its consequences for the structure of the quantum effective action of the higher-dimensional GUT and its power-like loop corrections. The central result here is that the power-like loop corrections can be understood entirely as the effect of higher-dimension operators in the effective action which are completely determined by the low-energy effective theory [34].

The second part concentrates on another area where higher-dimension operators can be important. In type IIB superstring compactifications they can serve to stabilize all moduli which are not fixed by fluxes in phenomenologically interesting 4d de Sitter (dS) vacua with a small cosmological constant. Higher-order α'-corrections are higher-dimension operators which are generically present in string theory. Together with the phenomenon of gaugino condensation in super Yang-Mills theory (SYM) they lead to a scalar potential which can stabilize all moduli. An analysis of this scalar potential demonstrates that one can have slow-roll inflation driven by the volume modulus of the compact Calabi-Yau manifold which ends in fully stabilized and phenomenologically interesting dS minima.

The first part begins with a short introduction (Chapter 1) into the issues of higher-dimensional field theories, their Kaluza-Klein reduction on compact extra dimensions to 4d and the problems of the so-called 'power law running' of gauge couplings in extra dimensions. Some of the original ideas will be summarized and it will be made clear why the naive power law scaling of gauge couplings in higher dimensions with the UV cutoff parameter cannot be used to discuss unification - and how this problem can be solved [33].

Chap. 2 implements these general ideas in the context of higher-dimensional Yang-Mills theory with a bulk GUT Higgs field Φ. Given an appropriate bulk potential, Φ will develop a symmetry breaking VEV leading to massive vector bosons and a number of massive physical scalars whose mass spectrum depends on the parameters of the GUT-Higgs scalar potential. In five dimensions, minimal symmetry assumptions suffice to forbid operators linear in Φ to obtain potentially sizeable, controlled power corrections [33] (cf. also [35]). The 1-loop power corrections are derived in this general setup both in a manifestly five-dimensional calculation and in the context of KK-mode summation [33]. Then general constraints of supersymmetry are analyzed in this context. For instance, potentially large and calculable corrections arise from GUT breaking by the adjoint scalar of the 5d vector multiplet [33]. The chapter finishes with investigating the basic phenomenological aspects of the above generic GUT scenario with or without SUSY for an SU(5) GUT compactified on an orbifold, an internal space with singularities. Orbifolds are attractive, since they break the higher SUSY of 5d to the level of 4d $\mathcal{N} = 1$ and may avoid the otherwise rapid proton decay. (This idea, already discussed in the present context in [6], has more recently been extensively used in the context of orbifold GUTs in 5d [36–38] and 6d [39, 40].)

Chapter 3 is concerned with the detailed structure of the supersymmetric case, where the application of the exact quantum effective action by Intriligator, Morrison and Seiberg [41] drastically improves the situation outlined above [34]. Here the crucial point is that the quantum effective action at the two-derivative level is completely known. Technically, this follows from the SUSY-based restrictions on higher-dimension operators, including the absence of two-derivative operators of mass dimension 6 and higher in the super Yang-Mills (SYM) lagrangian (see [41] and also [42, 43]). As a result, low-energy 4d gauge couplings may receive 100% corrections from higher-dimensional power-law effects which are nevertheless controlled within effective field theory. In the most extreme case, this allows scenarios with quantitative TeV-scale power-law unification [34]. It is shown how the by now familiar power corrections to gauge unification arise in the above framework. They correspond to higher-dimension operators which are, in general, non-analytic in the symmetry-breaking VEV [34]. Corrections induced by a bulk hypermultiplet become analytic (in fact, identical to a classical CS term) if the hypermultiplet mass is sufficiently large. However, the tuning of finite hypermultiplet masses comparable to the bulk VEV allows the realization of almost any desired power law effect. This ambiguity is avoided if the 5d model arises as the low-energy limit of a 6d construction because of the absence of massive hypermultiplets in 6 dimensions [34].

Chapters 4 and 5 discuss phenomenological aspects of realizing the above ideas in more concrete models. A realistic SU(5) model is introduced on an orbifold which breaks the GUT group down to the SM at its singular points. The bulk field Φ breaking the symmetry in the 5d bulk is the adjoint scalar of the 5d vector multiplet. Its VEV, which is stabilized, e.g., by boundary Fayet-Iliopoulos (FI) terms, induces large power-law corrections to gauge unification [33, 34]. At the same time, this VEV gives masses to the A_5 zero modes of X, Y gauge bosons. Power-law corrections to gauge unification are given in terms of the Φ-VEV, the bulk hypermultiplet masses, and the bulk CS term, the latter being fixed by brane anomaly cancellation. Intriguingly, a bulk field content of just the gauge multiplet and a massless **10** of SU(5) induces a power-law effect that is identical to the logarithmic running within the MSSM [34]. The extreme lightness of one of the three SM generations naturally emerges in this context. Then 6-dimensional unified models are considered [44,45] (for related earlier string theory results, especially including Wilson lines, see, e.g., [46] and [47,48]). The role of the Φ-VEV here is taken over by an A_6 Wilson line wrapping the cylinder-like central part of the compact space chosen to be an orbifolded 2-torus T^2/Z_2 with its two radii taken to be highly hierarchical (a long pillow). The value of the Wilson line is fixed by the orbifold breaking of the gauge symmetry at the 4d fixed points. As far as power-law corrections are concerned, such effectively 5d scenarios arising from the compactification of 6d theories are more predictive than pure 5d models because of the absence of massive gauged hypermultiplets and 6d anomaly constraints on massless bulk matter [34].

The second part begins with Chapter 6. Here the basic ideas of compactifying the type IIB superstring on a Calabi-Yau manifold in the presence of background fluxes of the higher p-form fields of the effective type IIB supergravity are introduced. The presence of Dp-branes in string theory allows for the appearance of warped geometries [49] with its simultaneous presence of p-form fluxes. These fluxes generate a superpotential for the complex structure, or shape moduli of the Calabi-Yau [21, 22], which can stabilize them [16–31]. Then the KKLT [15] construction of fully stabilized dS string vacua is reviewed: the remaining Kähler moduli, which are not fixed by the fluxes, are stabilized by non-perturbative effects such as D3-instantons or gaugino condensation on stacks of D7-branes. Afterwards the resulting stable and supersymmetric AdS_4-minima are lifted to metastable dS_4-vacua with a small cosmological constant by introducing anti-D3-branes as a source of SUSY breaking. Next a short introduction into inflation is given with emphasis on the conditions which scalar fields have to fulfill in order to become a successful candidate for driving inflation. The chapter closes with a review of some attempts to realize inflation in KKLT-like setups. One can use, for instance, moving D3-branes to do this [50] or introduce additional gaugino condensates [51] to generate sufficiently flat directions for the KKLT volume modulus.

Chapter 7 firstly states [52, 53] that the source of more or less explicit supersymmetry breaking, the anti-D3-branes usually used in KKLT-like models, can be replaced by higher-order α'-corrections [54] to the effective supergravity. In trying to realize inflation in such a setup one has to avoid the so-called η-problem of supergravity (for instance, the D3-brane models mentioned above run into this problem [50]). This problem arises because, unlike the superpotential, the Kähler potential of the inflaton candidate often gets corrections which give the field a mass comparable to the Hubble parameter during that epoch. Fortunately, the volume modulus considered in the model here has a leading order Kähler potential of the no-scale type. Therefore, there is no η-problem in this setup. In fact, it is shown that the α'-correction can generate a scalar potential for the volume modulus which contains both a stabilizing dS minimum and sufficiently flat saddle points where inflation can start. An explicit model is constructed where slow-roll inflation with about 130 e-foldings occurs. Inflation ends when the volume modulus rolls down into the metastable dS_4 minimum with a small cosmological constant. This model can accommodate the WMAP data of the cosmic microwave background (CMB) radiation. It yields the correct magnitude of the primordial density fluctuations with a reasonable value for the spectral index of the fluctuation spectrum $n_s \approx 0.93$.

The results of this investigation are discussed in the conclusion. More details can be found in the Appendices. App. A gives the general structure of the 1-loop corrections to the gauge couplings in 5d. It compares KK-mode summation with manifestly five-dimensional calculations. App. B deals with the properties of the prepotential of 5d SYM being at the very heart of the discussion of higher-dimension operators in gauge unification. App. C discusses analyticity properties of the higher-dimension operators induced by the power-like loop corrections. In App. D, the necessary details of the calculation of power corrections arising from symmetry-breaking Wilson lines in 6d are given. App. E deals with the structure of spinorial mass terms and the absence of massive gauged matter in 6d SUSY. Appendix F contains a short summary of facts regarding the moduli fields of Calabi-Yau compactifications.

Chapter 1

Gauge Coupling Unification in Higher Dimensions

Extra dimensions have been around in theoretical physics since the pioneering work of Kaluza and Klein in the 1920s. They provide a tool for unification by explaining gauge invariance as invariance under coordinate transformations along the internal directions of a compactified higher-dimensional space-time. Phenomenologically such extra dimensions must be smaller than TeV^{-1} or else one would have seen them, for instance, in collider experiments. In string theory extra dimensions were for a long time thought to be at the 4d Planck scale $M_p = 1.2 \cdot 10^{19}\,\mathrm{GeV}$. The existence of Dp-branes in string theory allowed for the possibility that some of six additional dimensions of string theory upon compactification might stay larger than M_p^{-1} contrary to what was hitherto assumed in the standard lore. This allowed low energy effective field theories embedded into string theory with extra dimensions far below the Planck scale to become realistic models. Moreover, this provided additional motivation to study higher-dimensional field theories by themselves (without necessarily embedding them into string theory).

In the context of this framework then Dienes, Dudas and Gherghetta (DDG) [6] proposed to realize the MSSM in a higher-dimensional space-time with one or more extra dimensions compactified on an S^1/Z_2 orbifold or a higher-dimensional orbifold, respectively. The process of orbifolding is necessary since already in 5d the minimal supersymmetry corresponds to $\mathcal{N} = 2$ in 4d language. Compactification from 5d to 4d on an orbifold S^1/Z_2 is a way to break the supersymmetry to 4d $\mathcal{N} = 1$. Upon a Kaluza-Klein reduction of this higher-dimensional field theory the MSSM fields acquire so-called KK-towers of equidistantly spaced massive KK-modes which all contribute to the running of the gauge couplings. Consider the vacuum polarization diagrams, which generate the 1-loop corrections to the gauge couplings. The effect of the KK-tower of fields running in these loops adds up to a power-law dependence of the gauge couplings on the UV-scale Λ in the higher-dimensional field theory. This so-called 'power law running' of the gauge couplings was then used in [6] to argue for the possibility of accelerated power law unification of the MSSM gauge couplings at $M'_{\rm GUT} \ll M_{\rm GUT} \approx 2\cdot 10^{16}$ GeV.

To clarify this idea, consider the KK reduction of a 5d U(1) gauge theory coupled to a 5d complex scalar field H on an S^1 to 4d. The 5d action is

$$S_{\rm 5d} = \int d^5x \left(-\frac{1}{4g_{\rm 5d}^2} F_{MN}F^{MN} + (D_M H)^* D^M H \right) \tag{1.1}$$

$$\text{with: } F_{MN} = \partial_M A_N - \partial_N A_M$$
$$D_M = \partial_M - iA_M \quad .$$

Now a function $f(x,y)$ has a Fourier decomposition on the S^1 given by

$$f(x,y) = \sum_{n=0}^{\infty} f^{(n)(+)}(x)\cdot \cos\left(\frac{ny}{R}\right) + \sum_{n=0}^{\infty} f^{(n+1)(-)}(x)\cdot \sin\left(\frac{(n+1)y}{R}\right) \tag{1.2}$$

where $(+)$ and $(-)$ refer to the intrinsic parity of the cosine and sine modes under $y \to -y$, respectively. R denotes the radius of the S^1 on which the fifth dimension has been compactified. The fifth coordinate has been called y and a 5d coordinate index is denoted by $M = 0\ldots 3,5$ while $\mu = 0\ldots 3$ denote 4d indices. Obviously the fields $A_M(x,y)$ and $H(x,y)$ split into so-called KK towers of 4d vector fields $A_\mu^{(n)(+)}(x)$, $A_\mu^{(n+1)(-)}(x)$ and 4d scalar fields $\phi^{(n)(+)}(x) = A_5^{(n)(+)}(x)$, $\phi^{(n+1)(-)}(x) = A_5^{(n+1)(-)}(x)$ and $H^{(n)(+)}(x)$, $H^{(n+1)(-)}(x)$. These modes get so-called KK masses because the action of ∂_y on a given KK mode produces a factor of n/R which is the KK mass of the mode. Note, that for each massive level of each field two KK modes (one sine and one cosine) are present. At low energies below the compactification scale $M_c = 1/R$ only the zero modes can be observed. Plug these definitions into the 5d action, retain only the zero mode of the 5d gauge field and integrate over the extra dimension

$$\int_0^{2\pi R} dy \quad . \tag{1.3}$$

The resulting 4d action is

$$S_{\rm 4d} = \int d^4x \Big[\sum_{n\geq 0} \Big(\big|D_\mu^{(0)} H^{(n)(+)}\big|^2 + \big|D_\mu^{(0)} H^{(n+1)(-)}\big|^2 + \frac{n^2}{R^2}\left(|H^{(n)(+)}|^2 + |H^{(n+1)(-)}|^2\right)\Big)$$
$$+V(\phi^{(0)(+)}, H^{(n)(+)}, H^{(n+1)(-)}) - \frac{1}{4g_{\rm 4d}^2} F_{\mu\nu}^{(0)} F^{(0)\mu\nu}\Big] \tag{1.4}$$

$$\text{with: } D_\mu^{(0)} = \partial_\mu - iA_\mu^{(0)(+)} \quad , \quad F_{\mu\nu}^{(0)} = \partial_\mu A_\nu^{(0)(+)} - \partial_\nu A_\mu^{(0)(+)} \quad .$$

V contains the couplings between the A_5-zero mode $\phi^{(0)(+)}$ and the H-KK modes. The 4d gauge $g_{4\mathrm{d}}$ coupling is given in terms of the 5d gauge coupling $g_{5\mathrm{d}}$ as

$$\frac{1}{g_{4\mathrm{d}}^2} = \frac{2\pi R}{g_{5\mathrm{d}}^2} \quad . \tag{1.5}$$

It is clear that the above KK-reduced 4d action contains with the A_5-zero mode $\phi^{(0)(+)}$ an unwanted massless scalar field which couples to the KK towers of scalars $H^{(n)(+)}$ and $H^{(n+1)(-)}$. One can remove this problem by compactifying on the orbifold S^1/Z_2 where the Z_2 denotes the transformation $Z_2 : y \to -y$ of the S^1. This orbifold is obtained by modding out the theory compactified on a circle S^1 by this Z_2 transformation. Upon identification of y and $-y$ on the circle the y-space of the theory becomes an interval $[0, \pi R]$ with the endpoints given by the fixed points of the S^1 under the Z_2. Modding out the field theory by the Z_2 means that the field space of the theory is restricted to the subset of fields which are consistent with the Z_2 action on the fields. Therefore, one has to specify the action of the Z_2 on the fields. One can do this here by demanding that the fields A_M and H transform under the Z_2 according to

$$Z_2 : \qquad \begin{aligned} A_\mu(x,y) &\to A_\mu(x,-y) = A_\mu(x,y) \\ A_5(x,y) &\to A_5(x,-y) = -A_5(x,y) \\ H(x,y) &\to H(x,-y) = H(x,y) \quad . \end{aligned} \tag{1.6}$$

The minus sign in the transformation of A_5 is required by the fact that this appears in the action in the combination $\partial_5 - iA_5$ and ∂_5 is odd under the Z_2. For the action Eq. (1.4) this means that the $A_\mu^{(n+1)(-)}$, $H^{(n+1)(-)}$ and the $\phi^{(n)(+)}$ modes are removed. Therefore, the orbifolding removes the unwanted massless scalar $\phi^{(0)(+)}$ from the low energy 4d theory. The lowest of the allowed modes $\phi^{(n+1)(-)}$ has $n = 1$ and therefore already a mass $1/R$.

In a similar way orbifolding reduces the higher supersymmetry of 5d. In 5d there are only 4-component Dirac spinors (no Majorana or Weyl condition is possible). Therefore the generator of the minimal 5d supersymmetry splits into the two generators of 4d $\mathcal{N} = 2$ SUSY with respect to 4 dimensions. Upon S^1/Z_2 orbifolding the two 4d Weyl spinors contained in the 5d Dirac spinor get opposite parities under the orbifold action. One of them has no (+)-KK modes and therefore no zero mode anymore. Thus, a 5d theory with minimal SUSY compactified on an S^1/Z_2 orbifold has chiral KK-zero modes which therefore have only 4d $\mathcal{N} = 1$ SUSY. Therefore, the authors of [6] proposed to compactify a 5d MSSM (which initially has $\mathcal{N} = 2$ SUSY in 4d language) on an S^1/Z_2 orbifold.

Consider now the 1-loop correction to the gauge coupling given by the vacuum polarization diagrams. For the scalar QED of Eq.s (1.1) and (1.4) above they are given by the

Figures A.1 and A.2 in Appendix A. At low energies the 4d gauge field $A_\mu^{(0)(+)}$ forms the external legs and thus the whole KK-tower of fields $H^{(n)(+)}$ runs in the loops (the $H^{(n+1)(-)}$-tower is removed by the orbifolding).

From here it is clear that in this effective 4d picture the 1-loop running of gauge couplings in the compactified 5d MSSM in [6] is described by an expression

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(\Lambda) + \frac{b_i}{2\pi} \ln\left(\frac{\Lambda}{M_Z}\right) + \frac{b_i'}{2\pi} 2\sum_{n=1}^{\infty} \ln\left(\frac{\Lambda}{nM_c}\right) \quad . \tag{1.7}$$

Here the $b_i = (33/5, 1, -3)$ denote the MSSM 1-loop beta function coefficients of the KK-zero mode for the gauge group $U(1)_Y \times SU(2)_F \times SU(3)_C$. The b_i' denote the corresponding beta function coefficients for the higher KK modes. They are different because for the massive KK-levels the supersymmetry still corresponds to 4d $\mathcal{N} = 2$. In more than four dimensions the gauge couplings carry negative mass dimension implying that higher-dimensional gauge theories are perturbatively non-renormalizable by power counting. Thus, such theories have to be viewed as effective field theories which at some UV scale Λ have to be embedded into some more fundamental theory. DDG proposed to solve this problem by retaining in the sum only the first N KK modes up the fundamental scale Λ where $N = \left[\frac{\Lambda}{M_c}\right]$. This leads to

$$\begin{aligned} \sum_{n=1}^{N} \ln\left(\frac{\Lambda}{nM_c}\right) &= N\ln\left(\frac{\Lambda}{M_c}\right) - \ln N! \\ &= N\ln\left(\frac{1}{N}\frac{\Lambda}{M_c}\right) + N - \frac{1}{2}\ln\left(\frac{\Lambda}{nM_c}\right) \\ &= \frac{\Lambda}{M_c} - \frac{1}{2}\ln\left(\frac{\Lambda}{nM_c}\right) + \mathcal{O}(1) \end{aligned} \tag{1.8}$$

where Stirling's approximation has been used to evaluate $N!$ and $N \approx \frac{\Lambda}{M_c}$. Thus one arrives at an expression for the gauge couplings given by

$$\alpha_i^{-1}(M_c) = \alpha_i^{-1}(\Lambda) + \frac{b_i - b_i'}{2\pi} \ln\left(\frac{\Lambda}{M_c}\right) + \frac{b_i'}{\pi}\frac{\Lambda}{M_c} \tag{1.9}$$

which motivated the term 'power law running'. If the KK beta function coefficients b_i' now fulfill the crucial unification constraints of the running of the gauge couplings relative to each other

$$\frac{b_1' - b_2'}{b_1' - b_3'} = \left.\frac{b_1 - b_2}{b_1 - b_3}\right|_{\text{MSSM}} = 1.4 \tag{1.10}$$

(which according to [6] they do up to a few %) then the gauge couplings unify accelerated by power law running - if they depend on precisely the same UV cutoff Λ.

However, as it is well known from quantum field theory, physical results of a calculation should not depend on the precise choice of the UV cutoff that regularizes the loop integrations. To illustrate this, look at the running couplings of the 4d MSSM. The running of the couplings is induced by logarithms

$$\ln\left(\frac{\Lambda}{M_Z}\right) \quad . \tag{1.11}$$

Changing the cutoff parameter by 100% from $\Lambda \to 2\Lambda$ produces a change

$$\ln\left(\frac{\Lambda}{M_Z}\right) \to \ln\left(\frac{2\Lambda}{M_Z}\right) = \ln\left(\frac{\Lambda}{M_Z}\right) + \underbrace{\ln 2}_{\mathcal{O}(1)} \tag{1.12}$$

which is small compared to the leading logarithm $\ln\left(\frac{\Lambda}{M_Z}\right)$. Compare that to the case of the 5d theory above: here

$$\frac{\Lambda}{M_c} \to \frac{2\Lambda}{M_c} = 2 \cdot \frac{\Lambda}{M_c} \tag{1.13}$$

which is a 100% correction to the original result! Imagine now a small dependence of the regulator on the gauge group factor or a higher-dimensional operator present at the high scale Λ which contributes to the value of the gauge coupling on the same level as the leading 1-loop correction. This will completely destroy the predictivity of the above scenario in generic cases. Therefore getting to work gauge coupling unification in this bottom-up approach via naive power law running appears to be beset with severe difficulties.

Fortunately, this situation is not the full story. One can change the whole picture completely if one starts instead in a top-down approach from a higher-dimensional grand unified theory with minimal symmetry assumptions. To see this, consider d-dimensional pure Yang-Mills theory, compactified to 4 dimensions on a torus of radius R. Scattering processes in the 4d theory at energies near the compactification scale $M_c \sim 1/R$ can be used to define a 4d gauge coupling $\alpha_4(M_c) = g_4^2(M_c)/(4\pi)$. In the following, this quantity will be considered as the basic physical observable of the low energy effective theory. It is linked to processes at energies far below M_c by conventional 4d logarithmic running. The relation to the coupling constant α_d of the d-dimensional theory is given by

$$\alpha_4(M_c)^{-1} \sim \alpha_d(\mu)^{-1} R^{d-4} + f_{\text{1-loop}}(\mu, R) + \text{higher orders}\,, \tag{1.14}$$

where μ characterizes the renormalization point of the higher-dimensional field theory (see, e.g., [55]). For $\mu \gg M_c$, the leading contribution from $f_{\text{1-loop}}$ is $\sim (\mu R)^{d-4}$. It describes the power-divergent loop-correction to the F^2 term in the bulk, multiplied by the extra-dimensional volume. Since the left hand side is μ-independent, one has $\alpha_d(\mu)^{-1} \sim M^{d-4} - \mu^{d-4}$, where M can be considered as the fundamental UV scale of the model, and $\mathcal{O}(1)$ numerical coefficients (which depend on the renormalization scheme) have been suppressed. It is convenient to assume $\mu \ll M$, so that $\alpha_d \sim M^{4-d}$.

Next, assume that the vacuum expectation value (VEV) of a bulk Higgs breaks the simple gauge group G of the fundamental theory to a subgroup $H = H_1 \times \cdots \times H_n$ (which is a direct product of simple groups and U(1) factors). The Higgs breaking is characterized by an energy scale M_B, related to the masses of vector bosons and physical scalars. For $M_c \ll M_B \ll \mu \ll M$, the 4d gauge couplings, labelled by the index $i = 1...n$, are now given by

$$\alpha_{4,i}(M_c)^{-1} \quad \sim \quad \alpha_d(\mu)^{-1} R^{d-4} + (\mu R)^{d-4} + f_{\text{1-loop},i}(\mu, R, M_B) + \text{higher orders}\,. \tag{1.15}$$

Here the 1-loop correction has been split into a universal (i-independent) piece carrying the leading divergence $\sim \mu^{d-4}$ and the non-universal piece $f_{\text{1-loop},i}$. To understand this structure, it is sufficient to observe that, while the bulk theory at energies below M_B possesses non-universal (with respect to i) power divergences of degree $d-4$, such divergences can not be present in the unbroken high-scale theory. Thus, their contribution to the coefficients of F_i^2 is suppressed by M_B^2. To be more specific, the function $f_{\text{1-loop},i}$ may be considered as arising from differences of one-loop integrals with massive and massless vector bosons,

$$\int^\mu \frac{d^d k}{(k^2 + M_B^2)^2} - \int^\mu \frac{d^d k}{(k^2)^2} \sim M_B^2 \mu^{d-6}\,, \tag{1.16}$$

which demonstrates the structure of the M_B-suppression. Appendix A gives this argument diagrammatically in Fig. A.3 and in a detailed calculation for a $U(1) \times U(1)'$ toy model. This estimate is, however, only valid for $d > 6$. For $d = 5$ this term is finite and calculable, so that Eq. (1.15) has to be replaced by

$$\alpha_{4,i}(M_c)^{-1} \sim \alpha_5(\mu)^{-1} R + \mu R + c_i M_B R + \cdots\,. \tag{1.17}$$

This structure was previously discussed in U(1) toy models [12–14]. Except for the non-universal numbers c_i, numerical coefficients have been suppressed in the above estimates. Furthermore, both higher-loop and volume-suppressed terms have been dropped in Eqs. (1.15) and (1.17).

For $d = 6$, the M_B suppressed term reads $c_i (M_B R)^2 \ln(\mu/M_B)$. This means that non-universal counterterms (corresponding to higher-dimension operators) have to be present for consistency of the theory. Thus, although an $\mathcal{O}(1)$ term coming with the log remains undetermined, the coefficients c_i and therefore the log-enhanced piece is calculable.

The above contributions proportional to c_i provide corrections to $\alpha_{d,i}^{-1}$ of relative size $(M_B/M)^{d-4}$. At first sight, the phenomenological relevance of these corrections appears to be limited by possible higher-dimensional operators, e.g., $\text{tr}[F^2 \cdot \Phi]$ (where Φ is the bulk field developing a symmetry-breaking VEV). In principle, such operators can give rise to non-universal corrections as large as the loop-effects discussed above.[1] However, as will be

[1]This has been pointed out in [12] in the context of a 5d toy model GUT with gauge group U(1)×U(1)′.

explained in detail below, in the simplest and most popular higher-dimensional scenarios, the leading dangerous operators are either automatically forbidden or can be forbidden by minimal symmetry assumptions. Furthermore, it turns out that the coefficients c_i are governed by the basic group theoretic structure of the theory and are therefore fairly model-independent. Thus, it can be concluded that power-like threshold corrections to gauge unification can and should be taken seriously at a quantitative level.

A simple example shall explicitly demonstrate the feasibility of this idea. For that purpose it will be sufficient to work in the framework of a 5d 'toy' GUT comprised of two complex scalar fields h, h' gauged under a $U(1) \times U(1)'$. Its lagrangian with the gauge couplings g, g' is subject to a Z_2 exchange symmetry $Z_2: \; h \to h', \; A_M \to A'_M, \; \phi \to \phi'$ which enforces $g = g'$. The action of this theory reads as

$$\begin{aligned}\mathcal{L}_{5\mathrm{d}}^{U(1)\times U(1)'} \;=\;& -\frac{1}{4g^2}F_{MN}F^{MN} - \frac{1}{4g'^2}F'_{MN}F'^{MN} + (D_M\boldsymbol{H})^\dagger D^M\boldsymbol{H} - \phi^2|h|^2 - \phi'^2|h'|^2 \\ & -V(\Phi) + |\partial_M\Phi|^2\end{aligned}$$

$$\text{where}: \; D_M = \partial_M - ig\begin{pmatrix} A_M & 0 \\ 0 & 0\end{pmatrix} - ig'\begin{pmatrix} 0 & 0 \\ 0 & A'_M\end{pmatrix}, \qquad (1.18)$$
$$\boldsymbol{H} = \begin{pmatrix} h \\ h'\end{pmatrix}, \; \Phi = \begin{pmatrix}\phi \\ \phi'\end{pmatrix} \text{ with: } \phi, \phi' \in \mathbb{R}$$

which is essentially scalar QED of two charged and uncoupled scalars which are, however, forced together by the supplementary Z_2. As a minimal symmetry assumption let $V(\Phi)$ now be invariant under $\Phi \to -\Phi$. Further, assume that $V(\Phi)$ is of a form such that Φ develops a VEV

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ \Phi_0\end{pmatrix} . \qquad (1.19)$$

This VEV breaks the Z_2-symmetry spontaneously and induces at the 1-loop level a splitting in the gauge couplings g, g' by giving h' a mass $m_{h'} = |\Phi_0|$. The competing higher-dimension operators ϕF^2, $\phi' F'^2$ mentioned above now have to vanish by virtue of the invariance under sign reversal of Φ, leaving as the first allowed operator containing F'_{MN} and ϕ'

$$\frac{1}{M} \cdot \phi'^2 F'_{MN}F'^{MN} . \qquad (1.20)$$

Here M denotes the UV-cutoff scale of the theory implying that the above operator remains suppressed as long as $|\langle\Phi\rangle| \sim |\Phi_0| \ll M$ holds. Explicitly, one has according to Eq. (1.17)

$$\left.\frac{1}{g_4'^2}\right|_{M_c} \sim \left.\frac{R}{g_5'^2}\right|_{\mu} + \mu R + c_i M_B R + \cdots \qquad (1.21)$$

where now $M_B \sim m_{h'} = |\Phi_0|$. This implies that at the 1-loop level the effective action contains at least the operators

$$\frac{1}{g_5'^2} F'_{MN} F'^{MN} \ , \ |\phi'| F'_{MN} F'^{MN}\big|_{1-\text{loop}} \ , \ \frac{1}{M} \cdot \phi'^2 F'_{MN} F'^{MN} \ . \tag{1.22}$$

From the absence of the tree level operator $\phi' F'_{MN} F'^{MN}$ by symmetry the calculability and phenomenological relevance of the 1-loop correction to the gauge couplings in the limit $|\Phi_0| \ll M$ is clear.

Using the results given in the Appendix A one can deduce the 1-loop corrections induced by the scalar fields running in the loops to the gauge couplings g_5 and g_5', respectively. Taking then the difference of their inverse values one arrives at one of the main results of this thesis (Eq.s (A.11) and (A.14) in the Appendix) - that this difference of the inverse gauge couplings

$$\alpha_{5d}^{-1}(M_c) - {\alpha'_{5d}}^{-1}(M_c) = b_h \cdot m_{h'} = \frac{1}{12\pi} \cdot m_{h'} = \frac{1}{12\pi} \cdot |\Phi_0| \tag{1.23}$$

is finite despite each single gauge coupling having a UV-divergent linear contribution.

Thus, in precisely this sense one may speak of well-defined and calculable power law unification in terms of power-like threshold corrections induced by softly broken higher-dimensional GUTs, and it is this idea which will be elaborated in the next chapters.

Chapter 2

Radiative Threshold Corrections to Gauge Unification in 5d

2.1 Calculable bulk threshold corrections

The previous introductory Chapter has outlined the idea of calculable power-like threshold corrections to gauge coupling unification which do not depend on a UV-definition of the naive dimensional power-law scaling of the gauge couplings in higher dimensions. The crucial ingredients necessary to achieve this were identified. One needs a higher-dimensional unified gauge theory which is softly broken in the bulk by the VEV of some Higgs field Φ. Further, the theory has to satisfy minimal symmetry assumptions with respect to the Higgs field (usually a symmetry $\Phi \to -\Phi$ is sufficient) which will guarantee the absence of the tree level operator $\Phi \cdot F^2$. Now one has to extend the $U(1) \times U(1)'$ toy GUT analysis of the last Chapter to the context of non-Abelian gauge theories which contain candidates like SU(5) as Grand Unified Theories.

Consider a d-dimensional Yang-Mills theory with simple gauge group G and a Higgs-field Φ transforming in some representation of G. The lagrangian reads

$$\mathcal{L} = -\frac{1}{2g_d^2} \cdot \operatorname{tr}\left(F_{MN}F^{MN}\right) - \left(D_M\Phi\right)^\dagger \left(D^M\Phi\right) - V(\Phi)\,, \tag{2.1}$$

where F_{MN} is the field strength tensor, D_M is the covariant derivative, and the indices M, N run over 0,...,3,5,...,d. Assume now that Φ develops a VEV breaking G to a subgroup $H = H_1 \times \cdots \times H_n$. Without supersymmetry, this can simply be realized by choosing an appropriate bulk potential $V(\Phi)$. Higher-dimensional supersymmetry restricts possible bulk interactions and different origins for a bulk VEV have to be considered (cf. Sects. 2.3 and 2.4).

At tree level, the couplings $\alpha_{d,i}$ of the group factors H_i are equal to the coupling α_d of G. At one loop, one has to calculate the contributions of the light and heavy vector bosons and the physical Higgs scalars to the coefficients of the F_i^2 terms, i.e., to the normalization of the field-strength terms of the unbroken subgroup factors. This calculation was done in the context of 4d GUTs in dimensional regularization [56, 57] (see also [58]), so that the d-dimensional result can simply be taken from [56]:

$$\alpha_{d,i}^{-1} = \alpha_d^{-1} + \frac{\Gamma(2-d/2)}{6(4\pi)^{d/2-1}} \left[-(25-d) \sum_{r_i} M_{V,r_i}^{d-4} T_{r_i} + \sum_{r_i'} M_{S,r_i'}^{d-4} T_{r_i'} + 2 s_d \sum_{r_i''} M_{F,r_i''}^{d-4} T_{r_i''} \right] . \tag{2.2}$$

Here r_i, r_i' and r_i'' label the representations under H_i of the vector, scalar and spinor particles and M_{V,r_i}, $M_{S,r_i'}$, and $M_{F,r_i''}$ stand for the corresponding masses. (Although the minimal setting discussed at the moment has no fermions, a possible fermionic contribution was included into the above equation for completeness. The number s_d characterizes the dimension of the relevant spinor.) Furthermore, T_{r_i} is defined by $\mathrm{tr}[T^a T^b] = \delta^{ab} T_{r_i}$, where $T^{a,b}$ are the generators in the representation r_i (and analogously for r_i' and r_i'').

Concerning the structure of Eq. (2.2), several comments are in order. The choice of α^{-1} (rather than α or g) as the basic quantity rests on its interpretation as the coefficient of the F^2 operator and hence the fact, that the further transition to the 4d theory proceeds simply by multiplication with the volume factor. Of course, this direct relation between Eq. (2.2) and Eq. (1.15) works only up to terms suppressed by a volume factor. Such terms will be discussed in more detail below. Note furthermore that Eq. (2.2) does not contain contributions from the gauge bosons of the unbroken subgroup. In the context here, the reason for this is the masslessness of these vector bosons. Because of the absence of a mass scale, the corresponding loop integrals have a pure power of the loop momentum in the integrand and therefore vanish in dimensional regularization.[1]

By power counting, one expects the one-loop correction to $\alpha_{d,i}^{-1}$ to diverge with the $(d-4)$th power of the cutoff. The fact that this does not show up in Eq. (2.2) is due to the use of dimensional regularization. However, this does does not restrict the validity of the conclusions in any way. On the one hand, this leading power-divergence is G-universal (independent of i) because of the symmetric structure of the UV theory and can thus be absorbed in a redefinition of α_d^{-1}. On the other hand, the main phenomenological implications depend only on the differences between the inverse gauge couplings of the group factors H_i and are therefore not affected by a G-universal correction.

The further analysis depends crucially on two closely related issues: the possible exis-

[1]This argument works only for $d > 4$, where the coupling can be defined at zero external momentum. In 4 dimensions, the relevant loop integrals require the external momentum as IR regulator, and as a result the familiar contribution $\sim \ln(\mu^2/Q^2)$ from massless gauge bosons appears.

tence of higher-dimension operators that can compete with the corrections on the r.h. side of Eq. (2.2) and the UV divergences that are present even in these non-G-universal loop corrections. To be specific, the lagrangian generically contains terms

$$\sim \frac{1}{M^k}\Phi^n (F_{MN})^2\,, \tag{2.3}$$

where it has been assumed that the relevant product of representations of G contains a singlet, and k is chosen to ensure the overall mass dimension d. When Φ develops a VEV $v = M_V/g_d$ (vector boson masses are generated as in the familiar 4-dimensional setting), the operator in Eq. (2.3) can lead to non-universal corrections to $\alpha_{d,i}^{-1}$ at tree-level. Since $g_d^2 \sim M^{4-d}$ (the fundamental UV scale of the theory), the relative size of this correction is given by

$$\frac{\Delta\alpha_{d,i}^{-1}}{\alpha_d^{-1}} \sim \left(\frac{M_V}{M}\right)^n\,. \tag{2.4}$$

This has to be compared to the correction from Eq. (2.2) which, focussing on the vector boson part, is of relative size $(M_V/M)^{d-4}$. Given the possibility that $n=1$, this appears to be discouraging. However, it is important to keep in mind that certain values of n may be forbidden by group theory or other symmetries. For example, for $d=5$ the leading calculable correction is of relative size M_V/M ($M_V/M \ll 1$), and a simple Z_2 symmetry $\Phi \to -\Phi$ is sufficient to forbid the competing $n=1$ term from Eq. (2.4). The $n=2$ term is of relative size $(M_V/M)^2$ and therefore negligible.

Higher-dimension operators can act as counter terms and are therefore intimately linked to the divergence structure of the non-universal corrections on the r.h. side of Eq. (2.2). In 5d, the non-universal part of the loop correction is finite, which is consistent with the possibility of forbidding the relevant operator by a Z_2 symmetry. For $d \to 6$, the Gamma function develops a pole, showing that the non-universal term is afflicted by a logarithmic divergence. Although this implies the existence of a higher-dimension operator $\sim F^2\Phi^2$ providing the counter term, predictivity is maintained at the leading logarithmic level. To be specific, it is assumed that the divergence is cured by a theory of higher symmetry at the scale M and that there are no anomalously large non-universal threshold effects associated with this transition. In short, one works at leading-log approximation in M/M_V. This logarithm can be extracted from Eq. (2.16), as is common in 4d, by setting $d = 6-2\epsilon$, introducing appropriate factors $\mu^{2\epsilon}$ to keep the correct dimensionality, expanding in ϵ, and letting $\epsilon \to 0$ and $\mu \to M$. Focussing on the vector contribution, the result reads

$$\alpha_{4,i}^{-1}(M_c) = V\alpha_6^{-1} + \frac{1}{3(4\pi)^2}\,19\sum_{r_i}(VM_{V,r_i}^2)T_{r_i}\ln\frac{M}{M_{V,r_i}}\,. \tag{2.5}$$

This should provide a good description if $M_c^2 \ll M_V^2 \ll M^2$ and scalar and fermion masses are small.

In more than 6 dimensions, power-counting suggests that there are non-universal power-divergences. More specifically, the explicitly calculated logarithmic divergence in $d = 6$ suggests a power-divergence of degree $d - 6$ in d dimensions, which would have to come with a factor M_V^2 for dimensional reasons. The corresponding counter term is provided by the operator $\Phi^2 F^2$, which should therefore always be included in the lagrangian. The term $\sim M_V^{d-4}$ in Eq. (2.2) is subdominant with respect to this operator. Thus, quantitative statements depend on a more detailed knowledge of the UV structure of the theory. However, it is likely that the group-theoretical specification of the VEV of Φ and a classification of the singlets contained in $\Phi^2 F^2$ will be sufficient to uniquely determine or strongly constrain the way in which $\Phi^2 F^2$ terms contribute to gauge coupling differences $\alpha_{4,i}^{-1}(M_c) - \alpha_{4,j}^{-1}(M_c)$.

Dangerous higher-dimension operators mixing Φ and F^2 may also reside on branes. But in this case their contribution to the observed effective 4d couplings is further suppressed by volume factors. For example, the contributions of operators on branes of co-dimension d_c are suppressed by the potentially small factor $(MR)^{-d_c}$, where R is the compactification radius.

To summarize, a generic and particularly predictive setup can be described as follows. Assume that there are no bulk fermions or at least no non-G-universal mass splitting of bulk fermions. Assume furthermore that $M_S \ll M_V$, i.e., the potential stabilizing the VEV of Φ is relatively flat (this is generic in supersymmetry which, however, will be discussed in more detail below). If dangerous higher-dimension operators are forbidden by appropriate symmetries, the leading power correction is calculable and the resulting 4d gauge couplings are obtained by multiplying Eq. (2.2) with the volume factor V:

$$\alpha_{4,i}^{-1}(M_c) = V\alpha_d^{-1} - \frac{\Gamma(2-d/2)}{6(4\pi)^{d/2-1}}(25-d)\sum_{r_i}(VM_{V,r_i}^{d-4})T_{r_i}\,. \tag{2.6}$$

Here α_d^{-1} on the r.h. side is defined in dimensional regularization, which makes it independent of the subtraction scale μ since the coefficient of the relevant G-universal power-divergence vanishes. One may think of α_d as the d-dimensional gauge coupling defined at zero momentum (in complete analogy with $1/M_P$ in 4d gravity). The T_{r_i} and the relative sizes of the M_{V,r_i} are determined by group theory (the representation of Φ and the direction of its VEV v), so that the power correction is proportional to Vv^{d-4}. The relative size of this correction is $(M_V/M)^{d-4}$. It has to be small enough so that even higher powers of M_V/M are suppressed. Nevertheless, it can be significantly larger than the usual 4d GUT threshold corrections of relative size $\alpha_{\rm GUT} \sim 1/25$. Jumping somewhat ahead one may speculate that higher supersymmetry or string theory may forbid or fix *all* dangerous higher-dimension operators and higher-loop corrections to the d-dimensional gauge couplings (cf. [59]), in which case one might hope to go to the region $M_V \sim M$ so that the relative sizes of low-energy gauge couplings are dominantly determined by power-law effects. In fact, exactly this situation will be discussed in Chap. 3 with a positive result because the higher supersymmetry in 5 dimensions indeed is strong enough to eliminate all but a finite number of higher-dimension operators on the two-derivative level.

2.2 Brane effects and the KK-mode approach

For now, however, the generic non-supersymmetric setup defined above will be further analyzed. All considerations so far used the 5d bulk point of view with the focus on true bulk effects. Therefore, terms suppressed by powers of the bulk-size R were completely neglected. However, it is clear that such contributions are generically present, e.g., on the r.h. side of Eq. (2.6). One can approach this issue using d-dimensional propagators in the full, compactified geometry. However, in the present investigation it appears to be simpler to discuss these effects using an effective 4d framework and summing KK modes. Clearly, these two methods are equivalent both conceptually and quantitatively.

To be specific, although being prepared to neglect terms down by full powers of MR (since these terms will in general be sensitive to unknown and largely unconstrained brane operators), one would like to take terms into account that are suppressed by powers of MR but enhanced by $\ln(MR)$. Such terms are known to be important in orbifold GUTs [37,38], where they give rise to the calculable 'differential running' [60] above the compactification scale.

For simplicity, first consider a toy example of one extra dimension compactified on an S^1. One starts with a theory with one unbroken gauge group G and considers only the contribution of a bulk scalar with mass $M_{S1} \sim M_c$ in a certain representation of G.

Further, compare this to a theory where the scalar mass is shifted to $M_{S2} \gg M_c$. The difference in the scalar contribution to the low-energy gauge couplings in these two models comes from the difference in log-contributions from the KK towers:

$$
\begin{aligned}
&\alpha_4^{-1}(M_c)_{\mathrm{model}\,2} - \alpha_4^{-1}(M_c)_{\mathrm{model}\,1} \\
&\qquad = \frac{T_r}{24\pi} \sum_{n=-\infty}^{\infty} \left[\ln \frac{\mu^2}{(nM_c)^2 + M_{S2}^2} - \ln \frac{\mu^2}{(nM_c)^2 + M_{S1}^2} \right] \qquad (2.7) \\
&\qquad \simeq \frac{T_r}{24\pi} \left[-\ln \frac{M_{S2}^2}{M_c^2} - 2 \sum_{n=1}^{\infty} \ln \left(1 + \frac{N^2}{n^2} \right) \right] , \qquad (2.8)
\end{aligned}
$$

where $N = M_{S2}/M_c$ and M_{S1} has been set to zero everywhere in Eq. (2.8) except for the zero-mode contribution, where it has been replaced by $M_c = 1/R$. This introduces only an $\mathcal{O}(1)$ error. The sum on the r.h. side of Eq. (2.8) can be estimated as

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{N^2}{n^2}\right) \simeq \pi N - \ln N + \mathcal{O}(1)\,, \tag{2.9}$$

for $N \gg 1$, so that the final result reads

$$\alpha_4^{-1}(M_c)_{\text{model}\,2} - \alpha_4^{-1}(M_c)_{\text{model}\,1} = -\frac{T_r}{12}\,\frac{M_{S2}}{M_c}\,. \tag{2.10}$$

Thus, model 2 differs from model 1 precisely by the power-like contribution $\sim M_{S2}$, which can also be obtained from Eq. (2.2) by setting $d = 5$. The important point here is that one finds no additional, log-enhanced contribution from the momentum region above M_c. In other words, the zero-mode log combines with the KK logs to give just a pure power.

The situation is different, however, if one compactifies on S^1/Z_2. In this case, the sum over positive and negative n in Eq. (2.7), corresponding to sines and cosines, is replaced by a sum over just positive n, corresponding to cosines only (assuming positive Z_2 parity of the scalar field). The zero mode still contributes with full strength. As a result, the cancellation of the zero-mode log is incomplete and Eq. (2.10) is replaced by

$$\alpha_4^{-1}(M_c)_{\text{model}\,2} - \alpha_4^{-1}(M_c)_{\text{model}\,1} = -\frac{T_r}{24}\,\frac{M_{S2}}{M_c} - \frac{T_r}{12\pi}\,\frac{1}{2}\ln\frac{M_{S2}}{M_c}\,. \tag{2.11}$$

If, on the other hand, the Z_2 parity of the scalar field is odd, there is no zero mode and only the sine modes contribute to the KK sum. One then finds

$$\alpha_4^{-1}(M_c)_{\text{model}\,2} - \alpha_4^{-1}(M_c)_{\text{model}\,1} = -\frac{T_r}{24}\,\frac{M_{S2}}{M_c} + \frac{T_r}{12\pi}\,\frac{1}{2}\ln\frac{M_{S2}}{M_c}\,. \tag{2.12}$$

This simple calculation allows for the following intuitive interpretation: Without branes, gauge coupling corrections are logarithmic below the compactification scale and purely power-like above it. Introducing 4d boundaries (branes) leads to typical 4d effects even above M_c, i.e., logarithmic corrections. For each brane at which a 5d field is non-zero (Neumann boundary conditions), one finds (1/4) times the usual log from 4d running. For each brane at which a 5d field is zero (Dirichlet boundary conditions), one finds $-(1/4)$ times this log. It can be easily checked that this rule extends to $S^1/(Z_2 \times Z_2')$, where a field can be zero at one brane and non-zero at the other.

While the extension of this rule to fermions is straightforward, the case of massive 5d vector fields requires some comments. The rule is that, if A_μ (where $\mu = 0, ..., 3$) is non-zero at a boundary, one finds a scalar log contribution with prefactor $(1/4)(-22)$. The factor -22 can be derived from Eq. (2.8) recalling that the zero mode (massive vector) has prefactor -21, while the KK tower (massive vectors and A_5-scalars) has prefactor -20. An intuitive understanding can be obtained if, guided by the scalar case above, one adds the A_μ contribution $(1/4)(-21)$ and the A_5 contribution $(-1/4)$ (the '$-$' arising since A_5 is zero if A_μ is non-zero). The rule extends in an obvious way to the case in which A_μ is zero at a boundary (orbifold breaking of the gauge group): one finds a scalar log with prefactor $(-1/4)(-22)$. In deriving this, it is important not to forget the A_5 zero-mode. Furthermore, there is a straightforward extension to the case of massless 5d vector fields, where the relevant prefactors of the boundary logs are $(\pm 1/4)(-23)$.

In fact, the above set of rules represents a simple and intuitive way of rederiving the 'differential running' in 5d orbifold GUTs above M_c because it relates 4d logs directly to the boundary conditions of fields (without any reference to the KK mode spectrum).

To illustrate the relevance of the above in the present context, now a more complete version of Eq. (2.6) in 5d is given. One works on S^1/Z_2 with A_μ and Φ non-zero at both boundaries. The result, which now includes both power-law and log-enhanced terms, reads

$$\begin{aligned} \alpha_{4,i}^{-1}(M_c) &= \pi R \alpha_5^{-1} + \frac{1}{24}\left[20\sum_{r_i}(RM_{V,r_i})T_{r_i} - \sum_{r_i'}(RM_{S,r_i'})T_{r_i'}\right] \\ &+\frac{1}{12\pi}\left[\frac{1}{2}(-22)\sum_{r_i}T_{r_i}\ln\frac{M}{M_{V,r_i}} + \frac{1}{2}\sum_{r_i'}T_{r_i'}\ln\frac{M}{M_{S,r_i'}} + \frac{1}{2}(-23)C_i\ln\frac{M}{M_c}\right]. \end{aligned} \tag{2.13}$$

Note, in particular, the appearance of contributions from the vector bosons of the unbroken subgroup (C_i is the adjoint Casimir of H_i) which, although irrelevant for the power-like terms, contribute to the boundary-driven logarithmic running above M_c. Furthermore, it should be observed that no non-universal logarithmic running occurs above the highest of the scales M_{V,r_i} and $M_{S,r_i'}$ since

$$\sum_{r_i} T_{r_i} + C_i = C_A(G) = i\text{-independent} \tag{2.14}$$

and

$$\sum_{r_i} T_{r_i} + \sum_{r_i'} T_{r_i'} = T_{\Phi-\text{repr.}}(G) = i\text{-independent}\,. \tag{2.15}$$

The next step will be the implementation of these results for a supersymmetric theory in higher dimensions. On the level of the above calculations the constraints of supersymmetry reside in the specific field content of the supersymmetric field multiplets.

2.3 Adding supersymmetry - general effects

The extension of the former results to supersymmetry is mostly straightforward. Supersymmetry requires equality of the number of fermionic and bosonic degrees of freedom which implies that all fields come in certain supersymmetric multiplets. This implies that Eqs. (2.2) and (2.13) simply require the inclusion of the additional degrees of freedom (fermions and scalars) that are present in the relevant SUSY multiplets. However, there are also some crucial new points that require a separate discussion. In particular, it is important to understand the possible origin of the bulk VEV and the resulting mass spectrum, both of which are strongly constrained by SUSY.

Firstly, focus on 5 and 6d, where the minimal SUSY corresponds to $N = 2$ in 4d language. This excludes all renormalizable (from the 4d point of view) interactions except those prescribed by gauge symmetry. In particular, the Higgs field Φ, which would have to come from a gauged hypermultiplet, can not have a conventional bulk potential with cubic and quartic terms. Although it appears conceivable that higher-dimension operators, consistent with 5d SUSY, generate a suitable potential[2] the simpler option of fixing the bulk Higgs VEV by an appropriate boundary potential was chosen. In doing so, one follows the method for breaking $U(1)_\chi$ in the 6d SO(10) model of [64]. Clearly, this has to rely on the existence of a D-flat direction in the bulk. (Here, D-flatness means that no potential arises from integrating out the SU(2)-R triplet of auxiliary fields of the gauge multiplet. For an explicit component lagrangian of a gauged 5d hypermultiplet see, e.g., [61].) In general, such a D-flat direction might not exist. This can, for example, be easily checked in the case of a single U(1) hypermultiplet. Assume now a representation or field content where a flat direction can be found. The non-zero VEV is stabilized only by a brane superpotential which will not be specified at the moment. In the bulk, the VEV will give masses to the whole 5d vector multiplet (in the broken directions) and to a whole hypermultiplet (in the directions corresponding to the would-be Goldstone-bosons). However, it is also known that in spontaneous gauge symmetry breaking a single scalar degree of freedom is transferred to the vector field. In the case of 5d SUSY, this is only possible if the masses of the vector multiplet and the hypermultiplet in the broken directions are the same. For the moment it is assumed that these two multiplets exhaust the set of heavy states.

This occurs, in particular if one chooses the hypermultiplet to be in the adjoint representation, which makes the model $\mathcal{N} = 4$ supersymmetric. One can then imagine the theory to arise via dimensional reduction from a SYM theory in 10d and think of the two complex

[2] A systematic analysis of such operators should be possible using the manifestly gauge-invariant formulation [61] of 5d SUSY in terms of 4d superfields [62, 63].

scalars of the hypermultiplet as $(A_7 + iA_8)$ and $(A_9 + iA_{10})$. It is now clear that flat directions exist (e.g. A_7 =const.) and that the whole hypermultiplet acquires a mass (from terms $\sim [A_7, A_8]^2$ etc.). Furthermore, it is immediately clear from the underlying gauge structure that all scalar and vector masses, and hence also the fermionic masses, corresponding to excitations of the broken directions are identical.

Thus, it has been argued that, after spontaneous symmetry breaking driven by bulk hypermultiplets, one finds the degrees of freedom of a vector multiplet and a hypermultiplet for every broken direction at the massive level. Simple counting of vector, scalar and fermionic fields according to Eq. (2.2) shows that no bulk loop correction arises. This does not come as a surprise since one is faced with the field content corresponding to $\mathcal{N} = 4$ SUSY.

However, the symmetry-breaking bulk VEV does not have to come from a Higgs. Instead, it is possible that, in a compact geometry, one of the extra-dimensional components of the vector field (e.g., A_5 in 5d; A_5 or A_6 in 6d) develops a VEV. Clearly, only adjoint breaking is possible in this case. However, it is a well-known and difficult problem to stabilize such a VEV. This is probably even more so if one requires the A_5 or A_6 VEV to be large enough to generate a large power correction.

A closely related and more immediately useful possibility exists in 5d. Consider, for example, a 5d SU(5) model. It is possible that the scalar partner of the gauge fields, from now on also called Φ, which is present in 5d SUSY, develops a bulk VEV in $U(1)_Y$ direction[3] (cf. [65]). Such a scalar VEV can arise in an S^1/Z_2 model where both boundaries break SU(5) and Fayet-Iliopoulos terms of the $U(1)_Y$ subgroup are present at both boundaries. As explained in [66] (see also [62, 67, 68]), in the 5d setup this term does not break SUSY or $U(1)_Y$, but instead drives a non-zero bulk VEV of Φ. More generally, whenever one has a 5d orbifold model where the bulk gauge symmetry is broken in such a way that an isolated U(1) factor survives on both branes, Fayet-Iliopoulos terms driving a bulk VEV of Φ can be introduced.

In the presence of a VEV of Φ, all the fields in the 5d vector multiplet corresponding to the broken directions acquire a bulk mass M_V. The formula for threshold corrections relevant to this case reads

$$\alpha_{4,i}^{-1}(M_c) = V\alpha_d^{-1} + \frac{1}{24\pi} 12 \sum_{r_i} (VM_{V,r_i}) T_{r_i} \,. \tag{2.16}$$

The prefactor 12 can be understood as the sum of 20 for a massive 5d vector and -8 for the spinor. The degree of freedom corresponding to Φ is absorbed in the massive vector field.

[3] The author is indebted to S. Groot Nibbelink for emphasizing this possibility in a very helpful conversation.

(As discussed above, it is also immediately clear that a hypermultiplet of mass M_V would precisely cancel this term.) The correction can be improved by including volume suppressed but log-enhanced terms using the discussion in the previous section. For simplicity, one works on S^1/Z_2 and assumes that both boundaries break the gauge group in the same way as the bulk VEV:

$$\begin{aligned} \alpha_{4,i}^{-1}(M_c) &= \pi R \alpha_5^{-1} + \frac{1}{24}\left[12\sum_{r_i}(RM_{V,r_i})T_{r_i}\right] \\ &\quad + \frac{1}{12\pi}\left[-\frac{1}{2}(-24)\sum_{r_i} T_{r_i}\ln\frac{M}{M_{V,r_i}} + \frac{1}{2}(-24)C_i\ln\frac{M}{M_c}\right] . \end{aligned} \tag{2.17}$$

Note that fermions do not contribute to the logarithmic terms since the two Weyl fermions contained in the 5d spinor have opposite boundary conditions at every brane.

For $d = 7$, the minimal vector multiplet again contains scalar adjoints that could acquire a VEV as Φ in the 5d case above. However, the minimal supersymmetry is $\mathcal{N} = 4$ in 4d language and no loop corrections to the gauge couplings are expected.

2.4 Analysis of the power-like thresholds in the context of simple SU(5) setups - both supersymmetric and non-supersymmetric

A preliminary analysis of phenomenological implications of the power-like threshold corrections calculated above is the next step to proceed to. This section restricts itself to 5d SU(5) models following, in essence, the construction principles of the simplest orbifold GUT models [36–38]. Proton decay is avoided by placing fermions on branes where SU(5) is not a good symmetry. The light SM Higgs doublet(s) can be localized on the same brane, as suggested in the minimal scenario of [38] (which can be dynamically realized using bulk masses as in [69]). Furthermore, as discussed in the previous section, one assumes that the scalar partner Φ of the gauge fields, which is present in 5d SUSY, develops a bulk VEV in $U(1)_Y$ direction. Such a scalar VEV can arise in an S^1/Z_2 model where both boundaries break SU(5) and Fayet-Iliopoulos terms are present. The usual problem with SU(5) models on S^1/Z_2, namely the existence of massless scalars with quantum numbers of the X, Y gauge bosons corresponding to their 5-components, is solved here automatically in presence of a bulk VEV of Φ, since the $A_5^{X,Y}$ then acquire a mass $\sim \Phi$.

It is important to note that in contrast to the case where the VEV comes from a hypermultiplet, the Φ-VEV can also couple linearly, $\sim F^2\Phi$, as in the super-Chern-Simons (CS) term discussed in [62]. This is clear since Φ in the 5d vector multiplet is necessarily accompanied by A_5 which implies together with ΦF^2 the presence of a term A_5F^2 which is part of the 5d Chern-Simons term. As it will be discussed in next Chapter, its contribution is completely fixed in terms of 5d supersymmetry and the constraints of anomaly cancellation on the boundaries of the orbifold. With the CS-term being $\sim \Phi F^2$ and thus of the same parametrical order as the loop corrections in question, any discussion of numerical predictions at this stage in the supersymmetric case can be only preliminary since the CS-term will add to the direct loop effects of the gauge and matter fields. Nonetheless, it is useful to continue deriving the gauge and matter correction formulae since they are needed later in this work anyway.

Assume a standard supersymmetric scenario in 4d, in which case the running between the electroweak scale and M_c is the familiar MSSM running. The low-energy data is taken to be $\alpha_i^{-1}(m_Z) = (59.0, 29.6, 8.4)$ and the effective SUSY breaking scale is set to m_Z. In this case, the relation between couplings at m_Z and M_c is given by

$$\alpha_{4,i}^{-1}(m_Z) = \alpha_{4,i}^{-1}(M_c) + \frac{1}{12\pi}(-18C_i + 12T_i)\ln\frac{M_c}{m_Z} + \text{SM matter contributions} \tag{2.18}$$

with $C_i = (0, 2, 3)$ (Casimirs of the SM gauge groups) and $T_i = (3/10, 1/2, 0)$ (SM Higgs representation). Furthermore, using the results of the previous sections and working on an S^1/Z_2, where the Z_2 breaks SU(5), one has

$$\begin{aligned}\alpha_{4,i}^{-1}(M_c) = \pi R\alpha_5^{-1} + \frac{1}{24}\left[12(RM_V)(5-C_i)\right] + \frac{1}{12\pi}\left[12T_i\ln\frac{M}{M_c}\right] \\ +\frac{1}{12\pi}\frac{1}{2}\left[24(5-C_i)\ln\frac{M}{M_V} + (-24)C_i\ln\frac{M}{M_c}\right] \\ +\text{SM matter contributions}\,.\end{aligned} \tag{2.19}$$

This follows immediately from Eq. (2.17), with the brane-localized Higgs contributing even above M_c.

The usual fairly precise MSSM unification is formally obtained in the limit $M = M_c = M_V = M_{\text{GUT}}$. One can now try to lower M_c and see whether gauge unification can be maintained at the cost of the power law term $\sim M_V/M_c$. This is not hopeless because the coefficients $-C_i$ coming with this term represent the main part of the usual MSSM running coefficients. Focussing on differences of 4d inverse gauge couplings, $\alpha_{ij} \equiv \alpha_i^{-1} - \alpha_j^{-1}$, the crucial gauge unification constraint can be characterized by

$$\frac{\alpha_{12}(m_Z)}{\alpha_{23}(m_Z)} = \frac{59.0 - 29.6}{29.6 - 8.4} = 1.39\ . \tag{2.20}$$

This has to be compared with the result obtained from combining the above running and threshold formulae:

$$\alpha_{ij}(m_Z) = \frac{1}{12\pi}\left(-18C_{ij} + 12T_{ij}\right)\ln\frac{M_c}{m_Z} \\ +\frac{1}{24}\left[-12C_{ij}\right]\frac{M_V}{M_c} + \frac{1}{12\pi}\left[12T_{ij} - 12C_{ij}\right]\ln\frac{M}{M_c} - \frac{1}{12\pi}12C_{ij}\ln\frac{M}{M_V}\,, \tag{2.21}$$

where $C_{ij} = C_i - C_j$ and $T_{ij} = T_i - T_j$.

The maximal value of M suggested by NDA [70] (cf. [38]) can be characterized by $M/M_c \sim 10^3$. The validity range of this calculation is $M_c \ll M_V \ll M$. It is amusing to observe that, if one sets $M_V \simeq \sqrt{M_c M}$ to realize this situation, the logarithmic terms from the energy range above M_c mimick precisely the MSSM contribution to coupling ratios. Thus, even if M_V does not have this precise value, the log terms will not affect MSSM-type unification significantly and, given the preliminary character of the present investigation, one focuses now on the power term. From Eq. (2.21) one can read off that just the logarithmic MSSM contribution would give $\alpha_{12}/\alpha_{23} = 1.4$ while just the power-like term would give $\alpha_{12}/\alpha_{23} = 2$ (cf. Eq. (2.20)). Thus, to maintain the above field content while lowering the unification scale significantly, one has to sacrifice precision. One can expect to find $\alpha_{12}/\alpha_{23} = 1.5$ at m_Z if about 1/6 of the log-running is traded for the power correction. This lowers the unification scale M to about 10^{14} GeV thus allowing, e.g., for a see-saw mechanism based directly on the GUT scale (without the usual mismatch by a factor $\mathcal{O}(10)$). One could also consider the possibility that there are no right-handed neutrinos and light neutrino masses are based directly on the appropriate higher-dimension operator suppressed by the new GUT scale. However, the price to pay is the extra $\mathcal{O}(1)$ threshold corrections to α_i^{-1} that are needed for consistency with the low-energy data. Although such corrections are not unnatural, given that a significant log-running continues all the way up to UV-scale M, they are certainly larger than what would be needed in the 4d MSSM.

Next, consider the possibility that power-like threshold corrections beyond those driven by the 5d gauge multiplet arise. This would not be possible if the bulk breaking was realized by a bulk Higgs field since 5d SUSY forbids the necessary coupling of this Higgs with other hypermultiplets. However, since one considers gauge breaking by the scalar adjoint, this possibility exists. If a bulk hypermultiplet is added, say in the 5 of SU(5), then the doublet and triplet part of it acquire different bulk masses due to the coupling to Φ. The ratio of M_V and these two masses M_d and M_t is prescribed by elementary group theory:

$$M_V : M_d : M_t \sim 5 : 3 : 2\,. \tag{2.22}$$

The power-like threshold corrections arising in this situation read

$$\Delta\alpha_{4,i}^{-1}(M_c) = \frac{1}{24}\frac{M_V}{M_c}\left[12(5-C_i) - 12(\frac{3}{5}T_i + \frac{2}{5}T'_i)\right] , \tag{2.23}$$

where $T'_i = (2/10, 0, 1/2)$ characterize the Higgs triplet representation. On the basis of just this power-law contribution one would have $\alpha_{12}/\alpha_{23} \simeq 2.27$, i.e., a situation worse than without the bulk 5.

However, this effect can be turned to its opposite by also introducing an SU(5)-invariant bulk mass M_f for the 5-hypermultiplet. In the presence of such a mass, quantified by $\xi = M_f/M_V$, with $\xi \simeq 0.4$ and with the sign chosen such that it almost compensates the Φ-driven doublet mass, Eq. (2.23) is transformed into

$$\Delta\alpha_{4,i}^{-1}(M_c) = \frac{1}{24}\frac{M_V}{M_c}\left[12(5-C_i) - 12\left(\left|\frac{3}{5}-\xi\right|T_i + \left|\frac{2}{5}+\xi\right|T'_i\right)\right] , \tag{2.24}$$

leading to $\alpha_{12}/\alpha_{23} \simeq 1.44$ just from the power-like correction. Now power-like threshold corrections can replace a significant part of the MSSM log-running without loss of precision of unification, but at the cost of tuning M_f. (This tuning can, of course, also be used to achieve perfect unification, including even the brane-driven log-running above M_c.) However, again in the light of the not-yet treated super-CS term these numbers serve only to clarify the effects of the presence of bulk hypermultiplet matter in that it can be used to arrange for a wide range of values for α_{12}/α_{23}. Precise predictions here await the discussion of the next Chapter.

Dangerous additional terms can come from higher-dimension operators. In particular, an operator $\sim F^2\Phi^2$ can contribute to α_4^{-1} at the level $M_V^2/(M_cM)$. If one requires this term not to be larger than $\mathcal{O}(1)$ and take $M \sim 10^3 M_c$, one finds the constraint $M_V \lesssim 30M_c$. (More optimistically, one could assume that this term is forbidden or at least uniquely specified in its structure by $N = 2$ SUSY, which, in fact, is shown to be true in the next Chapter.) From Eq. (2.24) one can now read off that nearly all of the low-energy value of, say, α_{12} can be due to power-like term, so that M and M_c can be lowered to $\sim 10^6$ GeV and $\sim 10^3$ GeV respectively. Given that these very crude estimates have produced this quite impressive result, a more detailed numerical study, including two-loop running and considering appropriate NDA factors, appears to be warranted. It certainly would be interesting to extend the above preliminary analysis to various other proposals involving gauge unification in extra dimensions where power-law effects can be important (see, for example, the discussion in [71] and [72]. However, this is beyond the scope of the this work.

At the end of this Chapter a look to the possible role of power-like threshold corrections in non-supersymmetric 5d SU(5) models is warranted. For this purpose, one chooses to accept an ad-hoc fine-tuning solution of the well-known problem of quadratically divergent Higgs mass corrections and focus exclusively on the precision of gauge coupling unification. Here, the presence of a CS-term is not dictated by supersymmetry, and thus in this case the calculated corrections can be used to derive numerical predictions on gauge coupling unification.

Consider an $S^1/(Z_2 \times Z_2')$ model with SM fermions and Higgs doublet on the SU(5)-breaking brane. The bulk Higgs field Φ is in the adjoint of SU(5) and develops a VEV in $U(1)_Y$ direction. The 4d running below M_c gives $\alpha_{12}/\alpha_{23} = 1.90$, in significant disagreement with data. To correct this, consider power-like threshold corrections from the bulk, introducing a set of fundamental fermions of SU(5) coupled to the adjoint Higgs by a standard Yukawa coupling $\sim \bar{\psi}\Phi\psi$. Since these fermions can also have an SU(5) symmetric bulk mass, one can treat the resulting doublet and triplet masses $M_{\psi,t}$ and $M_{\psi,d}$ as essentially independent parameters. Assuming that Φ has no or only a very small bulk mass, the non-SUSY analogue of Eq. (2.21) reads

$$\alpha_{ij}(m_Z) = \frac{1}{12\pi}\left(-22C_{ij} + 2T_{ij}\right)\ln\frac{M_c}{m_Z} \qquad (2.25)$$
$$+\frac{1}{48}\left[-20C_{ij}\frac{M_V}{M_c} - 8T_{ij}\frac{M_{\psi,d}}{M_c} - 8T'_{ij}\frac{M_{\psi,t}}{M_c}\right] + \frac{1}{12\pi}\left[2T_{ij} + \frac{1}{2}(-22)C_{ij}\right]\ln\frac{M}{M_c}\,.$$

For simplicity, assume $M_{\psi,d} \ll M_{\psi,t}$ so that the power-like contribution from the doublet can be neglected. Further, lower the compactification scale as far as possible according to the naive estimate based on the higher-dimension operator discussed above, $M_c \simeq 10^3$ GeV with $M_V \simeq 20M_c$ and $M \simeq 10^3 M_c$. One now finds that the moderate value $M_{\psi,t} \simeq 3M_V$ gives $\alpha_{12}(m_Z) \simeq 29.4$ and $\alpha_{23}(m_Z) \simeq 21.2$, in reasonable agreement with the data. Although, given the ad-hoc choice of several parameters, this certainly does not challenge the numerical superiority of the minimal SUSY framework, it is nevertheless interesting to see how easily a non-SUSY SU(5) unification can be achieved with the help of large power-like thresholds.

Given that the above exploratory study has shown the possibility of very low compactification scales $\sim 10^3$ GeV, it is tempting to speculate that the discussion of the structure of 5d SUSY will reveal viable scenarios with TeV scale precision unification.

Chapter 3

Exploiting Higher-Dimensional Supersymmetry

This Chapter will exploit the structure of the higher supersymmetry in 5 dimensions as regards its consequences for the power-like loop corrections to the gauge couplings derived in the previous Chapter. In 5d there are only 4-component Dirac spinors because no Majorana or Weyl conditions are possible. Thus, the generator of the minimal 5d supersymmetry contains two 2-component SUSY generators in 4d language which can be shown to fulfill the 4d $\mathcal{N} = 2$ supersymmetry algebra. Therefore, a minimally supersymmetric field theory in 5d has a $\mathcal{N} = 2$ SUSY in 4d language whose structure will be of much use in what follows.

3.1 Prepotential of the 5d SYM theory

This Section collects results of [41–43] which are relevant for the subsequent discussion and sets up the notation used in this and next two Chapters. Consider a 5d SYM theory with massive gauged hypermultiplet matter. In addition to the vector field and gaugino, the 5d vector multiplet contains an adjoint scalar field Φ. The theory is conveniently described as a 4d $\mathcal{N}=2$ SYM theory depending on the extra parameter x^5. Its low-energy effective action is thus completely characterized by its holomorphic prepotential $\mathcal{F}(\Sigma)$ [73] (see also [74]). The scalar component of the chiral superfield Σ is $\Phi + iA_5$, where the conventions of [61] (see also [62]) are used but the names Σ and Φ are interchanged to facilitate comparison with [41]. Given the prepotential, the lagrangian of a 4d $\mathcal{N}=2$ SYM theory can be written in conventional $\mathcal{N}=1$ superfield notation as

$$\mathcal{L} = \frac{1}{2}\left\{\int d^4\theta \, \frac{\partial \mathcal{F}(\Sigma)}{\partial \Sigma^a} \, \left(\bar{\Sigma} e^{2V}\right)^a + \int d^2\theta \, \frac{\partial^2 \mathcal{F}(\Sigma)}{\partial \Sigma^a \, \partial \Sigma^b} \, W^a W^b\right\} + \text{h.c.} \tag{3.1}$$

Here $\Sigma = \Sigma^a T_a$ and the generators of the gauge group G are normalized by $2\,\mathrm{tr} T_a T_b = \delta_{ab}$ (traces are taken in the fundamental representation unless otherwise specified).

Under the constraints of SUSY and 5d Lorentz invariance, the 4d lagrangian of Eq. (3.1) extends in a unique way to a 5d lagrangian. However, 5d gauge invariance now constrains the prepotential to be at most cubic in Σ. In the context here, this is crucial since it ensures the absence of higher-dimension operators beyond the CS term (see Appendix B for more details). Following [41], one can also write the prepotential as a function of Φ. Requiring the prepotential to be analytic, the most general form is now

$$\mathcal{F}(\Phi) = \frac{1}{2g_{5,cl.}^2}\,\mathrm{tr}\,\Phi^2 + \frac{c_{cl.}}{48\pi^2}\,\mathrm{tr}\,\Phi^3\,. \tag{3.2}$$

The coefficients of these two terms determine the coefficients of the classical F^2 term and of the classical CS term, all other terms in the component lagrangian then being fixed by supersymmetry. (The normalization is chosen such that, in the absence of charged matter, $c_{cl.}$ is integer due to the boundary anomaly constraint. This will become evident below.) In the present context of gauge coupling unification, it is crucial that the SUSY CS term includes an operator $\sim \Phi F^2$, which clearly has the potential of affecting low-energy gauge couplings if Φ develops a VEV. Thus, the most important two terms of the component lagrangian derived from Eq. (3.2) are

$$\mathcal{L} \supset -\frac{1}{2g_{5,cl.}^2}\,\mathrm{tr}\,F^2 - \frac{c_{cl.}}{16\pi^2}\,\mathrm{tr}\,\Phi F^2\,. \tag{3.3}$$

The field Φ has a flat potential and one can consider the low-energy effective field theory in the presence of a Φ-VEV. It will become clear from the discussion in Sect. 5 that such a Φ-VEV (rather than just hypermultiplet VEVs) is necessary in order for loop corrections to gauge unification to arise. Without loss of generality, one can write $\Phi = \phi^i H_i$, where H_i are the Cartan generators of the gauge group G and $i \in \{1, \cdots, r = \mathrm{rank}(G)\}$. Then choose the H_i to be the first r elements of the set of generators T_a. Since a generic VEV breaks G to $\mathrm{U}(1)^r$, the relevant quantity is the prepotential of this abelian gauge theory. Including quantum corrections induced by the vector and hypermultiplets and choosing counterterms such that $g_{5,cl.}$ and $c_{cl.}$ remain unchanged, it reads [41, 75]

$$\mathcal{F}(\Phi) = \frac{1}{4g_{5,cl.}^2}\delta_{ij}\phi^i\phi^j + \frac{c_{cl.}}{48\pi^2}d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{96\pi^2}\left(\sum_\alpha |\alpha_i\phi^i|^3 - \sum_f\sum_\lambda |\lambda_i\phi^i + m_f|^3\right)\,. \tag{3.4}$$

Given the definition

$$d_{abc} = \frac{1}{2}\,\mathrm{tr}\,T_a\{T_b, T_c\}\,, \tag{3.5}$$

it is clear that the first two terms of Eq. (3.4) are simply a restriction of Eq. (3.2) to the $\mathrm{U}(1)^r$ subgroup. The remaining terms are the 1-loop-effects resulting from integrating out the heavy vector multiplets (corresponding to the broken directions of G) and the hypermultiplets with masses m_f labelled by their 'flavour' f. The other sums run over the roots α of Lie(G) and the weights λ of the relevant matter representations[1]. This notation implies that

$$[H_i, E_\alpha] = \alpha_i E_\alpha \qquad \text{and} \qquad H_i|\lambda\rangle = \lambda_i|\lambda\rangle\,, \tag{3.6}$$

where E_α is the Lie algebra element (root) corresponding to the root vector α and $|\lambda\rangle$ is a representation vector with weight vector λ (see, e.g., [76]). It is important that Eq. (3.4) is interpreted as defining a locally *holomorphic* prepotential, i.e., the modulus-signs merely determine whether a given cubic term is to be multiplied by $+1$ or -1 in a given region of the multi-dimensional space parameterized by ϕ^i. Note also that the coefficient of the last term in Eq. (3.4) differs from Ref. [41] due to different normalization of $c_{cl.}$ used here.

For the purposes here, it is essential that Eq. (3.4) specifies the complete low-energy effective action – no higher-loop contributions arise and no other classical terms are allowed at the two-derivative level.

As done before in Eq. (3.3) for the classical non-abelian theory, one now gives the gauge-kinetic term of the component lagrangian for each of the surviving U(1) factors. For the U(1) group generated by H_i the relevant piece of the component lagrangian reads

$$\mathcal{L}_i \supset -\frac{1}{4}F_i^2 \left\{ \frac{1}{g_{5,cl.}^2} + \frac{c_{cl.}}{4\pi^2} d_{iij}\phi^j + \frac{1}{8\pi^2}\left(\sum_\alpha \alpha_i^2 |\alpha_j\phi^j| - \sum_f \sum_\lambda \lambda_i^2 |\lambda_j \phi^j + m_f| \right) \right\}\,. \tag{3.7}$$

3.2 Power-law corrections from higher-dimension operators

The above result now corresponds precisely to the power-like loop corrections to gauge unification which were derived in the first two Chapters. To see this, consider the one-loop correction to a U(1) gauge coupling induced by massive particles, Eq. (2.2), which lends itself to an immediate implementation in the 5d situation. Combining the effects two complex scalars and a Dirac fermion, as appropriate for a massive hypermultiplet with mass m and charge q, the correction reads

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2}\, m\,. \tag{3.8}$$

This dimensionally regularized result hides a mass-independent, linearly divergent piece (see App. A). However, this piece is irrelevant in the present context since it is universal with

[1] If several representation vectors have the same weight vector λ, this weight vector contributes with the appropriate multiplicity.

respect to the different U(1) subgroups emerging from a spontaneously broken simple group G. It gives rise to a renormalization of the original non-abelian gauge coupling.

In the context of the previous section, the hypermultiplet component corresponding to the weight λ has mass $|\lambda_j\phi^j + m_f|$ and, with respect to the U(1) subgroup generated by H_i, charge λ_i. Thus, Eq. (3.8) precisely reproduces the matter contribution in Eq. (3.7). Furthermore, the correction from massive vector multiplets in Eq. (3.7) is also unambiguously determined since, apart from the different charges and masses, its contribution must be equal and opposite in sign compared to the hypermultiplet. This is clear since a vector- and hypermultiplet with the same mass and charge combine to a 4d $\mathcal{N} = 4$ multiplet and therefore induce no gauge coupling correction.

Of course, the interest here is not really in the breaking of a gauge group G of rank r to $\mathrm{U}(1)^r$ but rather in its breaking to a set of simple subgroups and U(1) factors, for which one uses the common notation G_i. For example, G_i with $i = 1, 2, 3$ may be the three gauge groups of the SM. The relevant gauge coupling corrections can be immediately read off from Eq. (3.7) by choosing an appropriate Φ-VEV, i.e., appropriately degenerate ϕ^i. It is useful to present the corresponding result in a different form, using traces of representation generators. In this form, the correction to the low-energy gauge coupling of the subgroup G_i reads

$$\delta\left(\frac{1}{g_{5,i}^2}\right) = \frac{c_{cl.}}{4\pi^2}\,\mathrm{tr}\,H_i^2\Phi + \frac{1}{8\pi^2}\left(\sum_{r_i(a)} T_{r_i(a)}M_{r_i(a)} - \sum_f \sum_{r_i(f)} T_{r_i(f)}M_{r_i(f)}\right)\,. \tag{3.9}$$

Here H_i is one of the Cartan generators of G that fall into G_i. The G_i-representations emerging from the adjoint of G and from the representation of the hypermultiplet f are labelled by $r_i(a)$ and $r_i(f)$ respectively. As usual, T_{r_i} is defined by $\mathrm{tr}_{r_i}[T_aT_b] = \delta_{ab}T_{r_i}$, with the trace taken in the representation r_i. Furthermore, $M_{r_i(a)}$ and $M_{r_i(f)}$ denote the masses of the vector multiplet in the representation $r_i(a)$ and the hypermultiplet in the representation $r_i(f)$ respectively. Given these definitions and the relations

$$\sum_{\alpha\,\in\, r_i(a)} \alpha_i^2 = \mathrm{tr}_{r_i(a)}H_i^2 = T_{r_i(a)} \qquad , \qquad \sum_{\lambda\,\in\, r_i(f)} \lambda_i^2 = \mathrm{tr}_{r_i(f)}H_i^2 = T_{r_i(f)}\,, \tag{3.10}$$

the derivation of Eq. (3.9) from Eq. (3.7) is straightforward.

So far, the 5d threshold formulae of Chap. 2 have just been recovered [33], based on the 4d results of [56, 57], in the prepotential language of [41], which is based on the anomaly calculation of [75]. However, this deeper conceptual understanding of power-like threshold corrections is crucial for their phenomenological applicability. The main point here is that the above prepotential formulae are quantum exact, which implies that the by now familiar

1-loop power-law contributions to gauge unification are not subject to further corrections. More specifically, while higher-loop contributions are absent because of $\mathcal{N}=2$ SUSY, the only competing tree-level higher-dimension operator is the SUSY CS term, corresponding to the first term on the r.h. side of Eq. (3.9). (Note also that the holomorphic gauge couplings discussed here coincide with the canonical gauge couplings in $\mathcal{N}=2$ SUSY [77].) Moreover, in the phenomenologically relevant case of a compactification on an interval, the CS term induces anomalies at the boundaries [78] (see [79] for a recent review). These induced anomalies must precisely cancel possible boundary anomalies coming from gauged bulk or brane fields. Thus, the value of the coefficient $c_{cl.}$ is completely determined by the field content of the model. This will be worked out in more detail in Chap. 4.

The high predictivity of this scenario relies on the uniqueness of the tree-level dimension-5 operator, i.e., the SUSY CS term. This uniqueness is clearly based on the analyticity of the prepotential as a function of Φ and the uniqueness of the third-order symmetric invariant tensor d_{abc} [80] (in fact, such an invariant exists only for SU(N) groups). Furthermore, it is also clear that the quantum corrected prepotential is not globally analytic (it is only analytic away from points where certain charged particle masses vanish). This allows for the distinct group-theoretical structures appearing in the quantum part of Eq. (3.4). However, for a given Φ-VEV, any of the hypermultiplet contributions becomes analytic in the limit $|m_f| \to \infty$. In fact, because of the relations

$$2\sum_\lambda \lambda_i\lambda_j \sim \delta_{ij} \qquad \text{and} \qquad \sum_\lambda \lambda_i\lambda_j\lambda_k \sim d_{ijk}\,, \tag{3.11}$$

it simply corrects the already existing tree-level operators $\sim \delta_{ij}$ and $\sim d_{ijk}$. (For the fundamental representation the proportionalities in Eq. (3.11) become equalities.) In this sense, heavy matter effectively decouples from gauge unification corrections, i.e., its only trace is a contribution to the CS term which, however, is anyway fixed by low-energy anomaly constraints. (Appendix C contains a simple example where this decoupling of heavy bulk matter is seen explicitly.)

Finally, one may consider the following somewhat exotic possibility. If a certain hypermultiplet is in a large representation, then $\lambda_i\phi^i$ can balance even a very large m_f and a non-analytic contribution to Eq. (3.4) may result. However, this does not contradict the above claim of effective decoupling since, given the spread of the values of λ_i in a large representation, many relatively light states (with a mass comparable to $|\Phi|$) will automatically also be present. Thus, the presence of a large-representation hypermultiplet will be known to the low-energy effective field theorist even if its mass is very large.

Consider now this situation of a 5d SYM theory with hypermultiplet matter which is broken by the VEV of the scalar adjoint Φ of the vector multiplet. Then the conclusion here is that in this setup power-law corrections to gauge unification are calculable in low-energy effective field theory.

Chapter 4

5d GUT phenomenology

4.1 Basic structure

Here the results of the previous Chapter will be discussed in more realistic situations. A phenomenologically viable 5d GUT should provide 4d $\mathcal{N} = 1$ SUSY at low energies to get the MSSM. The simplest scenario in which the above power-law corrections to 5d low-energy gauge couplings become relevant for a realistic GUT model is that of a field-theoretic S^1/Z_2 orbifold [81] (see also the slightly different later models of [36–38]). Specifically, consider a 5d SYM theory with gauge group G and hypermultiplet matter compactified on an S^1 parameterized by $x^5 \in [0, 2\pi R)$ and restrict the field space by requiring invariance under the reflection $x^5 \to -x^5$. If the space-time action of this Z_2 is accompanied by an inner automorphism of G (characterized by an element $P \in G$ with $P^2 = 1$) acting in field-space, the gauge group is broken at both boundaries. In general, the surviving subgroup contains a U(1) factor which contains P, i.e., $G \supset G' \times \mathrm{U}(1)$. Assume now that boundary interactions stabilize a VEV of the adjoint scalar Φ which points in the direction of the U(1) generator (cf. [65]). This breaks the gauge group in the bulk in the same way as the orbifolding does at the two boundaries.

The Φ-VEV can, for example, be stabilized by introducing a FI term within the U(1) subgroup surviving at each boundary. This term is, in general, generated by loop effects [82] but may also be present at the classical level. Thus, one can treat its coefficient as a free parameter. However, to be consistent with 4d supergravity (which is of course required although, at the technical level, the present work uses only rigid SUSY), the coefficients at the two boundaries are assumed to sum up to zero. As discussed in detail in [66, 68], the FI terms induce the desired constant bulk VEV of the scalar adjoint Φ.

Alternatively, the 5d model may be considered as the small-R_6 limit of a 6d theory, in which case the Φ-VEV corresponds to a Wilson line wrapping the 6th dimension. It is stabilized by the boundary conditions at the conical singularities of the 6d model. A more detailed discussion will be provided in the next section.[1]

In the above setting, the 4d gauge couplings observed just below the compactification scale $M_c = 1/R$ read

$$\frac{1}{g_{4,i}^2(M_c)} = \frac{\pi R}{g_{5,i}^2} + \frac{1}{g_{bd.,i}^2}\,. \tag{4.1}$$

Here the 5d gauge couplings are defined at zero momentum (i.e., as in the low-energy effective action of Sect. 3.1) and the last term accounts for the (presumably sub-dominant) effect of boundary gauge-kinetic terms. From the results of the last two sections, it is now clear that power-law corrections to inverse 4d gauge couplings are of the order $\sim |\Phi| R$ and can thus be as large as the tree-level term $\sim R/g_{5,cl.}^2$.

To be more specific, the focus is placed on the situation where $G = \mathrm{SU}(5)$ and $P = \mathrm{diag}(1,1,1,-1,-1)$ so that G is broken to the SM gauge group at the branes. In this case, the above S^1/Z_2 orbifold of a pure 5d SYM theory gives, at the zero mode level, the SM gauge multiplet and a chiral superfield with the quantum numbers of the X, Y gauge bosons. The latter one becomes massive, as noted in Sect. 2.4, when Φ develops a VEV and is therefore phenomenologically harmless. SM matter and Higgs fields can be added at the branes and/or in the bulk making the model as realistic (and arguably even somewhat simpler and more generic) as the more widely discussed $S^1/(Z_2 \times Z_2')$ models of [36–38].

4.2 Power-law corrections and consistency with boundary anomaly cancellation

The power-law corrections will now be given in the above class of models explicitly for hypermultiplet matter in the smallest SU(5) representations **5**, **10** and **24**. (Recall that, for example, a hypermultiplet in the **5** contains, in 4d $\mathcal{N}=1$ language, one 4d chiral superfield in the **5** and one in the $\bar{\mathbf{5}}$.) For this purpose, boundary gauge-kinetic terms and the corresponding logarithmic running are treated as sub-dominant. Thus, the analysis is based entirely on Eq. (3.9). All the group theory one needs is the familiar decomposition of the simplest SU(5) representations under SU(3)×SU(2)×U(1):

$$\mathbf{5} \;=\; (\mathbf{3},\mathbf{1})_{-2} + (\mathbf{1},\mathbf{2})_{3} \tag{4.2}$$

$$\mathbf{10} \;=\; (\mathbf{3},\mathbf{2})_{1} + (\bar{\mathbf{3}},\mathbf{1})_{-4} + (\mathbf{1},\mathbf{1})_{6} \tag{4.3}$$

$$\mathbf{24} \;=\; (\mathbf{8},\mathbf{1})_{0} + (\mathbf{1},\mathbf{3})_{0} + (\mathbf{1},\mathbf{1})_{0} + (\mathbf{3},\mathbf{2})_{-5} + (\bar{\mathbf{3}},\mathbf{2})_{5}\,. \tag{4.4}$$

[1]This alternative possibility is interesting in view of the following possible criticism of FI-term-stabilization: As argued in [83], the FI-terms can be understood in supergravity as arising from a mixed gauge-graviphoton CS term in the bulk. However, in the constructions considered here the brane U(1) arises from a non-Abelian gauge symmetry in the bulk and the author is not aware that the required mixed CS term has been discussed in this case.

The U(1) charges q' given here, in the conventions of [76], correspond to charges $q = q'/\sqrt{60}$ if the U(1) generator is normalized consistently with the other SU(5) generators. For easy reference, in Table 4.1 the relevant group-theoretical factors $T_{r_i(f)}$ in a hopefully self-explanatory notation are collected.

			$T^{\mathrm{U}(1)}$	$T^{\mathrm{SU}(2)}$	$T^{\mathrm{SU}(3)}$	VEV-induced mass
$(\mathbf{3},\mathbf{1})$	of	$\mathbf{5}$	1/5	0	1/2	$-(2/5)M_V$
$(\mathbf{1},\mathbf{2})$	of	$\mathbf{5}$	3/10	1/2	0	$(3/5)M_V$
$(\mathbf{3},\mathbf{2})$	of	$\mathbf{10}$	1/10	3/2	1	$(1/5)M_V$
$(\bar{\mathbf{3}},\mathbf{1})$	of	$\mathbf{10}$	4/5	0	1/2	$-(4/5)M_V$
$(\mathbf{1},\mathbf{1})$	of	$\mathbf{10}$	3/5	0	0	$(6/5)M_V$
$(\mathbf{8},\mathbf{1})$	of	$\mathbf{24}$	0	0	3	0
$(\mathbf{1},\mathbf{3})$	of	$\mathbf{24}$	0	2	0	0
$(\mathbf{1},\mathbf{1})$	of	$\mathbf{24}$	0	0	0	0
$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	of	$\mathbf{24}$	5	3	2	M_V

Table 4.1: Group-theoretical factors $T_{r_i(f)}$ of the simplest SU(5) representations relevant for the evaluation of Eq. (3.9). The last column contains the masses which the different representations acquire in the presence of a Φ-VEV (parameterized by the mass M_V of the 5d X, Y gauge bosons).

At this point one can now write down explicitly the corrections $\Delta\alpha_1^{-1}$, $\Delta\alpha_2^{-1}$, and $\Delta\alpha_3^{-1}$ to the inverse couplings of the three SM gauge groups U(1), SU(2), and SU(3) (as usual, $\alpha_1 = g_1^2/(4\pi)$ etc.). For example, a **5** hypermultiplet with bulk mass $m_{\mathbf{5}}$ (parameterized by $\xi_{\mathbf{5}} = m_{\mathbf{5}}/M_V$) induces corrections

$$\Delta\alpha_1^{-1} = -\left(\tfrac{1}{5}\left|\xi_{\mathbf{5}} - \tfrac{2}{5}\right| + \tfrac{3}{10}\left|\xi_{\mathbf{5}} + \tfrac{3}{5}\right|\right)\tfrac{M_V}{2M_c}\ ,$$
$$\Delta\alpha_2^{-1} = -\tfrac{1}{2}\left|\xi_{\mathbf{5}} + \tfrac{3}{5}\right|\tfrac{M_V}{2M_c}\quad ,\quad \Delta\alpha_3^{-1} = -\tfrac{1}{2}\left|\xi_{\mathbf{5}} - \tfrac{2}{5}\right|\tfrac{M_V}{2M_c} \tag{4.5}$$

reproducing the results of Sect. 2.4 in a more systematic way. This and corresponding formulae for the **10** hypermultiplet and the **24** hypermultiplet or vector multiplet are easily read off from Table 4.1 and Eq. (3.9) after compactification on an interval with length $\pi R = \pi/M_c$.

Finally, one needs to deal with the effect of a classical SUSY CS term parameterized by $c_{cl.}$. This term is constrained by boundary anomaly cancellation [62, 78]. As can be seen explicitly from Eqs. (3.4) and (3.11), a bulk **5** in the limit $m_{\mathbf{5}} \to \pm\infty$ induces an effective CS term with $c_{cl.} = \mp 1/2$ [41]. The boundary anomalies induced by this term can be found as follows (see, e.g., [83, 84]):

Consider first two massless bulk hypermultiplets **5** and **5′**, each with the same boundary conditions at $x^5 = 0$ and $x^5 = \pi R$, but with the sign flipped between the two hypermultiplets.

The model is anomaly-free, not just at the zero-mode level but also at each of the two boundaries taken separately. This is clear since the zero-mode matter is vector-like, so that there is no 4d anomaly, and the boundary anomaly is simply 1/2 of the 4d anomaly. (Recall that there are no anomalies in 5d.) Furthermore, the consistency is not destroyed by continuously varying one of the mass parameters, e.g., taking $m_{\mathbf{5}} \to \infty$ while keeping $m_{\mathbf{5}'} = 0$.

Thus, the CS term induced by the infinitely heavy **5** precisely cancels the boundary anomalies coming from brane localized zero-modes emerging in the limiting procedure $m_{\mathbf{5}} \to \infty$ and from the massless $\mathbf{5}'$. The latter are half-integer-valued in units corresponding to a 4d chiral fermion in the **5**. This is obvious since, again, the zero-mode anomaly is split equally between the two identical boundaries. Postponing a more explicit discussion to the next subsection, one can now already conclude that $c_{cl.} = \mp 1/2$ induces half-integer boundary anomalies. Thus, in the absence of charged bulk matter, $c_{cl.}$ must be integer and, to achieve gauge invariance, appropriate brane fields cancelling the induced integer-valued anomalies must be present. This argument for the value of $c_{cl.}$ could have also been made on the basis of the $m_{\mathbf{10}} \to \infty$ limit of a **10** hypermultiplet, which induces a CS term identical to that induced by a **5**.

From Eqs. (2.24) and (4.5) and corresponding formulae for the matter in the **10** and **24**, it is clear that almost any ratio of low-energy gauge couplings can be realized by tuning appropriately the bulk masses of the matter fields. One therefore now focuses on the arguably more natural case where bulk fields are either massless or extremely heavy, i.e., contribute only via an analytic CS term. The relevant contributions to the differences of inverse 4d gauge couplings $\alpha_{ij} = \alpha_i^{-1} - \alpha_j^{-1}$ are given in Table 4.2.

massless fields or operator	$\alpha_{12} \times 2M_c/M_V$	$\alpha_{23} \times 2M_c/M_V$
24 vector	2	1
24 hypermultiplet	-2	-1
5 hypermultiplet	1/25	$-1/10$
10 hypermultiplet	$-27/25$	3/10
CS term with $c_{cl.} = \mp 1/2$	$\pm 1/5$	$\mp 1/2$

Table 4.2: Corrections to inverse gauge coupling differences (in units of $M_V/(2M_c)$) induced by massless fields in the simplest representations and by the smallest possible CS terms.

At this point, some basic phenomenological implications can already be derived. Note first that anomaly cancellation by boundary fields is only possible if the boundary anomalies induced by bulk fields and operators are integer-valued. Thus, the sum of the numbers of bulk **5**s, **10**s and "CS-term-quanta" (i.e., CS term contributions with $c_{cl.} = \pm 1/2$) has to be even.

Recall that 4d MSSM running gives $\alpha_{12}/\alpha_{23} = 7/5 = 1.4$, which is known to agree very well with the observed low-energy gauge couplings. The effect of just the gauge sector gives, both in the 4d logarithmic and in the above power-law case, $\alpha_{12}/\alpha_{23} = 2$. In the 4d case, this is then corrected by the contribution from the two Higgs doublets. As noted in [33], a single bulk hypermultiplet in the **5**, with $m_{\mathbf{5}}$ tuned such that, in the presence of the Φ-VEV, the doublet is massless in 5d, reproduces the approximately correct ratio $\alpha_{12}/\alpha_{23} = 1.2$ of [6]. However, as the anomaly argument above shows, such a single bulk **5** has to be supplemented with a CS term. Unfortunately, this destroys the approximately correct power-law effect of [6] (this important point was missed in [33]). Furthermore, it is now clear why the phenomenological analysis of the power-law corrections for the SUSY case in Section 2.4 was preliminary: on the level of the discussion given there the anomaly constraints were not taken into account.

Now, coming back to the more restrictive framework of Table 4.2, one can look for simple configurations which give the MSSM prediction of $\alpha_{12}/\alpha_{23} = 7/5$ as a power-law effect. It is interesting to observe that, indeed, the vector multiplet together with a massless bulk **10** and the minimal required CS term (choosing the negative sign, $c_{cl.} = -1/2$) gives precisely $\alpha_{12}/\alpha_{23} = 7/5$. Thus, this combination of bulk fields and operators generates a power-law effect mimicking MSSM 1-loop running. Furthermore, replacing the **10** with a **5** and changing the sign of the CS term, one finds $\alpha_{12}/\alpha_{23} = 46/35 \simeq 1.31$, which is also quite close to the desired value 1.4. For the moment, the two above examples are quite satisfying and the exploration of other, more complicated, matter field and CS term configurations is left for future work. Such a more extensive analysis should be performed in a context with tighter constraints, e.g., in the search for a realistic flavour model or in the framework of a first-principles string construction.

4.3 Low-energy field content

The construction of an anomaly free model on the basis of a given bulk matter content and CS term is the last step which remains. This procedure is illustrated using the particularly attractive scenario with an SU(5) vector multiplet, a **10** hypermultiplet and a CS term with $c_{cl} = -1/2$ in the bulk, where the power law effect is equivalent to logarithmic MSSM running.[2]

As before, one compactifies on S^1/Z_2 breaking SU(5) to the SM at both boundaries,

[2] It is interesting to speculate that this field content arises from a (possibly even-higher-dimensional) SO(10) model where the adjoint decomposes as $\mathbf{45} = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$ and the $\overline{\mathbf{10}}$ becomes heavy in the process of gauge-symmetry and SUSY breaking.

starting with no bulk CS term but with a **10** and **10′** bulk hypermultiplet with opposite boundary conditions. In this situation, the spectrum of fermionic fields which are non-zero at any of the two boundaries is vector-like, i.e., no boundary anomalies arise. By continuity, the consistency of this model is not destroyed if, while keeping $m_{\mathbf{10}} = 0$, the limit $m_{\mathbf{10}'} \to \infty$ is taken. One now has an anomaly-free model with the desired content of light bulk fields and a CS term with $c_{cl} = -1/2$. A specific brane field content arises from the **10′** in the limit $m_{\mathbf{10}'} \to \infty$ due to the presence of localized zero-modes [85] (see also [86]).

To discuss these brane fields explicitly recall that, in $\mathcal{N}=1$ language, the **10′** hypermultiplet contains two chiral superfields in complex-conjugate representations, which are denoted by **10′** and $\mathbf{10}'^c$. Assume that the sign-conventions of the 5d lagrangian are such that positive $m_{\mathbf{10}'}$ implies a localization[3] of the **10′** at $y = 0$ and of the $\mathbf{10}'^c$ at $y = \pi$. Furthermore, define the SU(5)-breaking boundary conditions such that the $(\mathbf{3}, \mathbf{2})'$ is non-zero while the $(\bar{\mathbf{3}}, \mathbf{1})'$ and $(\mathbf{1}, \mathbf{1})'$ vanish at both branes. It is now clear that, in the limit $m_{\mathbf{10}'} \to \infty$, the only light fields are the zero mode of $(\mathbf{3}, \mathbf{2})'$, completely localized at $y = 0$, and the zero modes $(\bar{\mathbf{3}}, \mathbf{1})'^c$ and $(\mathbf{1}, \mathbf{1})'^c$, completely localized at $y = \pi$.

Phenomenologically, it is also essential to know what zero modes arise from the **10** hypermultiplet and at which brane they are peaked. (Note that, in contrast to the complete localization of the zero modes arising from the **10′** hypermultiplet, one has strong but finite peaking characterized by $\exp[\pm ym]$.) Given the conventions used here for the relative sign between bulk mass and Φ-VEV, as specified by Eq. (4.5), and the signs in the last column of Table 4.1, the direction of the peaking of the various fields of the **10** hypermultiplet is easily determined. Recalling that the boundary conditions of the **10** hypermultiplet are opposite to those of the **10′** hypermultiplet, one finds a $(\mathbf{1}, \mathbf{1})$ zero mode peaked at $y = 0$ as well as $(\mathbf{3}, \mathbf{2})^c$ and $(\bar{\mathbf{3}}, \mathbf{1})$ zero modes localized at $y = \pi$.

To make the model realistic without destroying the MSSM-like power law contribution from the bulk, matter has to be introduced in the form of brane fields. One begins by localizing a **10** chiral superfield at $y = \pi$. Allowing all gauge-invariant mass terms and recalling that, from the previous construction, one also has a $(\mathbf{3}, \mathbf{2})^c$, $(\bar{\mathbf{3}}, \mathbf{1})$, $(\bar{\mathbf{3}}, \mathbf{1})'^c$ and $(\mathbf{1}, \mathbf{1})'^c$ peaked or localized at $y = \pi$, all fields except for a partnerless $(\bar{\mathbf{3}}, \mathbf{1})_{-4}$ are found to become massive. Together with the $(\mathbf{3}, \mathbf{2})_1$ and $(\mathbf{1}, \mathbf{1})_6$ left over at $y = 0$ from the previous construction, one now has a full **10** in zero modes. Of course, the introduction of a **10** chiral superfield at $y = \pi$ demands, by anomaly cancellation, the further introduction of a $\bar{\mathbf{5}}$ at

[3]By this it is meant that the relevant bulk equations of motion for the $\mathcal{N} = 1$ superfields are $(\partial_y + m_{\mathbf{10}'})\mathbf{10}' = 0$ and $(\partial_y - m_{\mathbf{10}'})\mathbf{10}'^c = 0$, implying bulk solutions $\mathbf{10}' \sim \exp(-m_{\mathbf{10}'}y)$ and $\mathbf{10}'^c \sim \exp(+m_{\mathbf{10}'}y)$. In particular, if a zero mode is allowed by the boundary conditions, it will then be localized as described in the main text above.

the same brane. Now one has a full SM generation, with all fields except for the left-handed quarks and right-handed electron peaked at $y = \pi$. Amusingly, this matter distribution excludes all (not exponentially suppressed) mass terms within this generation. Thus, this construction has produced an anomaly-free setting with MSSM-like power law correction and one naturally light generation. The two Higgs doublets and the heavy generations are now easily added at $y = \pi$ without affecting any of the attractive features achieved so far.

To be completely explicit, one now calculates the gauge couplings at the Z-pole in the above model as in Sect. 2.2, including the logarithmic terms. Below the compactification scale $M_c = 1/R$ there is conventional MSSM running; above that scale one has the power-like effects discussed here and further corrections associated with the logarithmic running of brane-localized gauge-kinetic terms (see, e.g., [9, 33, 37, 38, 60]). This logarithmic running above M_c is cut off at some UV-scale Λ where the singular boundary is resolved. For simplicity, one may assume $\Lambda/M_V = \mathcal{O}(1)$ and thus disregard logarithms of this ratio. In particular, this implies that only the Kaluza-Klein (KK) modes of the 5d vector multiplet within the SM gauge group contribute to the logarithmic running above M_c.

Note that, from the point of view of the bulk theory and the power-like terms, the existence of a UV scale Λ is immaterial since the calculation of inverse gauge coupling differences is entirely UV-insensitive. In fact, this was to be expected in view of the possible existence of a non-trivial UV fixed-point of the 5d theory discussed in [41], i.e., the possibility of taking $\Lambda \to \infty$ (see [87] for more general analyses, including in particular the 6d case, and [59] for a recent application in unified models). However, it must be emphasized that the calculations performed here, although quite consistent with the fixed point proposal, do not rely on it or on the limit $\Lambda \to \infty$ since all dangerous higher-dimension operators are forbidden by symmetries.

The low-energy inverse gauge couplings are given by

$$\alpha_{4,i}^{-1}(m_Z) = \pi R \alpha_{5,cl.}^{-1} + b_i \left(\frac{1}{10} \frac{M_V}{M_c} + \frac{1}{2\pi} \ln \frac{M_c}{m_Z} \right) + \frac{\tilde{b}_i}{2\pi} \ln \frac{M_V}{M_c} + \{i\text{-indep. terms}\} \qquad (4.6)$$

where $\alpha_{5,cl.} = g_{5,cl.}^2/(4\pi)$. The coefficients $b_i = (0, -6, -9) + 2\,(3/10, 1/2, 0)$ govern the familiar gauge and Higgs contributions to the MSSM running and, in this specific example, also the power-law term. Their S^1/Z_2 counterparts governing the modified running above M_c are $\tilde{b}_i = (0, -4, -6) + 2\,(3/10, 1/2, 0)$. Note that, to simplify Eq. (4.6), the "i-independent terms" have been chosen such as to ensure that the familiar coefficients b_i multiply both the power-law term and $\ln(M_c/m_Z)$. This is possible because the power-law corrections respect the MSSM relation $\alpha_{12}/\alpha_{23} = 7/5$.

The main technical statement to be made is the harmlessness of this modified logarithmic contribution, which is sufficiently similar to MSSM running and parametrically much smaller than the power law term. To see this explicitly, consider the most extreme case of $M_c \sim m_Z$ (i.e., disregard the term $\sim \ln(M_c/m_Z)$) and choose $M_V = 48.5\, M_c$. Equation (4.6) then gives $\alpha_{12}(m_Z) = 29.4$ and $\alpha_{23}(m_Z) = 21.3$ in almost perfect agreement with what is needed to accommodate the low-energy values $\alpha_i^{-1}(m_Z) = (59.0, 29.6, 8.4)$. At this point it is clear that precision unification at the TeV scale is a viable possibility in this model.

Chapter 5

6d GUT phenomenology

The previous two Sections emphasized that 5d GUT models with bulk matter which is either massless or extremely heavy (leaving only a CS term as its low-energy trace) are phenomenologically rather attractive. As argued below, such 5d models arise naturally as the low-energy limit of 6d models compactified on an even smaller S^1. Therefore, this Chapter discusses power-like corrections to gauge unification in 6d SYM theories. To begin, consider uncompactified, flat, 6-dimensional space with minimal SUSY (corresponding to $\mathcal{N} = 2$ in 4d), in which case the vector multiplet contains just the gauge field and a 6d-chiral spinor [88]. One may add 6d gauged hypermultiplets, the spinors of which must be of opposite 6d chirality relative to the gaugino. The reason for this is the presence of a Yukawa-like interaction term in the 6d lagrangian. This term combines the gaugino with the charged matter fermion, forcing them to have opposite chirality. (In this context, it is useful to recall that, unlike in 4d, in 6d complex conjugation does not change the chirality of a spinor. Thus, 6d chirality is an 'absolute concept' in the sense that it does not depend on whether one views the spinor or its complex conjugate as the basic degree of freedom. See App. E for details.)

The above implies that no mass terms connecting gauged hypermultiplets are allowed in 6d. Indeed, all the fermions involved have the same chirality making fermionic mass terms impossible. Independent of the gauging, the absence of masses in 6d simply follows from the fact that, in a hypermultiplet, the SUSY variation of a fermion is proportional to the SUSY generator, implying that all the fermions have the same chirality. This is very interesting from the model building perspective since it implies that the 5d hypermultiplet masses, which could in principle be used for an arbitrary tuning of 5d power-like unification corrections (cf. Eqs. (3.7) and (4.5)) have no 6d analogue.

However, it would be premature to conclude that there is no massive gauged matter in 6d. Indeed, mass terms linking a 6d hypermultiplet with a 6d vector multiplet, both charged under some gauge group, are possible. Such mass terms arise, for example, in the KK mode description of d-dimensional theories, where $d > 6$, compactified to 6d. They also appear in situations where a 6d gauge symmetry is broken by the VEV of the scalar component of one of the gauged hypermultiplets. Mass terms of this type, involving vector and hypermultiplet in the same representation, automatically produce a full $\mathcal{N} = 4$ multiplet at a given mass level. Thus, they are irrelevant in the present context of loop corrections to gauge coupling unification and one can from now on focus on massless 6d models.[1]

The 6d vector multiplet contains no scalar (the adjoint Φ of the corresponding 5d theory being promoted to the gauge field component A_6). Thus, soft gauge symmetry breaking in a 6d Lorentz-invariant setting has to rely on the VEV of one of the scalars of a gauged hypermultiplet. As explained above, massive fields can be collected in full $\mathcal{N}=4$ SUSY multiplets for any given mass and representation and no power corrections to gauge unification arise. This ends the discussion of the uncompactified 6d theory. What is more, it also implies that the only interesting situation in 5d is the one where the gauge symmetry breaking is driven by the adjoint scalar from the vector multiplet. Indeed, a 5d theory broken by a hypermultiplet VEV can be thought of as arising via dimensional reduction from a 6d theory, in which case the above argument demonstrates the absence of power-like loop corrections. This is the reason why the 5d analysis was focussed entirely on situations with gauge symmetry breaking by the adjoint scalar Φ. It may, however, be interesting to consider situations where bulk hypermultiplet VEVs are present in addition to the VEV of the adjoint scalar.

Given the absence of power-law corrections in the Lorentz-invariant 6d situation, one now focuses on 6d theories compactified on an S^1 of radius R_6 to 5 dimensions. Any possible further compactification (with compactification radius R_5) leading to a realistic 4d model is assumed to occur at a lower energy scale, $R_5 \gg R_6$. In the 5d effective theory, the gauge symmetry can be broken by the VEV of the adjoint scalar Φ. The latter has to be identified with the VEV of A_6, i.e., the Wilson line wrapping the S^1 [90]. Thus, one can straightforwardly apply the analysis of the previous sections and obtain the power-law corrections for any given 6d model. Important new features are the absence of a classical CS term and of hypermultiplet masses in 6d, which makes the setting more predictive, and the appearance of a tower of KK modes, the loop contributions of which have to be summed. The remainder of this Chapter is devoted to a detailed discussion of power-law effects in this effectively 5-dimensional situation.

Before coming to the actual calculation, another conceptual issue – the stabilization of

[1]More generally, according to Tables 4 and 5 of [89] all massive representations of minimal 6d SUSY with spin ≤ 1 automatically have a 4d $\mathcal{N}=4$ spectrum.

the Wilson line – has to be addressed. For the simplest geometrical setting, a rectangular torus T^2 with radii R_5 and R_6, the Wilson line in x^6-direction, which is the analogue of the Φ-VEV of the 5d models above, is a modulus protected by SUSY. However, in an appropriate orbifold of the type T^2/Z_2, $T^2/(Z_2 \times Z_2')$ etc., the Wilson lines have certain fixed, discrete values determined by the gauge twists associated with the various orbifold actions [39]. In fact, this is quite analogous to the discrete or quantized Wilson lines of string-theoretic orbifold models [91]. To be specific, recall that a T^2/Z_2 orbifold can be visualized as the surface of a 'pillow' [64]. It has the topology of a sphere and 4 conical singularities with deficit angle π. In various field- or string-theoretic orbifold constructions, gauge symmetry breaking on this space arises from the non-trivial gauge holonomy associated with loops surrounding the 'corners' of this pillow. By Gauss' theorem, two of these Wilson lines surrounding two adjacent corners combine into a Wilson line going around the center of the pillow, which will therefore in many cases have a non-zero, quantized value. It is now straightforward to imagine an extremely elongated pillow ($R_5 \gg R_6$) equipped with a fixed Wilson line in x^6 direction. The conical singularities are simply boundary effects (from the effective 5d point of view) stabilizing the Wilson line VEV. In fact, as discussed in [92], in field theory the Wilson lines surrounding each of the conical singularities do not have to be quantized but can vary continuously and each possible value can be stabilized by local physics at the fixed point (brane). An example for such a local stabilization mechanism is provided by brane-localized FI terms inducing locally a non-zero field strength [93].

5.1 Power corrections in a 6d theory compactified on a circle

As already mentioned at the beginning of this Chapter one has identified an interesting and realistic setting for 6d power corrections: the effectively 5-dimensional case of a 6d SYM on a T^2/Z_2 with a Wilson line wrapping the compactified 6^{th} dimension of length $2\pi R_6 \ll 2\pi R_5$. In fact, the corrections to gauge coupling unification arising in this setup could be extracted from the more general analysis of arbitrary tori with two Wilson lines performed in [44, 45] (including a discussion of the connection to string theory [48]). Terms linear in the Wilson line VEV appear, for example, in taking the appropriate limits of Eq. (27) in [45]. However, for the following it is useful to give an independent and extremely simple derivation, based on the 5d results obtained above, which adequately describes the dominant part of large, power-like corrections to gauge unification. It is important to note that the analysis of [44] supports the expectation (based, e.g., on the symmetry arguments or the UV fixed-point conjecture of [41]) that the field theory results for gauge coupling differences are recovered in string theory in the limit of infinite string tension.

Consider first, as at the beginning of Sect. 3.2, a supersymmetric 6d U(1) gauge theory with a gauged hypermultiplet of charge q. This will be a useful building block for the following realistic calculation although, without appealing to the Green-Schwarz mechanism, the simple U(1) model is inconsistent since it is anomalous.[2] After compactification, one has a KK tower of 5d hypermultiplets with masses $m_n = |n/R_6|$ with n integer. Turning on a Wilson line in x^6 direction, the former zero mode acquires a non-zero mass $m = qA_6$ (where a gauge with constant A_6-VEV has been chosen). A corresponding Wilson-line-induced effective mass correction is also added to the masses of the higher KK modes (which is particularly evident in the fermionic part of the lagrangian, see App. C, Eq.s (C.7) and (C.8), for details). The resulting KK spectrum is $m_n = |n/R_6 + m|$ with n running over all integers. Thus, the loop correction of Eq. (3.8) is replaced by

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2}\sum_{n=-\infty}^{+\infty}|nR_6^{-1}+m|\,. \tag{5.1}$$

As before, the interest is just in the mass dependence of this correction. This mass dependence is finite and can be easily extracted from the above divergent sum using dimensional regularization. It is convenient to introduce the dimensionless parameter $c = mR_6 = qA_6R_6$ assuming $0<c<1$ for the moment. The result, derived in Appendix D in Eq. (D.7), then reads

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2 R_6}\,c(1-c) = -\frac{q^2}{8\pi^2}\,m\,(1-c)\,, \tag{5.2}$$

where, to be emphasized again, an m-independent divergent contribution has been dropped.

This very simple formula has manifestly the correct limiting behaviour as $R_6 \to 0$ for fixed m. Furthermore, viewed as a function of R_6 and c, it is invariant under the substitution $c \to (1–c)$. This is a manifestation of the fact that the KK spectrum is completely determined once the lightest mode is known. In other words, the point $n = 0$ has no absolute meaning and a shift of the label n or a reflection $n \to -n$ do not affect the physics. This last comment makes it obvious that Eq. (5.2) is extended to values of c outside the interval $(0, 1)$ by simply demanding reflection symmetry with respect to any point where c is integer. This is illustrated in Fig. 5.1.

It is evident from Fig. 5.1 that, locally, the inverse gauge coupling squared depends quadratically on the Wilson line VEV and thus, from the 5d point of view, on Φ. However,

[2]For a 6d U(1) model, the anomaly induced by the box diagram is always non-zero since it is proportional to the sum of the fourth powers of the charges of the fermions, which all have the same chirality because of 6d SUSY. This will be different in non-abelian models (see below) since the gaugino is charged and has opposite chirality.

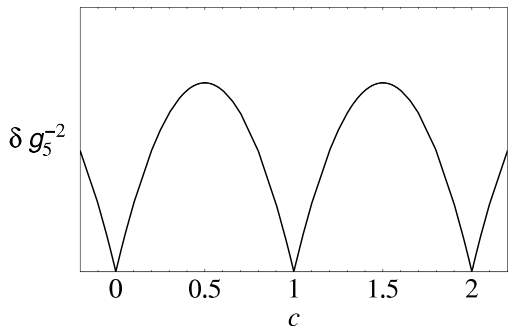

Figure 5.1: Illustration of the dependence of the 1-loop correction to the inverse gauge coupling in the 5d effective theory, g_5^{-2}, on the value of the Wilson line, parameterized by $c = qA_6R_6$.

it is known from SUSY and gauge invariance (cf. Sect. 3.1 and Appendix B) that the 5d prepotential is at most cubic and thus the Φ dependence is at most linear. This inconsistency is directly linked to the non-zero anomaly, as can be easily seen from Eq. (5.2). Indeed, for a model with several hypermultiplets the term quadratic in A_6 is proportional to the sum of the fourth powers of the charges, i.e., the anomaly, which is necessarily non-zero. It will become clear shortly that this problem disappears in an anomaly free, non-abelian model.

The non-abelian version of the above result can be written down without any further calculation. Recall that, at the beginning of Sect. 3.2, a rederivation of Eq. (3.7) on the basis of Eq. (3.8) and simple group theory was given. Following this line of reasoning, the 6d version of Eq. (3.7) can now immediately be given:

$$\mathcal{L}_i \supset -\frac{1}{4}F_i^2 \left\{ \frac{2\pi R_6}{g_{6,cl.}^2} + \frac{1}{8\pi^2}\left(\sum_\alpha \alpha_i^2\, |\alpha_j A_6^j| \left(1 - |\alpha_j A_6^j| R_6\right) - \sum_f \sum_\lambda \lambda_i^2\, |\lambda_j A_6^j| \left(1 - |\lambda_j A_6^j| R_6\right)\right)\right\} . \tag{5.3}$$

It is obtained from the original expression by identifying each 5d mass m and replacing it by $m(1-mR_6)$. This is the same procedure that leads from Eq. (3.8) to its 6d version Eq. (5.2). Of course in addition, the components ϕ^i of the field Φ are replaced by the corresponding components A_6^i of A_6 and the classical CS term as well as the hypermultiplet masses are dropped.

Similarly, the 6d analogue of Eq. (3.9), which is most directly useful for GUT phenomenology, reads

$$\delta\left(\frac{1}{g_{5,i}^2}\right) = \frac{1}{8\pi^2}\left(\sum_{r_i(a)} T_{r_i(a)} M_{r_i(a)} \left(1 - M_{r_i(a)} R_6\right) - \sum_f \sum_{r_i(f)} T_{r_i(f)} M_{r_i(f)} \left(1 - M_{r_i(f)} R_6\right) \right) . \tag{5.4}$$

It is clear from the structure of Eq. (5.3) that each of the 5d low-energy $U(1)$ gauge couplings depends on A_6 like a sum of functions of the type displayed in Fig. 5.1. In fact, both Eq. (5.3) and Eq. (5.4) can be taken at face value only in a certain neighbourhood of the point $A_6 = 0$. They are extended to all values of A_6 along a certain direction in the Cartan subalgebra by extending each of the terms of the form $m(1 - mR_6)$ as illustrated in Fig. 5.1. Locally, the sum of these terms must be a linear function since the 5d prepotential is at most cubic. The required cancellation of the coefficient of $(A_6)^2$ is indeed possible because of the relative sign between the vector multiplet and the hypermultiplet contributions in Eq. (5.3). This cancellation is intimately linked to the absence of 6d anomalies. To see this more explicitly, let A_6^i (with i fixed) be the only non-zero component of A_6 and consider the gauge coupling correction to the U(1) subgroup generated by H_i as specified by Eq. (5.3). The coefficient of $(A_6^i)^2$ is now manifestly proportional to the box anomaly coefficient. To see this, look at the pure gauge anomaly in 6d which is $\sim \mathrm{tr}_r\, F^4$. For a single spinor **16** of SO(10) one has according to [94] $\mathrm{tr}\, F^4 \sim \mathrm{Tr}_{\mathrm{symm.}}(T^a T^b T^c T^d) = A(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$. This is the box anomaly and like the coefficient of $(A_6^i)^2$ it is proportional to the fourth power of the charge if evaluated as above on a single generator. (Then it is $a = b = c = d = i$ on the root generator $T^i = \alpha_i$). Now if there is more than one field in the theory the coefficient of $(A_6^i)^2$ vanishes whenever the sum of the fourth powers of charges (specified by α_i) of fermions of the gaugino-chirality minus the sum of the fourth powers of charges (specified by λ_i) of fermions of matter-chirality is equal to zero. This fulfills the constraint of 6d gauge anomaly cancellation and is a nice consistency check of the present analysis.

5.2 A 6d SO(10) example

As an illustration of the general discussion above now the power corrections to gauge unification in a 6d SO(10) model compactified to 5d on an S^1 are explicitly calculated. The group is broken to SU(3)×SU(2)×U(1)×U(1)′ by an A_6 Wilson line along the hypercharge direction, which corresponds to the first of the two U(1)s above. (For a detailed discussion of the corresponding group theory and the various breaking possibilities see, e.g., Sect. 3.2 of [92].)

One possible special case of 5d effective theories of this type arises in the orbifold models of [39]. These models have a pillow-like fundamental space with gauge symmetries SO(10), SU(5)×U(1), SU(5)′×U(1)′ and SU(4)×SU(2)×SU(2)) at the four corners. One can now imagine stretching this space in one direction such that the SO(10) and the Pati-Salam fixed points are at one of the two boundaries of the resulting effectively 5-dimensional model (while the SU(5) and the flipped SU(5) fixed points are at the other boundary). Away from the boundaries, one has a cylinder wrapped by a Wilson line in hypercharge-direction, which is precisely the 6 to 5d compactification discussed above. In this specific orbifold realization, the Wilson line is quantized such that it corresponds to a Z_2 gauge twist and correspondingly the gauge symmetry in the 5d bulk is enhanced from the generic case, SM×U(1)′, to the Pati-Salam group (cf. the 5d the models of [72]). However, one can clearly imagine other similar constructions with different values of the A_6 Wilson line (see, e.g., the models of [92] and [93] where Wilson lines encircling conical singularities take on continuous values not related to the geometrical deficit angle).

Restricting ourselves to hypermultiplet matter in the **10** and **16** of SO(10), there is only one model without irreducible or reducible gauge anomalies. It contains, in addition to the vector multiplet in the **45**, 6 hypermultiplets in the **10** and 4 hypermultiplets in the **16** of SO(10) [95]. The existence and uniqueness of this solution is easily checked using the formulae of [94] (based on [96] and [97]). More possibilities exist if one only requires that the irreducible anomaly cancels, appealing to the Green-Schwarz mechanism [98] for the cancellation of the reducible anomalies. The investigation of power-law corrections in this context is left to future work. Also, 4d boundary anomalies arising at the conical singularities of the full model [99] will not be discussed, since they are not an intrinsic part of the effective 5d theory in which the power-corrections arise. However, it should be emphasized that an example of a realistic SUSY GUT with the above anomaly-free 6d bulk matter content has been given in [95].

In principle, the calculation of the power-law corrections in the anomaly-free 6d SO(10) model is a straightforward application of Eq. (5.4). The analysis becomes particularly simple if one uses the 5d results of Table 4.1 together with the familiar decomposition of SO(10) representations in SU(5) language. Specifically, the matter content of a vector **45** and hypermultiplets $6 \times \mathbf{10} + 4 \times \mathbf{16}$ of SO(10) corresponds to vector multiplets $\mathbf{24} + 2 \times \mathbf{10}$ and hypermultiplets $16 \times \mathbf{5} + 4 \times \mathbf{10}$ of SU(5). (Note that, as far as gauge coupling corrections are concerned, one does not need to distinguish between **5** and $\bar{\mathbf{5}}$ etc.) The effective 5d masses of the various fields are completely specified by these SU(5) representations since the symmetry-breaking Wilson line lies within the SU(5) subgroup. In particular, the two vector **10**s cancel the effect of two of the hypermultiplet **10**s of SU(5) because of effective $\mathcal{N} = 4$ SUSY in the spectrum.

Of course, the modification of corrections of the type displayed in Eq. (4.5) arising from the summation of the full KK tower has to be taken into account as described in Sect. 5.1.

For example, the fields of one 6d bulk hypermultiplet in the **5** give a correction

$$\Delta\alpha_2^{-1} = -\frac{1}{2}\left(\frac{3}{5}M_V\right)\left(1-\frac{3}{5}M_VR_6\right)\frac{R_5}{2}+\cdots = -\frac{1}{2}\left(\frac{3}{5}c\right)\left(1-\frac{3}{5}c\right)\frac{R_5}{2R_6}+\cdots\,, \tag{5.5}$$

where $c = M_VR_6$. Here it has been assumed that the effective 5d theory is further compactified to 4d on an interval of length πR_5 following as closely as possible the purely 5-dimensional situation of Sect. 4. As before, the typical A_6 dependence arising from the structure $m(1-mR_6)$ has to be continued to all values of A_6 as shown in Fig. 5.1. In the anomaly-free SO(10) model under discussion, one has contributions corresponding to a vector multiplet **24**, 2 hypermultiplet **10**s and 16 hypermultiplet **5**s in SU(5) language. Thus, the full correction reads

$$\Delta\alpha_2^{-1} = \frac{R_5}{2R_6}\left\{3c(1-c)-3\left(\frac{c}{5}\right)\left(1-\frac{c}{5}\right)-8\left(\frac{3}{5}c\right)\left(1-\frac{3}{5}c\right)\right\}\,, \tag{5.6}$$

with $c = M_VR_6$. Similar formulae for $\Delta\alpha_1^{-1}$ and $\Delta\alpha_3^{-1}$ are easily derived using the data of Table 4.1. For illustration, the inverse gauge coupling differences relevant to unification are plotted in Fig. 5.2. This figure nicely illustrates the piecewise linear functional dependence

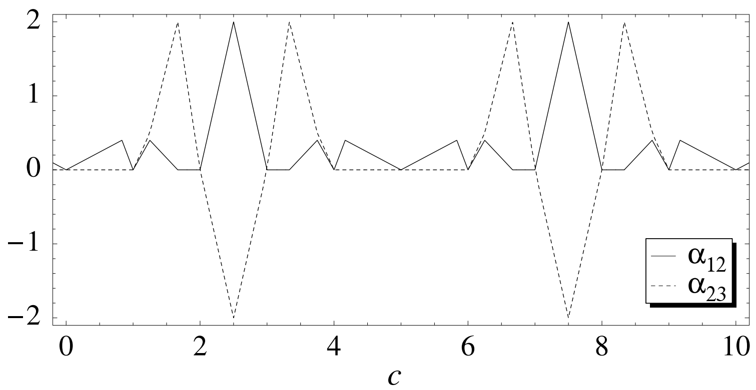

Figure 5.2: Power corrections to the inverse gauge coupling differences α_{12} and α_{23} in units of $(R_5/2R_6)$ as functions of A_6 (parameterized by $c = M_VR_6$, where M_V is the 6d X,Y gauge boson mass). Note the piecewise linear form related to 6d anomaly cancellation.

on A_6 that results from a sum of functions of the type displayed in Fig. 5.1 in an anomaly-free model. The figure also shows that, in the specific model under consideration, realistic gauge unification cannot be driven by just the power-law effect since the ratio $\alpha_{12}/\alpha_{23} \simeq 1.4$ is not realized for any value of A_6. This may be different for models with other matter content and corresponding Green-Schwarz anomaly cancellation. It may also be changed if other Wilson lines or bulk hypermultiplet VEVs affect the mass spectrum of the model. However, since the main aim of the present thesis is not the construction of realistic GUT models but rather the conceptual and technical understanding of power-law corrections to unification, this brief excursion into SO(10) phenomenology ends here.

Finally, it should be emphasized that the structure of Eq. (5.6) justifies, a posteriori, the assumption of an intermediate, effectively 5-dimensional theory, i.e., the assumption $R_5 \gg R_6$. Indeed, given that $c(1-c)$ and the other terms of this type are at most $\mathcal{O}(1)$, power corrections to inverse gauge coupling differences can only become parametrically larger than the familiar 4d threshold effects if $R_5/R_6 \gg 1$ (cf. [48]).

Chapter 6

KKLT - Higher-dimension operators and moduli stabilization

Field theories in a higher-dimensional space-time which are compactified to four dimensions on a so-called orbifold are conceptually and phenomenologically appealing. The simplicity and elegance of the symmetry breaking provided by orbifolding, for instance, already provides a sufficient motivation for the study of higher-dimensional field theories. The above results do strengthen this notion, since they state purely field theoretical and predictive power-law unification in higher dimensions. However, the structure of rigid supersymmetry in higher dimensions finally has to be embedded into gravitation. Gravitation means local supersymmetry and for supergravity, in particular in its unique form in 11 dimensions, it remains unclear whether it is controllable regarding possibly infinite loop corrections. Thus, we are left with superstring theory which up to now managed to upheld its claim of providing a way to unify the gauge interactions into a UV-finite theory of quantum gravity. There are five distinct superstring theories allowed by the constraints of 2d superconformal invariance on the string world sheet and 10d space-time supersymmetry: type I open strings with gauge group $SO(32)$, type IIA and IIB closed strings, and the two heterotic strings with gauge groups $E_8 \times E_8'$ and $SO(32)$. They have as their low energy effective theory in 10 dimensions different kinds of 10d supergravity coupled to 10d super Yang-Mills theory. The world, as we can observe it on scales between the Hubble scale (cosmic microwave background (CMB) radiation) and about $10^{-19}\,m$ ($= 100\,GeV^{-1}$, LEP 2 data), is neither supersymmetric nor higher-dimensional. Thus, the superstring and the 10d supergravities as its low energy description face the formidable tasks of breaking supersymmetry above the weak scale and and reducing ten to four dimensions. The latter problem is usually solved by KK-compactification from 10d to 4d on small compact internal manifolds [101]. Further, since the beginning of string theory several ways have been developed to solve he problem of breaking the large 10d supersymmetry and gauge symmetry. Choosing orbifolds or Calabi-

Yau manifolds for the internal spaces of KK-compactification can reduce the supersymmetry to 4d $\mathcal{N} = 1$ and break the gauge symmetry to Standard Model-like groups [100, 101]. The process of gaugino condensation can break the remaining supersymmetry and, further, it was used as a means to stabilize the string dilaton [101, 102]. However, string theory after compactification to 4d contains many massless scalar fields, the so-called moduli. The problem of finding compactifications to 4d which stabilize all moduli then has been adressed more recently in [15, 16] (partially, at least) by using certain properties of supersymmetry and string theory, such as the presence of non-trivial background fluxes [16–31], Dp-branes and gaugino condensation of strongly coupled sectors of super Yang-Mills theories.

Gauge coupling unification is the most prominent phenomenological success of the MSSM. Thus, it is desirable for string compactifications to arrive as a first step at effectively four-dimensional models with one surviving 4d supersymmetry ($\mathcal{N} = 1$). This can be done rather generically by compactifying the additional six spatial dimensions on an internal Kähler manifold of non-trivial topology with SU(3)-holonomy, a so-called Calabi-Yau threefold. The numerous continuous deformations of a given Calabi-Yau appear in the 4d effective field theory as massless scalar fields, called the moduli fields of the compactification. Further there is the dilaton, a massless scalar field predicted by all string theories to be present in its bosonic sector already in 10d which determines the string coupling of string perturbation theory. Therefore, one has to remove the dilaton and the moduli fields from the low energy effective theory in 4d by giving them large masses via a potential. This is required for two reasons. Firstly, these massless scalar fields would mediate long-range scalar interactions of roughly gravitational strength, which hitherto has not been observed. Secondly, fixing the moduli is necessary in order to choose a definite compact manifold. Otherwise one has huge classes of energetically equivalent compact manifolds to choose from.

The following sections will review how the moduli can be stabilized (that is, a potential can be generated for them) by different non-perturbative effects and several phenomenological aspects with the emphasis on inflation model building will be summarized. For this purpose, most of the considerations here will be presented in the framework of type IIB superstring theory and its type IIB low energy effective supergravity as being the example where up to now these structures have been understood best. Most of this review will be oriented along the lines of [15, 16, 100] in the first two Sections. Next, a short recollection of the basics of inflation will follow and some existing models of inflation in KKLT-like flux compactifications of string theory are discussed with further original sources given where necessary. This provides the background for the discussion of a new model of inflation constructed from the presence of higher-order α'-corrections [54] in string theory in Chapter 7.

Using the generic presence of such perturbative α'-corrections in string theory this model avoids introducing a source of (explicit) supersymmetry breaking like anti-D3-branes by hand which adds to its reliability.

6.1 AdS_5, fluxes and stabilization of the moduli

The appearance of the Kähler and complex structure moduli[1] in type IIB superstring compactifications on a Calabi-Yau threefold immediately leads to the question of how to stabilize them since they control both shape and size of the compact manifold. (For instance, the volume of the Calabi-Yau $\mathcal{M}_6$, given by $\mathrm{Vol}(\mathcal{M}_6) = \mathcal{K}/6$ where $\mathcal{K} = J \wedge J \wedge J$ and $J = g_{m\bar{n}} dy^m \wedge d\bar{y}^{\bar{n}}$ denotes the $(1,1)$-Kähler form, is clearly rescaled by fluctuations of $(1,1)$-form type.)

One way to lift the vacuum degeneracy represented by the moduli uses the presence of the other p-form fields in the low energy effective action of the type IIB superstring to give them, roughly speaking, a VEV. This is generalization of the geometrical KK-compactification on the Calabi-Yau. The compactification on a given Calabi-Yau is equivalent to the choice of a certain background metric. The metric arises as the symmetric 2-form field of the superstring. Therefore, the choice of a background metric means giving a VEV to the symmetric 2-form field of string theory. This process can be generalized to the other form fields in the effective action. Strictly speaking, one requires that the p-form flux given by the integral of a p-form field strength over a certain sub-manifold of the Calabi-Yau, a cycle, is quantized in units of the inverse string tension α'.

The possibility to have such a generalized flux compactification to 4d, however, is subject to severe constraints which can only be circumvented by the presence of typical stringy extended objects such as Dp-branes and orientifold planes. The following discussion of this procedure will be closely oriented along [16].

For this purpose begin with the bosonic part of the type IIB supergravity action given by

$$\begin{aligned} S_{\rm IIB} \;&=\; \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{\rm s}}\left\{e^{-2\phi}\left[R_{\rm s}+4\left(\partial\phi\right)^2\right]-\frac{F_{(1)}^2}{2}-\frac{1}{2\cdot 3!}\,G_{(3)}\cdot\bar{G}_{(3)}-\frac{\tilde{F}_{(5)}^2}{4\cdot 5!}\right\} \\ &\quad+\frac{1}{8i\kappa_{10}^2}\int e^{\phi}C_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)} \;+\; S_{\rm loc} \end{aligned} \tag{6.1}$$

where $S_{\rm loc}$ denotes additional terms in the action which arise from localized sources such as Dp-branes. Here the $F_{(2p+1)} = dC_{(2p)}$ denote the $2p+1$-form field strengths of the corresponding $2p$-form potentials $C_{(0)}$, $C_{(2)}$ and $C_{(4)}$ of the R-R sector of the type IIB superstring. The

[1] For the definition of the Kähler and complex structure moduli of a Calabi-Yau threefold see Appendix F.

type IIB axio-dilaton $\tau = C_{(0)} + ie^{-\phi}$ and the antisymmetric tensor $B_{(2)}$ of the NS-NS sector appear in the 3-form flux defined by $G_{(3)} = F_{(3)} - \tau \cdot H_{(3)}$ where $H_{(3)} = dB_{(2)}$. Furthermore one has defined

$$\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2}\, C_{(2)} \wedge H_{(3)} + \frac{1}{2}\, B_{(2)} \wedge F_{(3)} \quad . \tag{6.2}$$

This 5-form field strength is self-dual since the $(R+, R+)$-sector[2] of the type IIB superstring decomposes as $[1] + [3] + [5]_+$ under SO(1,9)-representations, demanding the presence of a self-dual 5-form field strength in the massless spectrum. The form of the action Eq. (6.1) is consistent with the self-duality of $\tilde{F}_{(5)}$ though this property does not follow from it. (It is known, that it is not possible to write a Lorentz invariant action for a self-dual 5-form field strength in 10d.)

For the next step one needs the equations of motion for the metric and $\tilde{F}_{(5)}$. For that purpose one rewrites the supergravity action in the Einstein frame defined by shifting to $g_{MN} = e^{-\phi/2} g_{sMN}$

$$\begin{aligned} S_{\mathrm{IIB}} \;&=\; \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ R - \frac{\partial_M \tau \partial^M \bar{\tau}}{2\,(\mathrm{Im}\,\tau)^2} - \frac{G_{(3)} \cdot \bar{G}_{(3)}}{12 \cdot \mathrm{Im}\,\tau} - \frac{\tilde{F}_{(5)}^2}{4 \cdot 5!} \right\} \\ &\quad + \frac{1}{8i\kappa_{10}^2} \int \frac{C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\mathrm{Im}\,\tau} \;+\; S_{\mathrm{loc}} \end{aligned} \tag{6.3}$$

Later it will turn out that in such compactifications [16, 49] the presence of fluxes and D3-branes or O3-planes is intimately linked to the warping of the geometry.

Varying S_{IIB} with respect to $\tilde{F}_{(5)}$ and using the self-duality of $\tilde{F}_{(5)}$ the equations of motion and the Bianchi identities for $\tilde{F}_{(5)}$ read

$$d * \tilde{F}_{(5)} = d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 \mu_3 \rho_3^{\mathrm{loc}} \quad , \tag{6.4}$$

where the last term describes the charge density of localized objects which are charged under the 4-form potential $C_{(4)}$ with some 5-form charge μ_3. For instance, a D3-brane, its low energy effective action in the bosonic sector to leading order being

$$S_{\mathrm{D3}} = -T_3 \int d^4\xi \sqrt{-g_4} + \mu_3 \int C_{(4)} \quad , \tag{6.5}$$

is a viable example for such a localized $C_{(4)}$-source.

The metric is governed by the 10d Einstein equations. Demanding 4d Poincaré invariance leads to a metric ansatz which in its compact part is conformally Calabi-Yau

$$ds_{10}^2 = \underbrace{e^{2A(y)} \eta_{\mu\nu}}_{(g_4)_{\mu\nu}} dx^\mu dx^\nu + \underbrace{e^{-2A(y)} \tilde{g}_{mn}}_{(g_6)_{mn}} dy^m dy^n \quad . \tag{6.6}$$

[2] '+' denotes the eigenvalue of the left- and rightmoving degrees of freedom, respectively, in the Ramond sector under the action of the worldsheet fermion number operator $\exp(iF)$

This requirement also forces one to consider the axio-dilaton $\tau = \tau(y)$ as well as the $G_{(3)}$ to depend only on the compact directions and $G_{(3)}$ to have only compact components. For $\tilde{F}_{(5)}$ the metric ansatz together with self-duality constrains its form to be

$$\tilde{F}_{(5)} = (1+*)[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3] \ , \ \alpha = \alpha(y) \ . \tag{6.7}$$

The equations of motion for the metric, the 10d Einstein equations follow as

$$\begin{aligned} R_{MN} - \frac{1}{2} g_{MN} R &= \kappa_{10}^2 \cdot T_{MN} \\ \Rightarrow \quad R_{MN} &= \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \ , \end{aligned} \tag{6.8}$$

where the energy-momentum tensor T_{MN} gets contributions from the various supergravity form fields as well as from the localized sources. The non-compact 4d components for bosonic fields then read

$$T_{\mu\nu} = -\frac{1}{2\kappa_{10}^2}\, g_{\mu\nu} \left(\frac{G_{mnp}\bar{G}^{mnp}}{12\,\mathrm{Im}\,\tau} + e^{-8A}\partial_m\alpha\partial^m\alpha \right) \underbrace{-\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{loc}}}{\delta g^{\mu\nu}}}_{T^{\mathrm{loc}}_{\mu\nu}} \ . \tag{6.9}$$

The other part of Eq. (6.8) is given by the 4d components of the Ricci tensor

$$R_{\mu\nu} = \partial_R \Gamma^R_{\mu\nu} - \partial_\mu \Gamma^R_{R\nu} + \Gamma^S_{\mu\nu}\Gamma^R_{RS} - \Gamma^R_{S\mu}\Gamma^S_{R\nu} \ . \tag{6.10}$$

Using that

$$\begin{aligned} \Gamma^\rho_{\mu\nu} &= \Gamma^R_{R\nu} = 0 \ , \ \ \partial_R \Gamma^R_{\mu\nu} = \partial_r \Gamma^r_{\mu\nu} = -\frac{1}{2}\,\eta_{\mu\nu}\partial_r \left(e^{2A}\tilde{g}^{rs}\partial_s e^{2A} \right) \\ \Gamma^S_{\mu\nu}\Gamma^R_{RS} &= \Gamma^s_{\mu\nu}\Gamma^r_{rs} = -\eta_{\mu\nu}e^{2A}g^{rs}\partial_r(\ln\sqrt{-g}\partial_s A) \\ \Gamma^R_{S\mu}\Gamma^S_{R\nu} &= \Gamma^\rho_{s\mu}\Gamma^s_{\rho\nu} = -\frac{1}{2}\,\eta_{\mu\nu}\tilde{g}^{rs}\partial_r e^{2A}\partial_s e^{2A} \end{aligned} \tag{6.11}$$

one finds

$$R_{\mu\nu} = -g_{\mu\nu}\nabla^2 A = -\eta_{\mu\nu}e^{4A}\tilde{\nabla}^2 A = -\frac{1}{4}\,\eta_{\mu\nu}\left(\tilde{\nabla}^2 e^{4A} - e^{-6A}\partial_m e^{4A}\partial^m e^{4A} \right) \tag{6.12}$$

where one defines

$$\nabla^2 f = \frac{1}{\sqrt{-g}}\,\partial_m(\sqrt{-g}g^{mn}\partial_n f) \tag{6.13}$$

and $\tilde{\nabla}^2$ likewise with g_{mn} everywhere replaced by $\tilde{g}_{mn}$. Finally the 4d components of the equations of motion of the metric follow from Eq.s (6.9) and (6.12) plugged into Eq. (6.8). Taking the 4d trace with $g^{\mu\nu} = e^{-2A}\eta^{\mu\nu}$ on them one arrives at

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\bar{G}^{mnp}}{12\,\mathrm{Im}\,\tau} + e^{-6A}\left(\partial_m e^{4A}\partial^m e^{4A} + \partial_m\alpha\partial^m\alpha \right) + \frac{\kappa_{10}^2}{2}\,e^{2A}(T^m_m - T^\mu_\mu)^{\mathrm{loc}} \ . \tag{6.14}$$

Next turn to the Bianchi identity of the 5-form field strength Eq. (6.4). The right hand side of Eq. (6.4) is given by

$$H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 \mu_3 \rho_3^{\rm loc} \quad . \tag{6.15}$$

This can be rewritten as

$$H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 \mu_3 \rho_3^{\rm loc} = i \cdot \frac{G_{mnp} \cdot *_6 \bar{G}^{mnp}}{12\,{\rm Im}\,\tau}\, dx^4 \wedge \ldots \wedge dx^9 + 2\kappa_{10}^2 \mu_3 \rho_3^{\rm loc} \quad . \tag{6.16}$$

Here the definition of the 3-form field strength $G_{(3)}$ has been used.

The left hand side of the Bianchi identity Eq. (6.4) after some technical calculations yields

$$d\tilde{F}_{(5)} = -\frac{1}{\sqrt{g_6}}\, \partial_m (g_4^{-1} \sqrt{-g} g^{mn} \partial_n \alpha)\, dx^4 \wedge \ldots \wedge dx^9 \quad . \tag{6.17}$$

For this calculation the self-duality of $\tilde{F}_{(5)}$ and the factors $\sqrt{-g}$ introduced by the Hodge dual $*$ are crucial. $d\tilde{F}_{(5)}$ can now be further rewritten to yield

$$\begin{aligned} d\tilde{F}_{(5)} &= \Big[\frac{1}{\sqrt{-g_4}\sqrt{-g}}\, \partial_m (\sqrt{-g} g^{mn} \partial_n \alpha) \\ &\quad + \frac{g_4}{\sqrt{-g_4}}\, \partial_m (g_4^{-1}) g^{mn} \partial_n \alpha \Big]\, dx^4 \wedge \ldots \wedge dx^9 \\ &= \Big[e^{-2A} \tilde{\nabla}^2 \alpha - 2 e^{-8A} \partial_m \alpha \partial^m \alpha \Big]\, dx^4 \wedge \ldots \wedge dx^9 \quad . \end{aligned} \tag{6.18}$$

Putting together Eq.s (6.16) and (6.18) one can rewrite the Bianchi identity of the 5-form field strength finally as

$$\tilde{\nabla}^2 \alpha = i e^{2A} \frac{G_{mnp} \cdot *_6 \bar{G}^{mnp}}{12\ {\rm Im}\,\tau} + 2 e^{-6A} \partial_m \alpha \partial^m \alpha + 2\kappa_{10}^2 e^{2A} \mu_3 \overline{\rho_3^{\rm loc}} \quad . \tag{6.19}$$

Here $\overline{\rho_3^{\rm loc}}$ is defined by

$$\int_{\mathcal{M}_6} d^6 y\, \overline{\rho_3^{\rm loc}} = \int_{\mathcal{M}_6} \rho_3^{\rm loc} = Q_3^{\rm loc} \quad . \tag{6.20}$$

As a last step take now the difference between this equation and Eq. (6.19) which yields a condition

$$\tilde{\nabla}^2 (e^{4A} - \alpha) = e^{2A} \frac{|i G_{mnp} - *_6 \bar{G}^{mnp}|^2}{6\ {\rm Im}\,\tau} + e^{-6A} \left| \partial (e^{4A} - \alpha) \right|^2 + 2\kappa_{10}^2 e^{2A} \left[\frac{1}{4} (T_m^m - T_\mu^\mu)^{\rm loc} - \mu_3 \overline{\rho_3^{\rm loc}} \right] . \tag{6.21}$$

The two constraints Eq.s (6.14) and (6.21) now connect the warping of the metric and the presence of 3-form and 5-form fluxes with the nature of the localized sources: firstly, integrate Eq. (6.14). The left hand side integrates to zero while the positive semi-definite form field contributions of the right hand side pick up every single non-zero piece of flux on the compact manifold. Therefore the fluxes have to vanish and the warp factor has to be constant unless there are localized sources present which manage to provide $T^m_m - T^\mu_\mu < 0$. Now integrate the second constraint Eq. (6.21). Again the left hand side integrates to zero. The right hand side of Eq. (6.21) upon integration in general does not give strong constraints, except if the last term of the right hand side of Eq. (6.21) is actually zero. If this stress term vanishes

$$(T^m_m - T^\mu_\mu)^{\rm loc} - \mu_3 \overline{\rho_3^{\rm loc}} = 0 \qquad (6.22)$$

then Eq. (6.21) implies that the 3-form field strength is imaginary self-dual (ISD) under the Hodge dual in the compact directions. Eq. (6.22) represents a BPS condition for the extended objects of the theory [16]. Further, if this BPS condition holds one has

$$\begin{aligned} iG_{(3)} &= *_6 G_{(3)} \\ e^{4A} &= \alpha \ . \end{aligned} \qquad (6.23)$$

At this point it is essential that string theory with its D3-branes and O3-orientifold planes contains objects which fulfill the condition Eq. (6.22) and have negative tension which implies $T^m_m - T^\mu_\mu < 0$ [16]. Regarding the last point note that the energy-momentum tensor of the localized objects is determined in terms of their 5-form charge $Q_3^{\rm loc}$. Now D3-branes (as well as D7-branes wrapped on 4-cycles of the Calabi-Yau) and O3-orientifold planes have $Q_3^{\rm loc} < 0$.

Note, that the above constraints are invariant under a rescaling of the compact metric $\tilde{g}_{mn} \to \lambda^2 \tilde{g}_{mn}$ [16]. According to this work the existence of a solution to the full supergravity equation of motions with the above properties is guaranteed if all the constraints Eq.s (6.14), (6.21) − (6.23) are fulfilled. Then the invariance of the constraints under the rescaling of the compact metric holds for the full solution, too. Since such a transformation rescales the overall volume of the Calabi-Yau and thus its radius, this property implies that there is at least one Kähler modulus of $(1,1)$-form type in the notation of Appendix F which has a flat direction.

A viable setup therefore is a Calabi-Yau orientifold compactification which has a set of O3-orientifold planes such that they carry an overall negative 5-form charge. O3-planes fulfill the BPS condition Eq. (6.22). Therefore, such a setup will enforce the presence of 3-form fluxes and a warped geometry such as to satisfy both the constraints of Eq.s (6.14) and (6.21). Thus, there are fluxes such that they cancel the integrated 5-form charge of the orientifold planes in the 5-form Bianchi identity

$$\frac{1}{2\kappa_{10}^2 \mu_3} \int_{\mathcal{M}_6} H_{(3)} \wedge F_{(3)} + Q_3^{\rm loc} = 0 \ . \qquad (6.24)$$

In addition, the $H_{(3)}$- and $F_{(3)}$-fluxes which combine to give the ISD $G_{(3)}$, are subject to certain quantization conditions with respect to the three cycles of the Calabi-Yau threefold chosen. Let C_I be a basis of the homology of three cycles on the Calabi-Yau. Then in turning on flux backgrounds the fluxes have to satisfy

$$\begin{aligned}\int_{C_I} F_{(3)} &= (2\pi)^2\alpha' \cdot M_I \ , \quad M_I \in \mathbb{N} \\ \int_{C_I} H_{(3)} &= -(2\pi)^2\alpha' \cdot K_I \ , \quad K_I \in \mathbb{N}\end{aligned} \tag{6.25}$$

which fixes and thus removes them as dynamical variables of the low energy effective theory.

Now consider this situation from the 4d point of view. Here one does not have a knowledge of the 10d constraints above which demand that $G_{(3)}$ is ISD. Since in the end one has to extract the potential for the moduli from the 'kinetic' term of the 3-form flux in the supergravity action Eq. (6.3) one has to separate it into purely topological and non-topological pieces. For this argument one needs the properties of the anti-ISD part $G^+_{(3)}$ of the 3-form flux. For that purpose look at the equations of motion for $G_{(3)}$ which read [16]

$$d\Lambda + \frac{i}{\operatorname{Im}\tau}\, d\tau \wedge \operatorname{Re}\Lambda = 0 \ , \quad \Lambda = e^{4A} *_6 G_{(3)} - i\alpha G_{(3)} \ . \tag{6.26}$$

For constant dilaton τ this becomes

$$de^{4A}(*_6 G_{(3)} - iG_{(3)}) = -2i \cdot de^{4A} G^+_{(3)} = 2 \cdot de^{4A} *_6 G^+_{(3)} = 0 \tag{6.27}$$

which is a constraint for the anti-ISD part of $G_{(3)}$. Thus, for constant warp factor $G^+_{(3)}$ must be a harmonic 3-form on the Calabi-Yau since the Laplacian $\Delta = *d*d + d*d*$ acting upon $G^+_{(3)}$ yields zero.

Splitting the 3-form flux into its ISD ($G^-_{(3)}$: $*_6 G^-_{(3)} = iG^-_{(3)}$) and anti-ISD ($G^+_{(3)}$: $*_6 G^+_{(3)} = -iG^+_{(3)}$) part and using that $*_6 G_{(3)} = iG_{(3)} - 2iG^+_{(3)}$ and $G^-_{(3)} \wedge \bar{G}^+_{(3)} = 0$ this term decomposes into

$$\begin{aligned} S_G &= -\frac{1}{24\kappa_{10}^2} \int_{\mathcal{M}_6} d^6y \sqrt{-\tilde{g}}\, \frac{G_{mnp}\bar{G}^{mnp}}{\operatorname{Im}\tau} \\ &= -\frac{i}{4\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{M}_6} G_{(3)} \wedge \bar{G}_{(3)} - \underbrace{\frac{1}{12\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{M}_6} d^6y \sqrt{-\tilde{g}}\, G^+_{mnp} \bar{G}^{+\widetilde{mnp}}}_{V} \ . \end{aligned} \tag{6.28}$$

While the first piece is purely topological, the second part can now be shown to represent a scalar potential V. $G^+_{(3)}$ was shown to be a harmonic 3-form and it is anti-ISD. Now on a Calabi-Yau threefold there are just two classes of non-vanishing anti-ISD harmonic 3-forms - the $(1,2)$-forms $\bar{\chi}_{\bar{A}}$ and the holomorphic 3-form Ω (see Appendix F). Thus, one may decompose the $G^+_{(3)}$ in the space of cohomologies as

$$G^+_{(3)} = a\Omega + \bar{b}^{\bar{A}} \bar{\chi}_{\bar{A}} \ . \tag{6.29}$$

Using now the inner product induced by the Hodge dual on the Calabi-Yau the decomposition coefficients (a complete analogon to the coefficients of linear combinations in a vector space spanned by a complete orthonormal system of basis vectors) follow to be

$$\begin{aligned} a &= \frac{\int_{\mathcal{M}_6} G_{(3)} \wedge \bar{\Omega}}{\int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}} \\ \bar{b}^{\bar{A}} &= \frac{\mathcal{G}^{\bar{A}B} \int_{\mathcal{M}_6} G_{(3)} \wedge \chi_B}{\int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}} \;,\;\; \mathcal{G}^{\bar{A}B} = \left(\frac{\int_{\mathcal{M}_6} \bar{\chi}_{\bar{A}} \wedge \chi_B}{\int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}} \right)^{-1} \;. \end{aligned} \tag{6.30}$$

Inserting these results into the term V of Eq. (6.28) allows one to write it as

$$\begin{aligned} V &= \frac{1}{12\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{M}_6} d^6y \sqrt{-\tilde{g}}\, G^+_{mnp} \bar{G}^{+\widetilde{mnp}} \\ &= \frac{\int_{\mathcal{M}_6} G_{(3)} \wedge \bar{\Omega} \int_{\mathcal{M}_6} \bar{G}_{(3)} \wedge \Omega + \mathcal{G}^{\bar{A}B} \int_{\mathcal{M}_6} G_{(3)} \wedge \chi_B \int_{\mathcal{M}_6} \bar{G}_{(3)} \wedge \bar{\chi}_{\bar{A}}}{2\kappa_{10}^2 \operatorname{Im}\tau \int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}} \;. \end{aligned} \tag{6.31}$$

Comparing this with the standard form of the scalar potential in supergravity

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3\,|W|^2 \right) \tag{6.32}$$

one finds that a Kähler potential

$$K = -3\ln(T + \bar{T}) - \ln(-i(\tau - \bar{\tau})) - \ln\left(-i \int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega} \right) \tag{6.33}$$

together with a superpotential [22]

$$W = \int_{\mathcal{M}_6} G_{(3)} \wedge \Omega \tag{6.34}$$

reproduces the above scalar potential V. Here a single volume modulus T has been included with a Kähler potential of no-scale type to account for the fact noted above, that the compactifications discussed here must possess at least one massless Kähler modulus. Since this no-scale piece cancels the $-3|W|^2$-piece in the potential, the supergravity scalar potential becomes

$$V = e^K K^{I'\bar{J}'} D_{I'} W D_{\bar{J}'} \bar{W} \;,\;\; I', J' = (A, \tau) \tag{6.35}$$

(A runs over the complex structure moduli) which gives precisely Eq. (6.31). One derives the supercovariant derivatives used here from Eq.s (6.33) and (6.34) to be

$$\begin{aligned} D_A W &= \partial_A W + W \partial_A K = \int_{\mathcal{M}_6} G_{(3)} \wedge \chi_A \\ D_\tau W &= \partial_\tau W + W \partial_\tau K = \frac{1}{\bar{\tau} - \tau} \int_{\mathcal{M}_6} \bar{G}_{(3)} \wedge \Omega \\ D_T W &= \partial_T W + W \partial_T K = -\frac{3W}{T + \bar{T}} \;. \end{aligned} \tag{6.36}$$

The conditions for unbroken supersymmetry $D_A W = D_\tau W = D_T W = 0$ thus imply that $G_{(3)}$ must be of $(2,1)$-type and $G^+_{(3)} = 0$ which reproduces the 10d condition Eq. (6.23) in the end. A $(0,3)$-piece in the G-flux would in general yield $W \neq 0$ at the point where $D_A W = D_\tau W = 0$ which would break supersymmetry.

As a result one is now in a situation where turning on a sufficiently generic 3-form flux $G_{(3)}$ of $(2,1)$- or $(0,3)$-form type generates a superpotential and thus a scalar potential that fixes all the complex structure moduli and the type IIB axio-dilaton at certain VEVs. Note, that from an analysis of the supersymmetry transformation of the gravitino in 10d it, too, follows that a $(0,3)$-form flux breaks the last $\mathcal{N} = 1$ SUSY while a flux of $(1,2)$-form type preserves it [16, 22, 103].

6.2 Volume stabilization and non-perturbative effects

The procedure described in the last Section provides a way towards stabilizing all except the Kähler moduli of Calabi-Yau compactifications of the type IIB superstring by turning on appropriate quantized fluxes of the p-form field strengths. However, this generically does not remove the flat directions of the Kähler moduli since the superpotential generated by the fluxes does not depend on them. For simplicity one may consider now compactification with just one remaining Kähler modulus, the volume modulus T (a generalization to the case of several Kähler moduli is possible [16]). As [15] have shown, it is possible to use non-perturbative corrections to the low energy effective supergravity action in order to break the no-scale structure of the Kähler potential of the volume modulus.

For instance, both fractional instanton effects on D3-branes and gaugino condensation of large gauge groups on stacks of D7-branes generate a non-perturbative superpotential for the T-modulus which takes the general form

$$W(T) = \sum_j A_j e^{-a_j T} \quad . \tag{6.37}$$

As an example a stack of N coinciding D7-branes may serve: such a stack may carry a super Yang-Mills theory of gauge groups up to SU(N). This theory then undergoes gaugino condensation on a scale below the 4d Planck scale which is set by the fast increase of the running gauge coupling towards the IR due to its large negative beta function. If the j^{th} contribution to Eq. (6.34) arises from such gaugino condensate then it is

$$a_j = \frac{2\pi}{N} \tag{6.38}$$

for a gauge group SU(N) on the D7-brane stack.

Imagine now that a sufficiently generic SUSY breaking $(0,3)$-form G-flux has been introduces by an appropriate setup of stacks of D7-branes and O3-orientifold planes, which fixes all the non-Kähler moduli. Then the superpotential, the Kähler potential and the resulting scalar potential for the volume modulus $T = X + iY$ read as

$$\begin{aligned} W(T) &= W_0\ , \quad K = -3\ln(T+\bar{T}) \\ V(T) = 0 & \end{aligned} \tag{6.39}$$

where W_0 denotes the flux induced superpotential of the non-Kähler moduli evaluated at their minimum. Now add the superpotential of a single large gauge group undergoing gaugino condensation on one of D7-brane stacks. The superpotential becomes

$$W(T) = W_0 + Ae^{-aT} \tag{6.40}$$

which implies that now a supersymmetric minimum of the scalar potential exists at

$$\begin{aligned} D_T W|_{T=T_{\rm cr}} &= 0 \\ \Leftrightarrow W_0 &= -Ae^{-aX_{\rm cr}}\left(1+\frac{2}{3}\,aX_{\rm cr}\right)\ ,\ Y_{\rm cr}=0\ . \end{aligned} \tag{6.41}$$

Thus the last remaining modulus now has been fixed at a supersymmetric AdS minimum which reoccurs periodically in Y with periodicity $2\pi a^{-1}$. In order to remove the negative cosmological constant of this minimum

$$V_{\rm AdS} = V(T_{\rm cr}) = e^K(-3|W(T_{\rm cr})|^2) = -\frac{A^2a^2e^{-2aX_{\rm cr}}}{6X_{\rm cr}} \tag{6.42}$$

one has to add an additional uplifting potential. This can come either from adding an anti-D3-brane [15] to the setup or from the presence of Fayet-Iliopoulos terms on the D7-branes [104]. The uplifting potential which for both cases can be described by the general form

$$\delta V = \frac{D}{X^\alpha} \tag{6.43}$$

($\alpha = 2$ for anti-D3-branes and $\alpha = 3$ for the FI D-term potentials from D7-branes) generically breaks supersymmetry and uplifts the former AdS minimum to a new de Sitter (dS) minimum at $X_{\rm dS} \approx X_{\rm cr}$. The positive cosmological constant of this dS-minimum can be fine-tuned to become very small by virtue of the large number of flux quanta at hand [15] (which in turn leads to the possibility to tune W_0 in very small steps). The dS-minimum is now separated from the decompactifying Minkowski minimum at $X \to \infty$ by a positive energy barrier with height

$$V_{\rm barrier} \approx \frac{D}{X_{\rm max}^\alpha} \approx -V_{\rm AdS}\ . \tag{6.44}$$

A numerical example for $\alpha = 2$ (the KKLT case [15]) given in Fig. 6.1 indicates this situation.

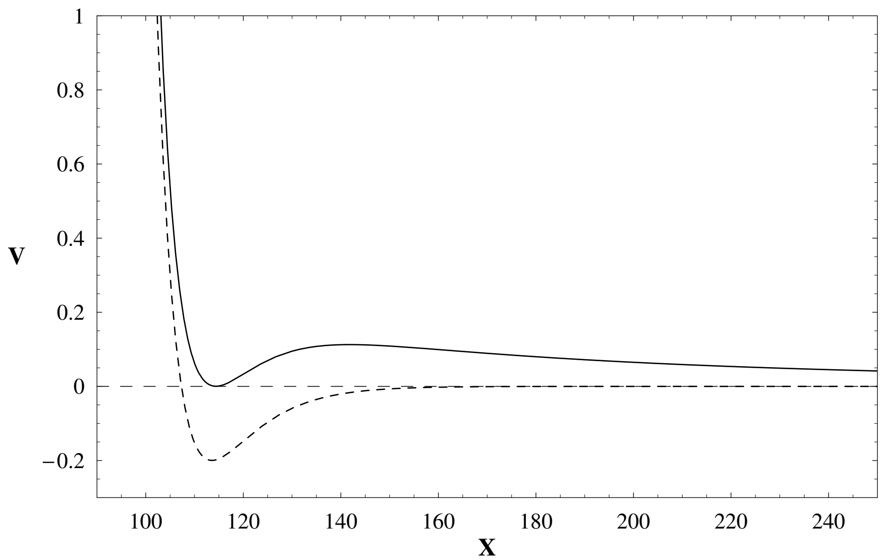

Figure 6.1: Scalar potential for the real part X of the T-modulus with fluxes only (dashed line) and in presence of an anti-D3-brane (solid line).

6.3 Basics of inflation

Within this procedure of moduli stabilization in the type IIB superstring one may now attempt to construct a model of inflation. Several suggestions have already been made here which rely either on the flatness of the potential for the position of a D3-brane viewed as a scalar field in 4d or on the axionic direction Y of the T-modulus which can be tuned to be sufficiently flat. Before entering a short review of these models one should firstly state the necessary basics about inflation in general.

Inflation describes a de Sitter stage of the early universe characterized by a phase of exponential expansion. Consider a Friedmann universe described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right\} \tag{6.45}$$

with the curvature of its space-like hypersurface given by k/R^2. This form of the metric is predicted once one adopts the Cosmological Principle. This principle states that the universe looks the same independently from where you look or into which direction - which on the largest scales has been verified to a very good accuracy by astronomical observations. This means that the 3d spatial hypersurfaces of our 4d space-time have to be maximally symmetric spaces which, in turn, dictates the form of the metric above. Its dynamics is fully controlled by the scale factor $R(t)$ which, in turn, is governed by the Friedmann equation (the Einstein equations for the scale factor R(t) as the relevant physical degree of freedom of the metric)

$$H^2 = \frac{\dot{R}^2}{R^2} = \frac{1}{3}\rho - \frac{k}{R^2} \quad . \tag{6.46}$$

Here H denotes the expansion rate of the universe (the Hubble parameter, its present day value is about $72 \pm 6\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [105]) and ρ its energy density.

An inflationary de Sitter stage of expansion is characterized by a situation where the energy density is dominated by the contribution from a positive cosmological constant $\rho \approx \Lambda$. Then the scale factor grows exponentially

$$R(t) \propto e^{Ht}$$

with a fixed Hubble parameter $H = \sqrt{\Lambda/3} = \text{const.}$

The necessity of such an inflationary phase in the very early universe arises from long-standing problems of the standard hot big bang cosmology. One of the most prominent contradictions, the so-called 'horizon' problem, provides a good example: the microwave picture of the deep sky as seen by the WMAP satellite shows the 2.73K microwave radiation that originated when light decoupled from matter about 300.000 years after the big bang. At this time a causally connected and thus thermalized region had a diameter of roughly $300.000\,ly \approx 100\,kpc$. On the scale of our today's horizon[3] of size $H^{-1} \approx 3\,Gpc$ these causally connected regions from the time of decoupling thus make up regions of about $1^\circ \approx 0.01\,\mathrm{rad}$ in angular diameter. Compared with the full sphere of $4\pi\,\mathrm{rad}$ angular size this shows that our sky today consists of approximately 10^5 causally disconnected regions. If true this would render the observed large-scale homogeneity and isotropy of the universe extremely improbable. Now imagine an early inflationary phase which increases the scale factor by at least $50\ldots60$ e-foldings. This would expand even a Planck length sized patch of FRW space-time so enormously that after the end of inflation our whole observed universe can have easily grown out of this single patch.

The inflationary scenario now can be described by substituting a true cosmological constant with a situation, where the energy density of the universe at some very early stage is dominated by the potential energy of a scalar field. During this phase the scalar field, called inflaton, rolls sufficiently slowly so that its potential energy dominates its kinetic one. To be specific, the dynamics of a single real scalar field with action

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\}$$

[3] The horizon denotes the largest distance into the past from which any (light-like) signal can have reached us until now and is $\sim H^{-1}$.

in an FRW background is governed by an equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ . \tag{6.47}$$

The energy density of the scalar field is then given by the time-time component of the energy-momentum tensor

$$\begin{aligned} T_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - \mathcal{L}g_{\mu\nu} \\ \Leftarrow \rho &= T_{00} = V(\phi) \end{aligned} \tag{6.48}$$

which implies that the Friedmann equation becomes

$$H^2 = \frac{1}{3}V(\phi) - \frac{k}{R^2} \ . \tag{6.49}$$

Slow-roll of the scalar field now is defined by the conditions

$$\ddot{\phi} \ll 3H\dot{\phi} \ , \ \ddot{\phi} \ll V'(\phi) \tag{6.50}$$

which leads to the slow-roll equation of motion

$$3H\dot{\phi} = -V'(\phi) \ . \tag{6.51}$$

Upon differentiation of this equation and use of the Friedmann equation the conditions Eq. (6.50) can be shown to be equivalent to the standard inflationary slow-roll conditions [106, 107]

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \ , \ \eta = \frac{V''}{V} \ll 1 \tag{6.52}$$

which in addition define the so-called slow-roll parameters of scalar field inflation, ϵ and η. Assume now that one is initially close to a stationary point of the scalar potential where $\epsilon = 0$. The closer to zero the values of the slow-roll parameters are in this case at the begin of inflation, the longer the inflationary stage will last, which is to say, the more e-foldings are produced referring to the ratio

$$\frac{R(t)}{R_0} = e^{H(t-t_0)} = e^N \ , \ N: \text{ Number of e-foldings} \ .$$

Inflation ends if the scalar has rolled to a region of the potential where at least one of the slow-roll parameters becomes $\gtrsim 1$ since then the energy density is no longer dominated by the potential energy of the scalar field.

This scenario of slow-roll inflation has another important feature in that it can generate the nearly scale invariant adiabatic power spectrum of primordial density fluctuations seen in the microwave sky by WMAP. This property has its origin in the fact that a scalar field in a de Sitter background spacetimes generates field perturbations with a scale invariant power spectrum [108]. The magnitude of the field fluctuations generated at each comoving scale k is given by

$$\delta\phi_k = \sqrt{\langle\Delta\phi^2\rangle} = \frac{H}{2\pi} \quad . \tag{6.53}$$

As shown, e.g., in [109] linear perturbation theory of the Einstein equations leads to a curvature perturbation $\mathcal{R}$ generated by the scalar field fluctuations

$$\mathcal{R}_k = \left(\frac{H}{\dot{\phi}}\,\delta\phi\right)_k \quad . \tag{6.54}$$

Imagine now a perturbation of comoving scale k. Its comoving (i.e. measured in the 'comoving' coordinates r, θ, ϕ of the FRW metric above) wavelength λ corresponds to a physical wavelength

$$\lambda_{\text{phys.}} = \lambda \cdot R(t) \tag{6.55}$$

which grows exponentially fast with the scale factor $R(t)$ during inflation. However the horizon, i.e. the causally connected volume of space-time, which is $\sim H^{-1}$ is constant during inflation since H is constant there. If this physical wavelength becomes larger than the horizon

$$\lambda_{\text{phys.}} = H^{-1} \;\Leftrightarrow\; k = R \cdot H \tag{6.56}$$

(called 'it leaves the horizon') one finds [107, 109] that a perturbation of this comoving scale does no longer fluctuate. Instead after leaving the horizon it behaves as a classical field distribution which now only stretches further by the expansion of space-time. After inflation ends the Hubble parameter decreases like

$$H \sim t^{-1} \tag{6.57}$$

which implies that the horizon now grows linearly with time. Therefore a curvature perturbation $\mathcal{R}_k$ that left the horizon during inflation will re-enter it some time after inflation has ended. This happens because its physical wavelength grows now with $R(t) \sim t^{\alpha}$ and $\alpha < 1$ in a matter- or radiation dominated FRW universe. After re-entry this perturbation acts as a seed for density perturbations since a perturbation of the curvature background implies a perturbation of the gravitational field there. It can now be shown [109] that the density perturbations $(\delta\rho/\rho)_{k=RH}$ generated by the inflationary curvature perturbations at horizon crossing are given by

$$\left(\frac{\delta\rho}{\rho}\right)_{k=RH} = \sqrt{\frac{4}{25}\mathcal{P}_{\mathcal{R}}(k)} = \left.\sqrt{\frac{4}{25}\left(\frac{H}{\dot{\phi}}\,\delta\phi\right)^2}\right|_{k=RH}$$
$$= \left.\sqrt{\frac{1}{50\pi^2}\frac{H^4(\phi)}{\mathcal{L}_{\rm kin}(\phi)}}\right|_{k=RH}$$
$$= \left.\sqrt{\frac{1}{150\pi^2}\frac{V}{\epsilon}}\right|_{k=RH} \tag{6.58}$$

Here $\mathcal{P}_g(k)$ denotes the power spectrum of the fluctuations of a quantity g at the comoving scale k and in the last line the slow-roll parameters Eq. (6.52) have been used.

Now the largest observable fluctuation structures in the microwave sky which are accessible to both the COBE and WMAP satellites (about 90° angular diameter) correspond to the comoving scale of about $k_0 \sim 10^3\,Mpc$. At this scale k_0 that magnitude of the primordial density fluctuations is measured to be $(\delta\rho/\rho)_{k_0} = 1.91 \cdot 10^{-5}$ by both COBE and WMAP [105]. One can show [109] that this scale k_0 left the horizon during inflation about $55\ldots60$ e-foldings before the inflationary phase ended. This COBE normalization constraint on the magnitude of density perturbation then any model of inflation must fulfill if one wants it to generate the primordial density fluctuations.

Finally there is the possibility that the inflationary density fluctuations generated after inflation (unlike the scalar field perturbation in itself) are not perfectly scale-invariant. One defines [109] a so-called spectral index

$$n_s = 1 + \frac{d\ln\mathcal{P}_{\mathcal{R}}(k)}{d\ln k} \tag{6.59}$$

which is exactly unity if the power spectrum $\mathcal{P}_{\mathcal{R}}$ of the curvature perturbations does not depend on k. However, in slow-roll inflation H changes slowly during the inflationary phase implying that here the Hubble parameter and thus $\mathcal{P}_{\mathcal{R}}$ is a function of k. In slow-roll inflation the spectral index can be calculated [109] to leading order in ϵ and η

$$n_s = 1 + 2\eta - 6\epsilon \;\; . \tag{6.60}$$

Then the spectral index may deviate from unity and the WMAP probe data has placed bounds on its value $n_s = 0.97 \pm 0.03$ [105].

6.4 Moduli stabilization and inflation

One way to construct inflation in the KKLT approach of moduli stabilization has been to use the fact that the D3-branes in this setup are BPS states which preserve the last

$\mathcal{N} = 1$ supersymmetry. Thus their position in the Calabi-Yau, viewed as a scalar field in 4d, represents a flat direction which renders this scalar field to be a candidate for the inflaton [50]. The geometric picture of this idea is sketched in Fig. 6.2. The geometry of

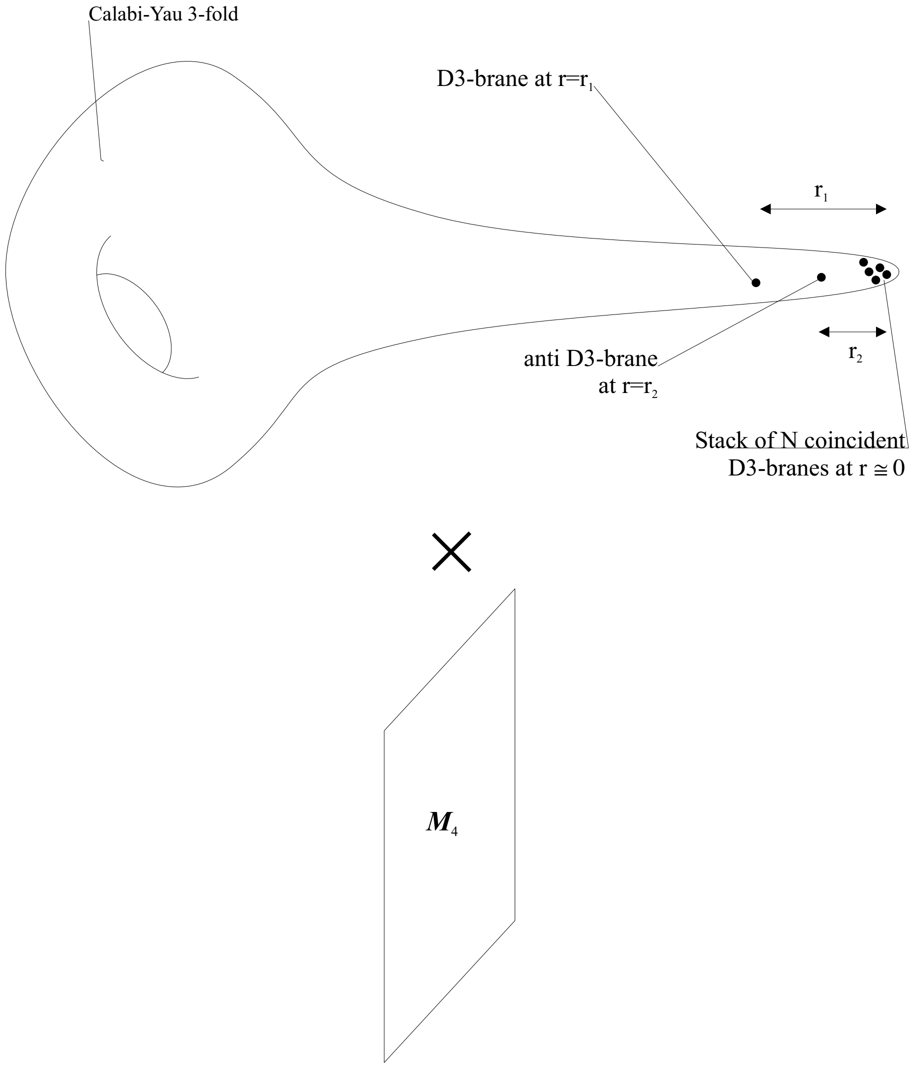

Figure 6.2: The approximate AdS_5-throat between the stack of coincident D3-branes providing the source of warping and the Calabi-Yau forming the UV-'brane' in RS I language.

this setup inside the throat is described approximately by $AdS_5 \times \mathcal{M}_5$, i.e., the metric can be written as

$$\begin{aligned} ds^2_{10} &= h^{-1/2}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + h^{1/2}\left(dr^2 + h^{-1/2}\tilde{\mathcal{G}}_{ij}dy^i dy^j\right) \\ h(r) &= \frac{R^4}{r^4} \;\;,\;\; R^4 = 4\pi a g_S \alpha'^2 N = \frac{1}{2\pi^2}\,a\,\frac{N}{T_3} \;\;. \end{aligned} \tag{6.61}$$

Here $h(r)$ denotes the AdS_5 warp factor which is generated by N coincident D3-branes carrying 5-form charge $\mu_3 = T_3$ (they are BPS states) at the end of the throat at $r \approx 0$ and

$$T_3 = \frac{1}{(2\pi)^3 g_S \alpha'^2} \tag{6.62}$$

denotes the D3-brane tension. $\tilde{\mathcal{G}}_{ij}$ with $i, j = 5 \ldots 9$ denotes the metric of the compact 5-manifold which is an Einstein manifold [50].

Imagine now an additional D3-brane placed at radial position r_1 close the end of the throat. From Eq. (6.5) the action of a D3-brane with its BPS property implying $T_3 = +\mu_3$ is

$$S_{\mathrm{D3}} = -T_3 \left\{ \int d^4\xi \sqrt{-\gamma} - \int C_{(4)} \right\} \quad . \tag{6.63}$$

Here γ denotes the induced metric on the worldsheet. The fact that this object is BPS implies that $S_{\mathrm{D3}} = 0$, i.e., there is no potential for a static D3-brane.

Consider now fluctuations of the D3-brane in the AdS_5 r-direction. These fluctuations induce analogous fluctuations of the world sheet metric of the brane

$$\gamma \to \gamma + \delta\gamma = \gamma(1 - \gamma^{\mu\nu}\delta\gamma_{\mu\nu}) = \gamma \left(1 - \gamma^{\mu\nu} g_{rr} \frac{\partial r_1}{\partial \xi^\mu} \frac{\partial r_1}{\partial \xi^\nu}\right) = g_4 \left(1 - h\, \tilde{g}^{\mu\nu} \frac{\partial r_1}{\partial \xi^\mu} \frac{\partial r_1}{\partial \xi^\nu}\right) \tag{6.64}$$

where the world sheet has been identified with the 4d space-time of our world $\gamma_{\mu\nu} = g_{\mu\nu}$. This is necessary if the Standard Model lives on a stack of $n << N$ D3-branes placed at $r \approx r_1$ instead of the one model D3-brane considered here. Insert this expansion now into the D3-action and expand up to two-derivative order. Denoting contractions with $\tilde{g}^{\mu\nu}$ as $\partial^{\tilde{\mu}}$ the result is

$$S_{\mathrm{D3}} = \frac{T_3}{2} \int d^4\xi \sqrt{-\tilde{g}_4} \partial_\mu r_1 \partial^{\tilde{\mu}} r_1 - \underbrace{T_3 \left\{ \int d^4\xi \sqrt{-\tilde{g}_4} h^{-1} - \int C_{(4)} \right\}}_{=0} \tag{6.65}$$

Here one uses the fact that the 4-form potential generated by the stack of N coincident D3-branes at the end of the throat is linked to the warp factor via

$$\langle C_{(4)} \rangle = h^{-1} \sqrt{-\tilde{g}_4}\, d^4\xi \tag{6.66}$$

which leads to the cancellation in the last term of the action. Thus, a single D3-brane moves inside the throat without any potential as expected from its BPS property above.

Next, add a single anti-D3-brane at $r_2 \ll r_1$ even closer to the end of the throat. According to [50] the background metric it will feel is changed due to the previously added D3-brane to have a warp factor

$$\begin{aligned} h(r) &= \frac{R^4}{r^4} + \delta h(r) = \frac{R^4}{r^4} \left(1 + \frac{1}{N} \frac{r^4}{r_1^4}\right) \\ &\text{with: } \delta h(r) = \frac{1}{N} \frac{R^4}{(r - r_1)^4} \approx \frac{1}{N} \frac{R^4}{r_1^4} \ , \ r \ll r_1 \quad . \end{aligned} \tag{6.67}$$

Now by the same method of expanding around the unperturbed world sheet metric the action of the anti-D3-brane in this changed background becomes

$$\begin{aligned} S_{\overline{\text{D3}}} &= \frac{T_3}{2}\int d^4\xi\sqrt{-\tilde{g}_4}\partial_\mu r_2\partial^{\tilde{\mu}}r_2 - T_3\left\{\int d^4\xi\sqrt{-\tilde{g}_4}h^{-1} + \int C_{(4)}\right\} \\ &= \frac{T_3}{2}\int d^4\xi\sqrt{-\tilde{g}_4}\left\{\partial_\mu r_2\partial^{\tilde{\mu}}r_2 - \frac{2r_2^4}{R^4}\left(1-\frac{1}{N}\frac{r_2^4}{r_1^4}\right)\right\} \quad . \end{aligned} \tag{6.68}$$

Call now the position r_1 of the formerly added D3-brane the inflaton ϕ. Then the second piece of the anti-D3-brane action describes a potential for the inflaton

$$V(\phi) = \frac{2r_2^4}{R^4}\left(1-\frac{1}{N}\frac{r_2^4}{\phi^4}\right) \quad . \tag{6.69}$$

This potential is extremely flat for large $\phi \gg r_2$, i.e., $\epsilon, \eta \ll 1$ and thus can provide for a sufficiently long inflationary phase generating enough e-foldings. Inflation in this model ends when either the potential for ϕ rolling to smaller values becomes too steep and thus $\epsilon > 1$, or the D3- and the anti-D3-brane collide. The latter process of anti brane-brane collision also provides a mechanism of reheating. Reheating denotes the process of transferring the potential energy of the inflaton into that of thermalized matter and radiation which then initiate the ordinary hot big bang.

If this model of inflation is now to be merged with the KKLT proposal of moduli stabilization, it has to be embedded into an effective supergravity description. According to [50] one then runs into the usual η-problem of inflation in supergravity: the argument given in [50] assumes the above inflaton field ϕ to be a part of a chiral superfield $\mathbf{\Phi}$. Then the Kähler potential is assumed to be minimally extended as

$$K = K_{\text{KKLT}} + \overline{\mathbf{\Phi}}\mathbf{\Phi} \quad . \tag{6.70}$$

The scalar potential induced now by this Kähler potential, however, reads

$$V(T,\phi) = V_{\text{KKLT}}(T)\left(1+\bar{\phi}\phi+\ldots\right)$$

which implies the $\eta = V_{\bar{\phi}\phi}/V \sim 1$ spoiling slow-roll inflation. However, it is not clear in which way this extension of the Kähler potential is minimal or why precisely this form is required. Possible ways around this problem have been suggested by searching for cancellations among the $\bar{\phi}\phi$-piece of the scalar potential and higher-order corrections [50] or invoking additional symmetries (such as a shift symmetry) [110].

Another way of constructing inflation within the KKLT setup is to use the axionic direction Y of the T-modulus. This idea [51] can be realized by just extending the original KKLT ansatz to a superpotential of the racetrack type consisting of the flux term and two gaugino condensate contributions

$$W(T) = W_0 + Ae^{-aT} + Be^{-bT} \tag{6.71}$$

and the Kähler potential as in the KKLT setup $K = -3\ln(T+\bar{T})$. The resulting scalar potential

$$V(X,Y) = \frac{e^{-2(a+b)X}}{6X^2}\Big\{aA^2(3+aX)e^{2bX} + bB^2(3+bX)e^{2aX} + 3aAW_0\,e^{(a+2b)X}\cos(aY) + 3bBW_0\,e^{(2a+b)X}\cos(bY) + AB\,[3(a+b)+2abX]\,e^{(a+b)X}\cos[(a-b)Y]\Big\} \tag{6.72}$$

amended by the usual uplift Eq. (6.43) for an anti-D3-brane

$$\delta V = \frac{D}{X^2}$$

for the choice of parameters

$$W_0 = -4\cdot 10^{-5}\ ,\ A = \frac{1}{50}\ ,\ B = -\frac{35}{1000}\ ,\ a = \frac{2\pi}{100}\ ,\ b = \frac{2\pi}{90}\ ,\ D = 4.14668\cdot 10^{-12}$$

leads to a potential shown in Fig. 6.3. The Y-direction of the T-modulus possesses close

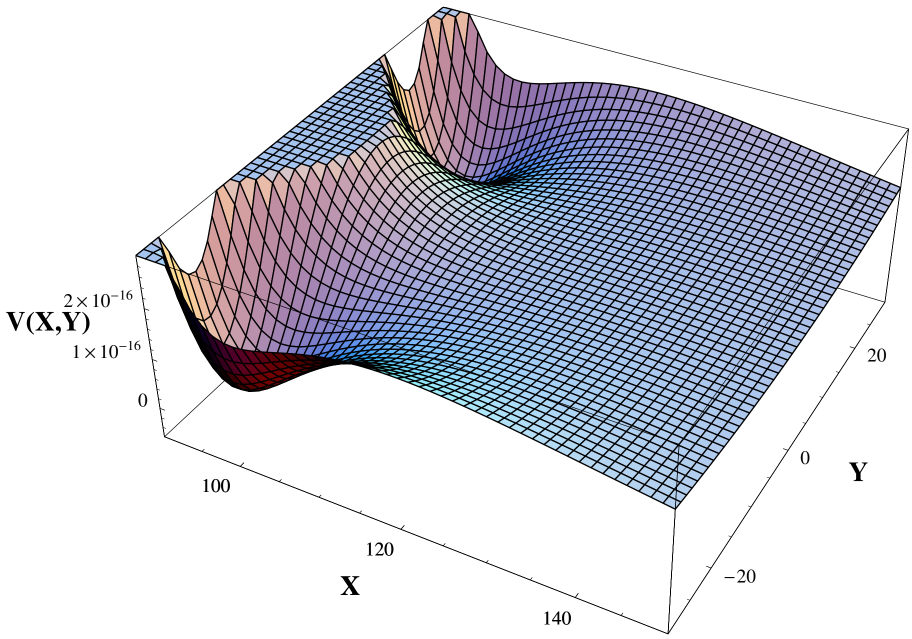

Figure 6.3: The scalar potential of the T-modulus for the above choice of parameters. The uplift parameter D has been tuned to lift the two degenerate minima at about $X = 96.130$ and $Y = \pm 22.146$ to $V \approx 0$. Clearly visible is the saddle point that provide in its vicinity a very flat direction for the axionic direction Y.

to the saddle point at $X_{\text{saddle}} = 123.22$, $Y_{\text{saddle}} = 0$ connecting the two degenerate minima a very flat potential with slow-roll parameters evaluated at the saddle point $\epsilon_Y = 0$ and

$\eta_Y = -0.0067$. Depending on the initial parameters slow-roll inflation can take place here with $\gtrsim 100$ e-foldings.

This discussion provides one with some background for the inflationary model presented in the following chapter. As seen above both models existing in the literature have some problems. For the D3-brane model [50] the embedding into a supersymmetric setup is unclear. Both this model and the racetrack model of [51] have to introduce an anti-D3-brane. This object is non-BPS and the effective F- or D-term description of this source of SUSY-breaking is unclear questioning the controllability of its effects. As discussed in the next Chapter one can solve this problem by taking into account higher-dimension operators induced by higher-order α'-corrections of string theory. The introduction of anti-D3-branes then is no longer necessary.

Chapter 7

Application of α'-corrections and inflation

7.1 α'-corrections

Concerning KKLT inspired setups like those described above one may now ask which of the ingredients used there is least controlled with respect to the constraints of perturbativity and negligible backreactions. Clearly, such a question arises with the use of anti-D3-branes as uplifts for given volume-stabilizing AdS minima. The presence of either D3-branes or anti-D3-branes by themselves does not pose a problem. Each kind viewed for itself is a BPS state that preserves half of the original $\mathcal{N} = 8$ supersymmetries in 4d ($\mathcal{N} = 2$ in 10d), which, in turn, can be arranged to contain the 2 supersymmetries preserved by the Calabi-Yau compactification. However, an anti-D3-brane in the presence of a compact geometry with D3-branes is non-BPS with respect to the supersymmetries preserved by the BPS condition of the D3-branes. Thus, it breaks SUSY, and it is not clear whether this SUSY breaking is explicit or has a description in terms of F-term or D-term breaking. If anti-D3-branes break SUSY explicitly, the use of the supergravity approximation to calculate the effect on the scalar potential may be questionable.

In view of these difficulties it is appealing that there are other possibilities to provide uplifting effects by means of perturbative α'-corrections [54] in the type IIB superstring. KKLT have argued that these higher-order corrections in the string tension are not relevant in the large volume limit [15]. The non-perturbative effects invoked by KKLT vanish exponentially fast in this limit. In contrast, the perturbative corrections usually depend on a power of the volume. This motivates the discussion of these effects as an alternative to anti-D3-branes.

Higher-order α'-corrections which usually lift the no-scale structure of the Kähler potential of the volume modulus (and generate 1-loop corrections to the gauge kinetic functions) are not known in general. However, there is one known perturbative correction [54] given by a higher-derivative curvature interaction on Calabi-Yau threefolds of non-vanishing Euler number χ. Its relevant bosonic part (cf. Eq. (6.1)) is given as

$$S_{\rm IIB} \;=\; \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{\rm s}}\; e^{-2\phi}\left[R_{\rm s}+4\,(\partial\phi)^2+\alpha'^3\,\frac{\zeta(3)}{3\cdot 2^{11}}\,J_0\right]\quad . \tag{7.1}$$

Here J_0 denotes the higher-derivative interaction [54]

$$J_0=\left(t^{M_1N_1\cdots M_4N_4}t_{M_1'N_1'\cdots M_4'N_4'}+\frac{1}{8}\,\epsilon^{ABM_1N_1\cdots M_4N_4}\epsilon_{ABM_1'N_1'\cdots M_4'N_4'}\right)R^{M_1'N_1'}{}_{M_1N_1}\cdots R^{M_4'N_4'}{}_{M_4N_4}$$

which after Calabi-Yau compactification to 4d yields a correction to the Kähler potential of the volume modulus T [54]

$$\begin{aligned} K \;&=\; -2\cdot\ln\left(\mathcal{V}+\frac{1}{2}\,\hat{\xi}\right)\quad,\ \hat{\xi}=\xi e^{-3\phi/2}\ \ ,\ \xi=-\frac{1}{2}\,\zeta(3)\chi \\ &=\; \underbrace{-3\cdot\ln\left(T+\bar{T}\right)}_{K^{(0)}}-2\cdot\ln\left(1+\frac{\hat{\xi}}{2(2\;{\rm Re}\;T)^{3/2}}\right)\quad . \end{aligned} \tag{7.2}$$

Here the volume modulus T is related to the Calabi-Yau volume $\mathcal{V}$ as $\mathcal{V}=(T+\bar{T})^{3/2}$ (see, e.g., [111])[1]. χ denotes the Euler number of the Calabi-Yau under consideration which can be of both signs and in its absolute value can be at least as large as 2592 [112]. From the general expression for the scalar potential in 4d $\mathcal{N}=1$ supergravity Eq. (6.32) the potential for the T-modulus is

$$V(T)=e^{K}\left(K^{T\bar{T}}D_TWD_{\bar{T}}\bar{W}-3\,|W|^2\right)\quad . \tag{7.3}$$

This leads to a correction to the scalar potential of T which to $\mathcal{O}(\alpha'^3)$ reads [54]

$$\delta V=-\frac{\hat{\xi}}{(2\;{\rm Re}\;T)^{3/2}}\;V_{\rm tree}+\frac{3}{8}\;e^{K^{(0)}}\frac{\hat{\xi}}{(2\;{\rm Re}\;T)^{3/2}}\;\left|W+(\tau-\bar{\tau})\;\tilde{D}_\tau W\right|^2 \tag{7.4}$$

where $\tilde{D}_\tau W=\partial_\tau W+W\partial_\tau K^{(0)}$. $V_{\rm tree}$ denotes the full scalar potential for the volume modulus T except the effects of the α'-correction under discussion.

This correction, which breaks the no-scale structure of the Kähler potential of the volume modulus, can be used as a replacement for the anti-D3-brane or D-terms on D7-branes to provide the uplift necessary for realizing the KKLT mechanism. Combining the KKLT ansatz for the superpotential Eq. (6.40) with the α'-correction is sufficient to realize de Sitter vacua with all the moduli stabilized [52, 53]. One can show now that a combination of the mechanism of uplifting by α'-corrections with the racetrack superpotential Eq. (6.71) generates dS-minima with full moduli stabilization. Simultaneously, the same potential contains regions where T-modulus inflation with roll-off into the desired dS-minima is realized. There is no η problem in this setup because the leading order Kähler potential of the volume modulus is of the no-scale type.

[1] $\mathcal{V}$ is defined here in the Einstein frame [54] where also the relation with T holds [111].

7.2 T-modulus inflation in the simplest KKLT setup

Before analyzing the setup sketched at the end of the last Section, one should clarify why the original KKLT setup with just the superpotential Eq. (6.40) and one uplifting correction δV does not allow T-modulus inflation. For this purpose, note that the types of uplift considered so far can be written as in Eq. (6.43)

$$\delta V = \frac{D}{X^\alpha} \quad . \tag{7.5}$$

Strictly speaking, the above α'-correction behaves as a mixture of additive and multiplicative corrections. However, from the general form of the potential Fig. 6.1 it is clear that the above α'-correction in the vicinity of the maximum can be written locally in the same additive form

$$\delta V = \frac{D}{X^{3/2}} \ , \quad D = \frac{\hat{\xi}}{2\sqrt{2}} \left(-V_{\text{tree}} + \frac{3}{8}\, e^{K^{(0)}} \left| W + (\tau - \bar{\tau})\ \tilde{D}_\tau W \right|^2 \right) \Bigg|_{T=T_{\max}} \quad . \tag{7.6}$$

Thus, one may consider the following general setup: take the superpotential Eq. (6.40) to fix the T-modulus after the flux part W_0 has fixed all the other non-Kähler moduli. Add one uplifting term Eq. (6.43) with $\alpha > 0$ being general. Such a setup generically generates a maximum in the X-direction separating the dS-minimum from infinity. Since this maximum simultaneously forms a minimum in the Y-direction, one has the situation that inflation would have to start from a saddle point with direction towards the dS-minimum. For this purpose, two ingredients are necessary: firstly, a definition of the slow-roll parameters for a scalar field with a non-canonically normalized kinetic term. Secondly, an analysis of the scalar potential's stationary points with respect to whether slow-roll can be satisfied on the saddle or not.

Start with a general non-canonically normalized action for a set of real scalar fields

$$S_\phi = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2}\, G_{ij} g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j - V(\phi) \right\} \quad . \tag{7.7}$$

Here G_{ij} denotes the metric in the target space of scalar fields. With

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \phi^k} &= \sqrt{-g} \left(-\frac{\partial V}{\phi^k} + \frac{1}{2}\, \frac{\partial G_{ij}}{\partial \phi^k}\, g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j \right) \\ \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^k)} \right) &= G_{jk} \sqrt{-g}\, \nabla_g^2 \phi^j \end{aligned}$$

and $\nabla_g^2 \phi^j = \ddot{\phi}^j + 3H\dot{\phi}^j$ for a sufficiently homogeneous scalar field, the equations of motion for non-canonically normalized scalar fields [113–116] read

$$\ddot{\phi}^l + 3H\dot{\phi}^l + \Gamma^l_{ij} \dot{\phi}^i \dot{\phi}^j + G^{lk}\, \frac{\partial V}{\partial \phi^k} = 0 \ , \quad \Gamma^l_{ij} = -\frac{1}{2}\, G^{lk} \frac{\partial G^{ij}}{\partial \phi^k} \quad . \tag{7.8}$$

For the T-modulus this implies

$$G_{T\bar{T}} = K_{T\bar{T}} = \frac{3}{4X^2} \;\Leftarrow\; \mathcal{L}_{\rm kin} = \frac{3}{4X^2}\left(\partial_\mu X\partial^\mu X + \partial_\mu Y\partial^\mu Y\right) \tag{7.9}$$

and thus the equations of motion become

$$\begin{aligned} \ddot{X} + 3H\dot{X} + \frac{1}{X}\dot{X}^2 + \frac{2}{3}X^2\frac{\partial V}{\partial X} &= 0 \\ \ddot{Y} + 3H\dot{Y} + \frac{1}{X}\dot{Y}^2 + \frac{2}{3}X^2\frac{\partial V}{\partial Y} &= 0 \ . \end{aligned} \tag{7.10}$$

Now recall that the dS-minimum of the KKLT setup and the maximum separating it from infinity are close to each other compared to their positions $X = \mathcal{O}(100)$. Thus, the normalization of the kinetic term of T in Eq. (7.9) changes slowly during a possible slow-roll phase. Assume that T starts on the maximum at $X_{\rm max}$. Then, by field rescaling one can go to a lagrangian with canonically normalized kinetic term

$$\begin{aligned} X &= \lambda\tilde{X}\ ,\ Y = \lambda\tilde{Y} \text{ with: } \lambda = \sqrt{\frac{2}{3}X_{\rm max}^2} \\ \mathcal{L}_{\rm kin} &= \frac{1}{2}\left(\partial_\mu\tilde{X}\partial^\mu\tilde{X} + \partial_\mu\tilde{Y}\partial^\mu\tilde{Y}\right)\ . \end{aligned} \tag{7.11}$$

From the potential rewritten in terms of the rescaled fields $V(\tilde{X},\tilde{Y})$ the slow-roll parameters of, e.g., $\tilde{X}$ are

$$\epsilon_{\tilde{X}} = \frac{1}{2}\left(\frac{1}{V}\frac{\partial V}{\partial\tilde{X}}\right)^2\ ,\ \eta_{\tilde{X}} = \frac{1}{V}\frac{\partial^2 V}{\partial\tilde{X}^2}\ . \tag{7.12}$$

In terms of the original fields X, Y this becomes

$$\epsilon_X = \lambda^2\frac{1}{2}\left(\frac{1}{V}\frac{\partial V}{\partial X}\right)^2 = \frac{X_{\rm max}^2}{3}\left(\frac{V'}{V}\right)^2\ ,\ \eta_X = \lambda^2\frac{1}{V}\frac{\partial^2 V}{\partial X^2} = \frac{2X_{\rm max}^2}{3}\frac{V''}{V} \tag{7.13}$$

where $'$ denotes differentiation with respect to X.

The next step is to analyze the scalar potential. Including the uplift this follows from Eq. (7.3) to be

$$V(T) = \frac{1}{4X^2}\left\{2aA^2e^{-2aX}\left(1+\frac{1}{3}aX\right) + 2aAW_0e^{-aX}\cos(aY)\right\} + \frac{D}{X^\alpha}\ . \tag{7.14}$$

The extrema of this potential are determined by the conditions

$$\frac{\partial V}{\partial Y} = 0 = -\frac{a^2A}{2X^2}e^{-2aX}W_0\sin(aY) \ \Rightarrow\ Y_{\rm extr} = 0 \ \text{ for: } AW_0 < 0 \tag{7.15}$$

$$\begin{aligned} \frac{\partial V}{\partial X} &= 0 = -aAe^{-2aX}\left\{-\frac{aA}{3X^2} - \frac{a^2A}{3X} - \left(\frac{a}{2X^2}+\frac{1}{X^3}\right)\left(\frac{A}{3}+W_0e^{aX}\right)\right\} - \alpha\frac{D}{X^{\alpha+1}} \\ \Leftrightarrow 0 &= \frac{3\alpha D}{aA}X^{2-\alpha} + \left(3W_0 + Ae^{-aX}\right)\left(1+\frac{aX}{2}\right)e^{-aX} + A\left(aXe^{-aX}\right)^2\left(1+\frac{1}{aX}\right)\ . \end{aligned} \tag{7.16}$$

This implies that all extrema in X are found along the direction $Y = 0$ with replications at $Y = \frac{2\pi n}{a}\ \forall n \in \mathbb{Z}$. The other equation determining the X-values can be simplified using the fact that one works in the regime of large volume

$$aX \gg 1 \ , \ X \gg 1 \ \Rightarrow \ aA \gg \frac{A}{X} \gg \frac{W_0}{X} \ , \ \frac{A}{X}\, e^{-aX} \ll \frac{W_0}{X} \tag{7.17}$$

in order to trust the effective potential which takes into account just the leading order perturbative and non-perturbative corrections. In this region one therefore has

$$\frac{3\alpha D}{aA}\, X^{2-\alpha} + \frac{3}{2}\, W_0 \cdot \lambda + A\lambda^2 = 0 \ , \ \ \lambda = aXe^{-aX} \ . \tag{7.18}$$

Expanding the solutions to this quadratic equation in $X^{2-\alpha}D/W_0^2 \ll 1$ leads to two extrema at

$$\begin{aligned} \frac{aX_{\rm max}e^{-aX_{\rm max}}}{X_{\rm max}^{2-\alpha}} &= -\frac{2\alpha D}{aAW_0}\left(1+\mathcal{O}\Big(X_{\rm max}^{2-\alpha}\frac{D}{W_0^2}\Big)\right) \\ aX_{\rm min}e^{-aX_{\rm min}} &= -\frac{3W_0}{2A}\left(1-\frac{4\alpha D}{3aW_0^2}\,X_{\rm min}^{2-\alpha}+\mathcal{O}\Big(\Big(X_{\rm min}^{2-\alpha}\frac{D}{W_0^2}\Big)^2\Big)\right) \end{aligned} \tag{7.19}$$

as long as $AW_0 < 0$, which a posteriori justifies the use of this condition in extremizing the potential in Y above. The position of the minimum is largely independent on the strength of the uplifting potential D while the maximum in X-direction, the saddle point, moves to infinity for $D \to 0$ and disappears there as expected. One may now calculate

$$\left.\frac{\partial^2 V}{\partial X^2}\right|_{X=X_{\rm max},Y=0} = \frac{a^3AW_0}{2X_{\rm max}^2}e^{-aX_{\rm max}}\left[1+\mathcal{O}\left(\frac{1}{aX_{\rm max}}\right)\right] = -\frac{\alpha\, aD}{X_{\rm max}^{\alpha+1}} \tag{7.20}$$

and calculate from that the slow roll parameters of the saddle

$$\epsilon_{X,\rm saddle} = 0 \ \ , \ \ \eta_{X,\rm saddle} = \frac{2}{3}\,X_{\rm max}^2\ \frac{1}{V}\left.\frac{\partial^2 V}{\partial X^2}\right|_{X=X_{\rm max},Y=0} = -\frac{2}{3}\,\alpha\, aX_{\rm max} \ \ . \tag{7.21}$$

Thus, except for $\alpha \lesssim 0.1$ (for which no known realization exists) or $aX_{\rm max} \lesssim 1$, which violates the large volume and perturbativity assumptions, slow-roll inflation with the T-modulus on the saddle point of this most simple class of KKLT-like setups does not work.

7.3 T-modulus inflation with α'-corrections

The above result forces one to look for minimal extensions of the setup that may lead to saddle points with sufficiently small negative curvature. One example that can be shown to contain saddles of sufficient flatness is given by the following modified setup: the superpotential is given as in Eq. (6.71) by

$$W(T) = W_0 + Ae^{-aT} + Be^{-bT} \ \ . \tag{7.22}$$

Departing from [51] the uplift of the two degenerate AdS-minima present in the corresponding scalar potential will now be provided by the α'-corrected no-scale breaking Kähler potential of Eq. (7.2)

$$K = -3 \cdot \ln\left(T + \bar{T}\right) - 2 \cdot \ln\left(1 + \frac{\hat{\xi}}{2(2\,\mathrm{Re}\,T)^{3/2}}\right) \tag{7.23}$$

inducing the contribution Eq. (7.4) to the scalar potential.

Assume now, that the flux contribution W_0 has stabilized the dilaton τ in a minimum given by $\tilde{D}_\tau W = 0$. Then the resulting scalar potential can be written as

$$V(T) = \left(1 - \frac{\hat{\xi}}{(2\,\mathrm{Re}\,T)^{3/2}}\right) V_{\mathrm{tree}} + \frac{3}{8}\, e^{K^{(0)}} \frac{\hat{\xi}}{(2\,\mathrm{Re}\,T)^{3/2}}\, |W|^2 \tag{7.24}$$

where

$$K^{(0)} = -3\ln(T + \bar{T}) \ . \tag{7.25}$$

V_{tree} denotes the scalar potential induced by the above superpotential. It is given by Eq. (6.72) as

$$\begin{aligned} V_{\mathrm{tree}}(X,Y) \;=\; & \frac{e^{-2(a+b)X}}{6X^2} \Big\{ AB\left[3(a+b) + 2abX\right] e^{(a+b)X} \cos[(a-b)Y] \\ & + aA\left[3\left(A + W_0\, e^{aX}\cos(aY)\right) + aAX\right] e^{2bX} \\ & + bB\left[3\left(B + W_0\, e^{bX}\cos(bY)\right) + bBX\right] e^{2aX} \Big\} \ . \end{aligned} \tag{7.26}$$

Finally, $|W|^2$ reads

$$\begin{aligned} |W|^2 \;=\; & W_0^2 + A^2 e^{-2aX} + B^2 e^{-2bX} + 2AW_0 e^{-aX}\cos(aY) + 2BW_0 e^{-bX}\cos(bY) \\ & + 2ABe^{-(a+b)X}\cos[(a-b)Y] \ . \end{aligned} \tag{7.27}$$

Compared to an anti-D3-brane uplift, the structure of this scalar potential is changed considerably, since, as noted before, the α'-uplift can only be written locally as a purely additive contribution of the type D/X^α. The saddle at $Y = 0$ which connects the two degenerate AdS-minima at $Y_{\mathrm{min}}^{(1)} = -Y_{\mathrm{min}}^{(2)} \neq 0$ of the scalar potential induced by the above superpotential is rather flat and extended in X and Y. Therefore, unlike an anti-D3-brane uplift, the α'-contribution will not just lift the two minima to $V > 0$ while leaving the form of the saddle practically unchanged. The saddle upon α'-uplifting will get deformed and in general one will have three different local minima with the properties

$$X_{\mathrm{min}}^{(1)} = X_{\mathrm{min}}^{(2)} \ , \ Y_{\mathrm{min}}^{(1)} = -Y_{\mathrm{min}}^{(2)} \neq 0 \ ; \ X_{\mathrm{min}}^{(3)} > X_{\mathrm{min}}^{(1)} \ , \ Y_{\mathrm{min}}^{(3)} = 0 \ . \tag{7.28}$$

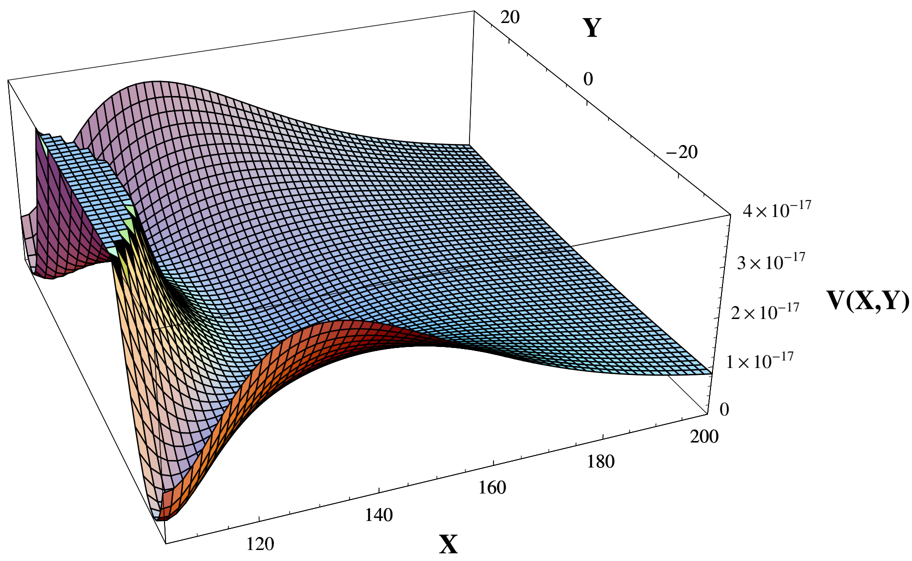

Figure 7.1: The scalar potential of T-modulus with α'-correction for a generic choice of parameters. Clearly visible are the three minima connected by two off-X-axis saddle points.

Two of them, (1) and (2), are connected to the third one via a saddle point. Fig. 7.1 shows this situation for a generic choice of parameters. The two saddle points have the properties

$$X^{(1)}_{\rm saddle} = X^{(2)}_{\rm saddle} = X_{\rm saddle}\ ,\ Y^{(1)}_{\rm saddle} = -Y^{(2)}_{\rm saddle} \neq 0$$

and furthermore

$$X^{(1)}_{\rm min} = X^{(2)}_{\rm min} < X_{\rm saddle} < X^{(3)}_{\rm min}\quad . \tag{7.29}$$

This structure now allows for a new possibility of tuning the scalar potential in order to find sufficiently flat saddle points: since the uplift of the α'-correction scales with a negative power of X, the two degenerate minima (1) and (2) will get more strongly lifted than the saddle points connecting them to minimum (3) at $Y = 0$. This third minimum, in turn, gets even more weakly lifted than the saddle points. Hence, the potential can be tuned in such a way, that the minimum (3) remains approximately Minkowski while the two degenerate minima rise as a function of the uplift parameter $\hat{\xi}$. Therefore, the saddle between minimum (3) and, say, minimum (1) has very small negative curvature shortly before minimum (1) disappears. The mechanism is quite generic for a superpotential consisting of the flux piece and two gaugino condensate contributions with its two degenerate AdS-minima: it depends mainly on the hierarchy of the positions in X of the three minima and the two saddles that arise upon uplifting. Thus, even with further α'-corrections one expects this picture to remain qualitatively the same, though the numerical values will change.

As an example, consider the parameter choice

$$W_0 = -5.55 \cdot 10^{-5} \;,\; A = \frac{1}{50} \;,\; B = -3.37461131 \cdot 10^{-2} \;,\; a = \frac{2\pi}{100} \;,\; b = \frac{2\pi}{91}$$
$$\hat{\xi} = -\frac{1}{2}\zeta(3) e^{-3\phi/2}\chi \;,\; \chi = -4209 \;. \tag{7.30}$$

Note that the 4d gauge coupling on a stack of D3-branes is $\alpha_{D3} = e^{\phi}/2$ [102]. Phenomenologically $\alpha_{\rm GUT} = 1/24$ and thus $e^{-\phi} \sim 12$. This implies that in the weakly coupled string theory $\exp(-3\phi/2) \gtrsim 1$ and for simplicity this quantity has been set to one. Then the desired value of $\hat{\xi}$ implies that one has to choose Calabi-Yau manifolds of large negative Euler number with $\chi = -10^3 \ldots - 10^4$ which, in general, appears to be possible [112]. Otherwise, one may choose $|\chi|$ smaller which will move the above structure of three minima towards smaller X-values.

For the example given above one finds the minimum (3) at approximately

$$X^{(3)}_{\rm min} = 132.398 \;,\; Y^{(3)}_{\rm min} = 0 \tag{7.31}$$

being weakly de Sitter. The other two degenerate minima reside at

$$X^{(1)}_{\rm min} = X^{(2)}_{\rm min} = 116.724 \;,\; Y^{(1)/(2)}_{\rm min} = \pm 19.431 \;. \tag{7.32}$$

The two saddle points one finds very close by at

$$X_{\rm saddle} = X^{(1)}_{\rm saddle} = X^{(2)}_{\rm saddle} = 116.728 \;,\; Y^{(1)/(2)}_{\rm saddle} = \pm 19.428 \;. \tag{7.33}$$

As a consistency check one may calculate the ratio

$$\frac{\hat{\xi}}{(2X)^{3/2}} \tag{7.34}$$

at the three minima. This ratio is the expansion parameter used in deriving Eq. (7.24) from Eq. (7.23). One finds $\hat{\xi}/(2X)^{3/2} \approx 0.5 < 1$ for the minimum (3) and $\hat{\xi}/(2X)^{3/2} \approx 0.7 < 1$ for the other two degenerate minima (1) and (2). This implies, that the region around the three minima still resides in the perturbative regime of the effective potential.

One may now calculate the Hesse matrix of curvatures

$$\mathcal{H} = \begin{pmatrix} \frac{\partial^2 V}{\partial X^2} & \frac{\partial^2 V}{\partial X \partial Y} \\ \frac{\partial^2 V}{\partial X \partial Y} & \frac{\partial^2 V}{\partial Y^2} \end{pmatrix} \tag{7.35}$$

diagonalize it and calculate from it the matrix of slow-parameters on one of the saddle points to yield

$$\mathcal{H}_\eta = \frac{2}{3} X^2_{\rm saddle} \mathcal{H}^{\rm diag} \approx \begin{pmatrix} 1222.83 & 0 \\ 0 & -0.069 \end{pmatrix} \;. \tag{7.36}$$

Therefore, on these two saddle points, slow-roll inflation can take place if the T-modulus starts from the saddle with initial conditions fine-tuned to some amount. For example, for initial conditions given by

$$X_0 = X^{(1)}_{\rm saddle} + 10^{-6} \ , \ \ Y_0 = Y^{(1)}_{\rm saddle} \ , \ \ \dot{X}_0 = \dot{Y}_0 = 0 \tag{7.37}$$

one gets slow-roll inflation with some 130 e-foldings and rolling-off into the dS-minimum (3) of our world, as seen in Fig. 7.2. Here the equations of motion for the T-modulus Eq. (7.10)

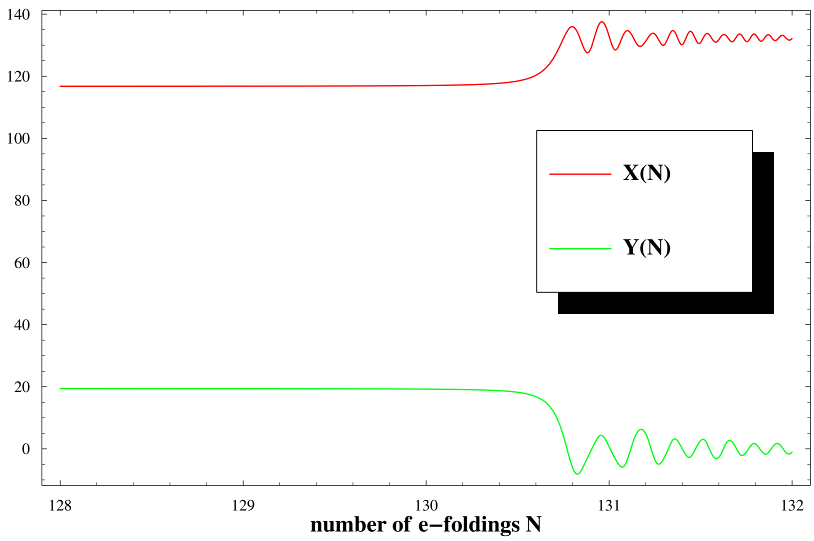

Figure 7.2: Evolution of the inflaton $T = X + iY$ as a function of time measured by the number of e-folding N.

have been rewritten using

$$\begin{aligned} \frac{\partial}{\partial t} &= H \frac{\partial}{\partial N} \ , \quad \text{where: } R(t) = e^{Ht} = e^N \\ H^2 &= \frac{1}{3}\left[\frac{3}{4X^2}(\dot{X}^2 + \dot{Y}^2) + V(X,Y)\right] \\ &= \frac{1}{3} V(X,Y) \cdot \left(1 - \frac{X'^2 + Y'^2}{4X^2}\right)^{-1} \end{aligned} \tag{7.38}$$

to yield [51]

$$\begin{aligned} X'' &= -\left(1 - \frac{X'^2 + Y'^2}{4X^2}\right)\left(3X' + 2X^2 \frac{1}{V}\frac{\partial V}{\partial X}\right) + \frac{X'^2 - Y'^2}{X} \\ Y'' &= -\left(1 - \frac{X'^2 + Y'^2}{4X^2}\right)\left(3Y' + 2X^2 \frac{1}{V}\frac{\partial V}{\partial Y}\right) + \frac{2X'Y'}{X} \end{aligned} \tag{7.39}$$

and $'$ denotes $\partial/\partial N$. The structure of the potential and the initial part of the inflaton trajectory in field space close to the saddle point can be found in Fig. 7.3. The Hubble

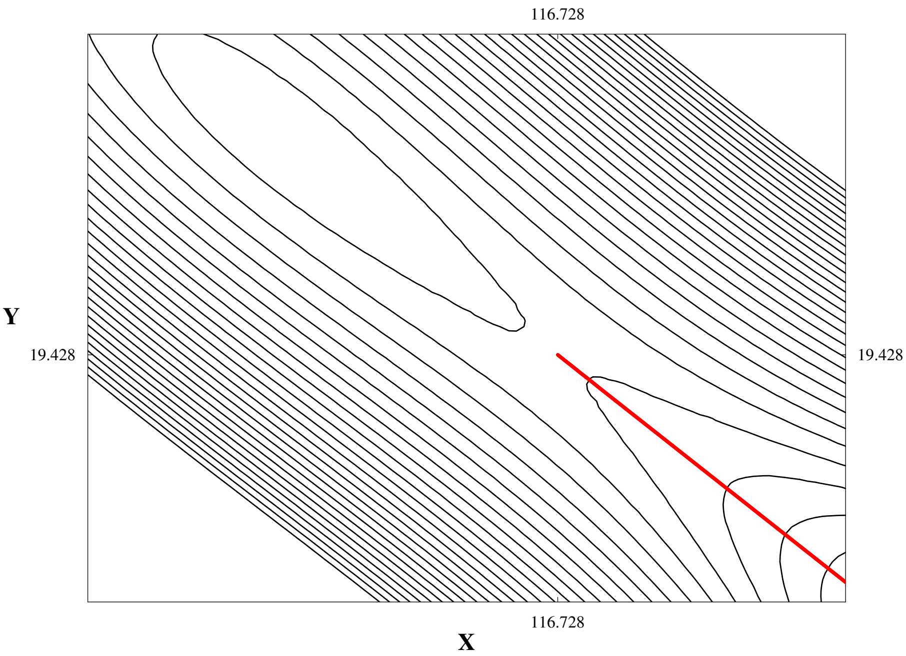

Figure 7.3: Contour plot of the potential close to the saddle point (1) and the evolution of the inflaton trajectory (red) in field space. The contour lines curving away from the starting point of the inflaton clearly indicate the saddle point nature of this region. The long thin ellipse in the upper left encloses the local minimum (1).

parameter at the saddle point

$$H_{\rm saddle} = \sqrt{\frac{1}{3}V_{\rm saddle}} \approx 10^{-9} \tag{7.40}$$

is much smaller than the initial fine-tuning of the inflaton on the saddle. Thus, the scalar field fluctuations generated during inflation being of order $H/2\pi = \mathcal{O}(10^{-10})$ here [107] will not destroy the slow-roll motion of the field.

One should mention here, that by stronger fine-tuning in the potential the slow-roll parameter η of the saddle points can be made much smaller than in the above numerical example. In this case, the amount of fine-tuning in the initial conditions of the inflaton necessary to achieve sufficiently many e-foldings can be relaxed. Thus, one may trade fine-tuning of the initial conditions for fine-tuning of the potential. Since the latter, however, is tuned discretely by the fluxes, one may consider this an advantage compared to purely field theoretic inflation models, where the potential can be fine-tuned continously in its parameters.

Next, note that the setup under discussion possesses certain scaling properties. Choose a different Calabi-Yau manifold with Euler number χ' and define the ratio

$$\lambda = \frac{\chi'}{\chi} \ . \tag{7.41}$$

Then one can show that upon rescaling

$$a \to \frac{a}{\lambda^{2/3}} \ , \ b \to \frac{b}{\lambda^{2/3}} \tag{7.42}$$

while leaving the values of W_0, A and B unchanged the whole structure of the three minima and two saddle points found above shifts along the X-axis. In the rescaled model the stationary points reside at

$$\begin{aligned} X'^{(i)}_{\text{saddle/min/max}} &= \lambda^{2/3} \cdot X^{(i)}_{\text{saddle/min/max}} \\ Y'^{(i)}_{\text{saddle/min/max}} &= \lambda^{2/3} \cdot Y^{(i)}_{\text{saddle/min/max}} \end{aligned} \tag{7.43}$$

respectively. The quantities $\hat{\xi}/(2X)^{3/2}$ of Eq. (7.34) and aX, bX, for instance, are clearly invariant under the rescaling. The potential itself rescales as

$$V \to \frac{V}{\lambda^2} \ . \tag{7.44}$$

The eigenvalues of the slow-roll parameter matrix $\mathcal{H}_\eta$ are invariant under this rescaling as long as W_0, A and B remain fixed (this has been checked numerically).

This rescaling property will be of use shortly when the density fluctuations generated during inflation are analyzed. A realistic model of inflation has to generate a nearly scale-invariant power spectrum of density fluctuations of the right magnitude. Using the results of Section 6.3 one can derive the magnitude of the density fluctuations at the COBE normalization point from the above numerical results. According to Eq. (6.58) the density fluctuations at about 55 e-foldings before the end of inflation, i.e. at $N \approx 80$ in the model above are given by $(\delta\rho/\rho)_{k_0} \approx 3 \cdot 10^{-4}$ which is about one order of magnitude too large. Furthermore, from the value of $\eta = -0.069$ on the saddle point and Eq. (6.60) one expects a spectral index $n_s = 0.86$ which is too small to be consistent with the WMAP data.

Therefore, one has to tune the model to have saddle points with a smaller slow-roll parameter η. And using the above rescaling property one has to shift the relevant part of the scalar potential along the X-axis in order to search for a region where the density fluctuations become smaller. Firstly, choose a new value for B

$$B = -3.37461130541 \cdot 10^{-2} \ . \tag{7.45}$$

This results in the two saddle points now having

$$\eta = -0.0064 \tag{7.46}$$

which should be small enough to reduce the red-tilt of the spectral index sufficiently. Secondly, one rescales the model to an Euler number[2]

$$\chi' = -42090 \;\Rightarrow\; \lambda = 10 \;\; . \tag{7.47}$$

Solving the equations of motion for this rescaled model with initial conditions given by

$$X_0 = X^{(1)}_{\rm saddle} + \lambda \cdot 2.7 \cdot 10^{-4} \;\;,\;\; Y_0 = Y^{(1)}_{\rm saddle} \;\;,\;\; \dot{X}_0 = \dot{Y}_0 = 0 \tag{7.48}$$

leads to about 137 e-foldings of inflation with the X and Y fields behaving very similar to the first case shown in Fig. 7.2.

Now calculate again the magnitude of the density fluctuations at the COBE normalization point. The result at about 55 e-foldings before the end of inflation corresponding to $N \approx 80$ is now

$$\left(\frac{\delta\rho}{\rho}\right)_{k_0} \approx 2 \cdot 10^{-5} \tag{7.49}$$

yielding this time the correct magnitude. Proceeding now to the spectral index note that in Eq. (6.60) one can replace $d\ln k \simeq dN$ because k is evaluated at horizon crossing $k = RH = He^N$. Then one arrives at

$$n_s = 1 + \frac{d\ln \mathcal{P}_{\mathcal{R}}}{dN} \tag{7.50}$$

which results in the curve shown in Fig. 7.4. The spectral index at the COBE normalization point therefore yields a value of

$$n_s \approx 0.93 \tag{7.51}$$

which is at 1σ marginally consistent with the WMAP result $n_s = 0.97 \pm 0.03$ [105]. Thus the future PLANCK mission with a target accuracy of ± 0.01 in measuring n_s will either confirm or reject this model.

Note that for the parameters chosen the rescaling places the post-inflationary 4d dS-minimum of our universe at $X^{(3)}_{\rm min} = 614.54$ and $Y^{(3)}_{\rm min} = 0$. If one were to place the Standard Model on a stack of D7-branes where the 4d gauge coupling in this dS minimum would be given by $\alpha \sim 1/X^{(3)}_{\rm min}$ the resulting value in this 4d dS-minimum clearly comes out wrong ($X^{(3)}_{\rm min} \sim 24$ would be phenomenologically required). Thus, in this model one has to place the Standard Model elsewhere, for instance, on a stack of D3-branes where the gauge coupling is controlled by the dilaton instead of T.

[2] A short comment is useful regarding the large negative value of the Euler number. According to [112] the construction of Calabi-Yaus with Euler numbers in that range is possible. Moreover, realistic values of the dilaton imply $e^{-3\phi/2} \sim 50$. For the dilaton fixed at $e^{-3\phi/2} = 61$, for instance, the above two examples are realized for $\chi = -69$ and $\chi' = -690$. Thus, the model does not have to rely on the existence of Calabi-Yaus with $\chi < -1000$.

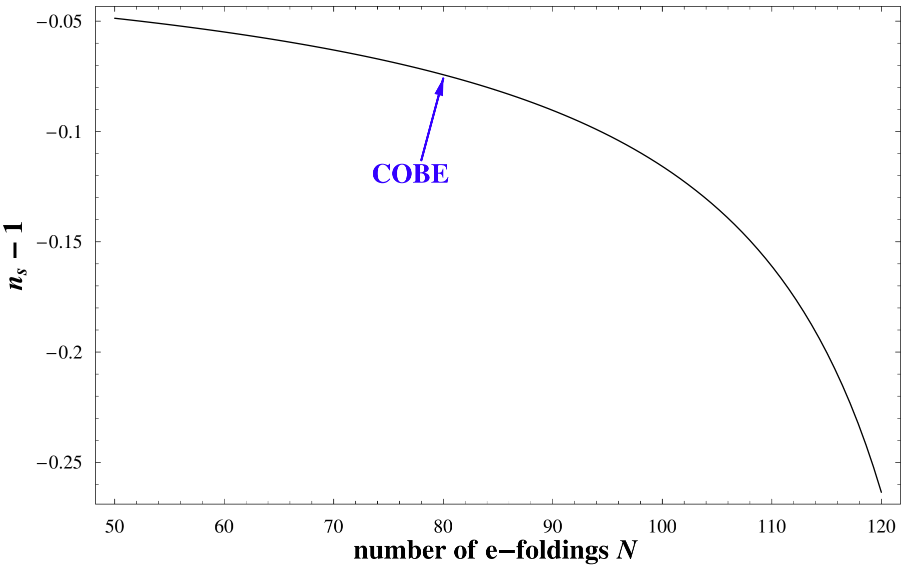

Figure 7.4: The deviation of the spectral index from unity $n_s - 1$ as a function of the number of e-foldings N. The COBE normalization point ($N \approx 80$) is given in blue.

7.4 Saddle point inflation

A check of the above numerical results is warranted. Therefore, one should study the equations of motion Eq. (7.10) of the non-canonically normalized field T in such KKLT-like setups in the vicinity of a saddle point. For simplicity just concentrate on the equation of motion for the X-component. Next assume, that the saddle point at X_s is tachyonic with negative curvature in the X-direction. Then in its vicinity the potential can be approximated by

$$V(X) = V_s - \frac{1}{2}|V_s''|(X - X_s)^2 \quad . \tag{7.52}$$

Here $'$ denotes differentiation with respect to X. For a canonically normalized scalar field the properties of inflation caused by the scalar field rolling down from the saddle point have been studied in [117]. Following the lines of the analysis given there, one first rewrites the equation of motion for X in terms of the field $\phi = X - X_s$. The field will roll down from the saddle into a local minimum with $|X_{\min} - X_s| << X_s$. Thus, ϕ obeys

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{X_s}\dot{\phi}^2 - \frac{2}{3}X_s^2|V_s''|\phi = 0 \quad . \tag{7.53}$$

Using the ansatz

$$\phi(t) = \phi_0 e^{\omega t} \tag{7.54}$$

this becomes

$$\omega^2 + 3H\omega + \frac{\omega^2\phi}{X_s} - \frac{2}{3}X_s^2|V_s''| = 0 \quad . \tag{7.55}$$

Since one will analyze a regime where the Hubble parameter is still dominated by the potential energy of ϕ and ϕ is very slowly moving, one may assume $\omega^2\phi \ll X_s$. This will be justified in the end. Now focus on the exponentially growing solution given by

$$\begin{aligned}\omega &= \frac{3}{2}H\left(-1+\sqrt{1+\frac{8}{9}X_s^2\frac{|V_s''|}{3H^2}}\right)\\ &= H\cdot|\eta_s| \end{aligned} \tag{7.56}$$

where the slow-roll parameter is again defined as above

$$|\eta_s| = \frac{2}{3}X_s^2\frac{|V_s''|}{V_s} \quad . \tag{7.57}$$

As a check of the approximation made, plug in the example of KKLT above: there it is $|\eta_s| = \frac{4}{3}aX_s$ and $V_s \sim \frac{D}{X_s^2}$. Thus

$$\omega = H|\eta_s| \sim \frac{4}{3\sqrt{3}}\frac{a\sqrt{D}}{X_s} \approx 10^{-9} \ll X_s \ , \quad \text{for: } a \approx 0.1 \text{ and } X_s \approx 130 \;\; D \approx 10^{-12} \tag{7.58}$$

which satisfies the assumption $\omega^2\phi \ll X_s$ a posteriori (the value of the field at the end of inflation is at most $\phi_{\text{end}} = \mathcal{O}(10)$ in the KKLT example above).

Denoting now the value of field at the time where inflation ends with ϕ^* one can derive the number of e-foldings in this fast-roll inflation scheme as given by

$$N = \frac{1}{|\eta_s|}\ \ln\left(\frac{\phi^*}{\phi_0}\right) \quad . \tag{7.59}$$

The final value ϕ^* here is determined either by the fact that the potential and thus the Hubble constant have decreased significantly (this works if the potential is very well described by the quadratic approximation even for large ϕ) or that at ϕ^* one has reached $|\eta| = \mathcal{O}(10)$. The last condition arises from Eq. (7.59). For $|\eta| = 6\ldots10$ even a very large ratio $\phi^*/\phi_0 \sim M_p/M_{EW} \sim 10^{17}$ does not generate more than about 10 additional e-foldings.

As a check of the numerical results of the last Section one may apply now these results. The number of e-foldings there is given by Eq. (7.59) in terms of the initial deviation of the inflaton field from the saddle point ϕ_0, the final value ϕ^* when inflation ends and the saddle curvature in its tachyonic direction η_s as

$$N = \frac{1}{|\eta_s|}\ \ln\left(\frac{\phi^*}{\phi_0}\right) \quad . \tag{7.60}$$

Now in the first example of the last Section $\phi_0 = 10^{-6}$ (see above). Further, one has $|\eta_s| = 0.069$. It remains to determine ϕ^* as the end point of the inflationary phase. For this purpose one has to analyze the potential $V(X(N), Y(N))$ along the inflationary trajectory above and to calculate the η-values along the trajectory. One finds that when the T-modulus has moved to a distance of about 0.01 from the saddle, $\eta \approx -10$ which means that inflation effectively ends there. Plugging this now in the above formula one obtains

$$N = \frac{1}{|\eta_s|}\ \ln\left(\frac{\phi^*}{\phi_0}\right) \approx 133 \ . \tag{7.61}$$

This is sufficiently close to the purely numerical results above, which indicates that the numerical solution is stable and closely resembles the true one.

As a last comment note that in this model each of the two rather flat saddle points still connects two minima ((1) and (3) or (2) and (3), respectively). In such a situation, where a sufficiently flat saddle point connects two minima along a certain direction in field space, inflation may also arise from inflating topological defects, namely, domain walls [118]. It is therefore tempting to speculate, that besides slow-roll inflation also eternal topological inflation arises on the saddles constructed here, which would relieve the question of fine-tuning the initial conditions of the inflaton [51, 119]. The original literature [118, 119] uses a saddle point connecting two degenerate minima in deriving the conditions for topological inflation: the saddle curvature has to be small enough that $\eta_{\rm saddle} \ll 1$, which corresponds to domain walls whose wall thickness is large compared to their gravitational radius. As an illustration consider the example of static domain walls of the Z_2-symmetric theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) \ \ , \ V(\phi) = \frac{\lambda}{4}(\phi^2 - \beta^2)^2 \tag{7.62}$$

which are given by the solution

$$\phi(x) = \beta \tanh\left(\sqrt{\frac{\lambda}{2}}\,\beta x\right) \tag{7.63}$$

for a wall in the yz plane. The thickness of the wall δ is determined by the equilibrium of gradient and potential energy density as

$$\rho_{\rm grad}\big|_{x\sim\delta} \sim \frac{\beta^2}{\delta^2} \sim \rho_{\rm pot} = V(0) \sim \lambda\beta^4 \ \Rightarrow \ \delta \sim \frac{1}{\beta\sqrt{\lambda}} \ \ . \tag{7.64}$$

The gravitational radius of the wall is $R = 2M_{\rm wall} \sim 8\pi\rho\delta^3/3$ where the energy density is $\rho = \lambda\beta^4/2$ (the sum of the potential energy density and the gradient energy density). Gravitational effects become important once the gravitational radius exceeds the wall thickness, i.e, for

$$\delta < R \ \Rightarrow \ \beta > \frac{3}{4\pi} \tag{7.65}$$

in Planck units. If one calculates the slow-roll parameter η at the center of the wall the result is $\eta_{x=0} = V''(0)/V(0) = 4/\beta^2$. Requiring $\eta < 1$ therefore corresponds to the previous 'importance of gravity' condition. The above static wall solution would never inflate since the potential and gradient energy density are of the same order near the wall. However, if inflation started in a small patch of space-time with $\phi = 0$ then the fluctuations $\delta\phi \sim H$ with wavelength H^{-1} generated after each time interval H^{-1} have a gradient energy $\sim H^4 \sim V^2 << V$ as long as $V << 1$ in the wall. In this case, an initially inflating wall which fulfills Eq. 7.65 will continue to inflate forever near to the wall center [118, 119].

The analysis has been carried out in the symmetric potential of the example above. Since in the cases under consideration in the last Section the saddles connect two highly non-degenerate minima

$$\frac{V^{(1)}_{\rm saddle} - V^{(1)}_{\rm min}}{V^{(1)}_{\rm saddle}} = \mathcal{O}(10^{-11}) \ll \frac{V^{(1)}_{\rm saddle} - V^{(3)}_{\rm min}}{V^{(1)}_{\rm saddle}} \tag{7.66}$$

it is not clear whether this derivation, which otherwise would imply the existence of eternal topological inflation on these saddles, remains valid. For such highly asymmetric minima of the potential the determination of the wall thickness, for instance, is not clear. The possibility of eternal topological inflation in the T modulus inflation model of the last Section would be an interesting subject of further investigations.

Conclusion

This thesis shows on the basis of several examples that higher-dimension operators can have a significant impact on the phenomenological aspects of higher-dimensional field theories.

The first part leads to two main statements. Firstly, power-like loop corrections to gauge coupling unification arising in generic supersymmetric 5d unified models are exactly calculable in the framework of the 5d low-energy effective field theory. Secondly, even without SUSY these power corrections remain calculable as long as the softly broken higher-dimensional GUT respects a hierarchy $M_c \ll M_B \ll M$. (Here M_c denotes the compactification scale, M_B the bulk symmetry breaking scale and M the UV scale of the GUT.) Such power-law corrections are induced, for example, by the loop effects of charged bulk matter fields. Higher-dimension operators, which contain the symmetry-breaking bulk Higgs field together with the square of the field strength tensor, introduce the same kind of power-law corrections. In fact, it is equivalently possible to view the loop effects of bulk matter as arising from higher-dimension operators introduced when these fields are integrated out. These operators then change low-energy gauge couplings at the tree level.

The essential points which underlie the exact calculability in the supersymmetric case are the following: on the one hand, minimal 5d SUSY, which corresponds to $\mathcal{N}=2$ SUSY in 4d language, ensures that no corrections arise beyond the one-loop level. On the other hand, possible higher-dimension operators are extremely restricted by the combination of 5d SUSY and 5d gauge invariance. In fact, there is only one globally analytic higher-dimension operator at the two-derivative level, which is the SUSY version of the Chern-Simons (CS) term. Knowledge of the light 5d field content and the coefficient of the CS term determines the low-energy gauge couplings completely.

A realistic 5d model has to reduce the higher supersymmetry of five dimensions to 4d $\mathcal{N}=1$. This is possible by compactification on an interval, e.g., as an S^1/Z_2 orbifold. The 5d CS term induces boundary anomalies in a theory compactified on an interval. Once a bulk and brane field content is given the requirement of boundary anomaly cancellation determines the coefficient of this higher-dimension operator uniquely. Thus, power-like corrections to gauge coupling differences are completely fixed. Because of the absence of higher-loop effects or other higher-dimension operators, this calculability extends to strong coupling, i.e., if the gauge symmetry is broken at a scale where the 5d gauge theory is strongly coupled. In this case, power-law corrections are parametrically large and can be of the same size as the conventional logarithmic running from GUT scale to weak scale. In particular, a 5d SU(5) model with a single **10** hypermultiplet in the bulk and the CS term required by anomaly cancellation generates a power-law effect which is group-theoretically equivalent to the MSSM running. Thus, calculable TeV-scale unification is possible.

The next point of consideration was the possibility that a 5d model arises as the low-energy effective theory of a 6d model compactified on an S^1. In this case, the 5d bulk breaking, realized in all interesting cases by the bulk VEV of the scalar adjoint from the vector multiplet, has its origin in a 6d Wilson line wrapping the S^1. This situation is known from familiar 6d T^2/Z_2 constructions where the two torus radii, R_5 and R_6, are highly hierarchical. Here power-like gauge coupling corrections are calculable in close analogy to the 5d case and produce contributions to differences of inverse gauge couplings of the order $\sim R_5/R_6$. Such an effective 5d theory coming from 6d is highly constrained by two requirements: firstly, 6d anomaly cancellation is highly restrictive and, secondly, gauged hypermultiplets can not have bulk masses in 6d. For $d \geq 7$ the minimal SUSY corresponds to $\mathcal{N}=4$ in 4d language and no loop corrections to gauge coupling unification arise.

Thus, this investigation comes to the conclusion that large and fully calculable power-like loop corrections to gauge unification arise in the context of 5d and 6d grand unified theories. Their phenomenological relevance may be as striking as a very low unification scale reduced by many orders of magnitude or as modest as an interesting field theoretic contribution to the detailed GUT dynamics in a string-derived high-scale model. In any case I believe that the calculability of such power-like loop corrections within field theory, based on higher-dimensional SUSY, gauge symmetry, and anomaly cancellation, is an interesting phenomenon.

The second part of the thesis dealt with higher-order α'-corrections in type IIB superstring theory. The α'-corrections which were studied are higher-order curvature corrections and thus higher-dimension operators appearing in the Kähler potential of the effective action. In this thesis it was shown that the generic ability of these higher-dimension operators to lift stable AdS_4 type IIB string vacua to the desired metastable dS-minima for the T modulus (the volume modulus) can also be used to provide slow-roll inflation using the same T modulus. Such a setup has no η-problem because the leading order Kähler potential for the T modulus is of the no-scale type. A concrete model using fluxes and a racetrack superpotential was constructed which upon inclusion of the α'-corrections yields T-modulus inflation on saddle points of the potential with some 130 e-foldings. At the end of inflation the T-modulus rolls from the saddle point down into a dS-minimum with a small positive cosmological constant where the modulus is stabilized. The model has certain scaling properties allowing one to shift the inflationary region of the potential to different values of the real part of T while leaving the slow-roll parameters of the inflationary saddle points invariant. It was argued that these saddle points might be generically present if racetrack superpotentials and α'-corrections are both taken into account. The model can accommodate the WMAP data of the CMB radiation. It yields primordial density fluctuations of the right magnitude with a spectral index of these fluctuations $n_s \approx 0.93$. Finally the possibility of eternal topological inflation on the saddle points of the model was briefly discussed.

There are still many open questions in this area, and I believe that the further analysis of the rich vacuum structure of string theory will uncover more of the highly interesting cosmological dynamics of such a multi-vacua universe.

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Appendix A

Power-like loop corrections in 5d

In detail, the 1-loop corrections to the gauge couplings in the $U(1) \times U(1)'$-toy model arise from the scalar loops shown in Fig.s A.1 and A.2.

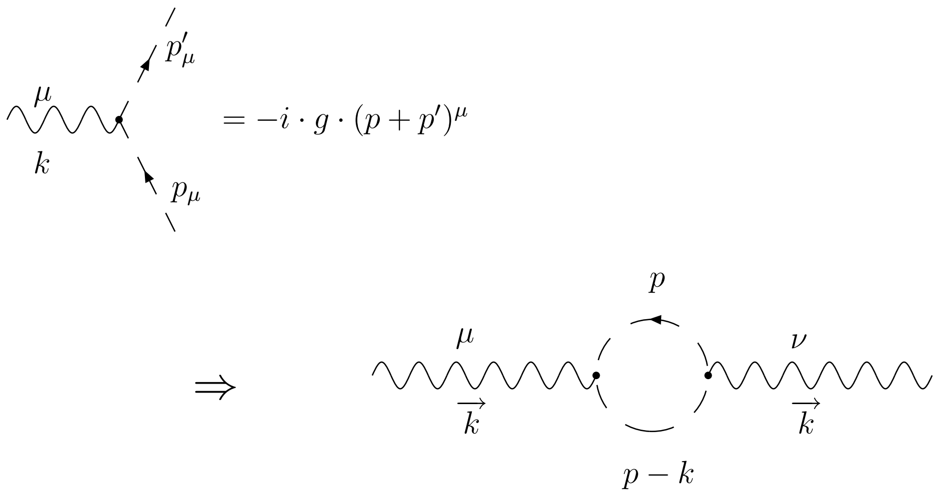

Figure A.1

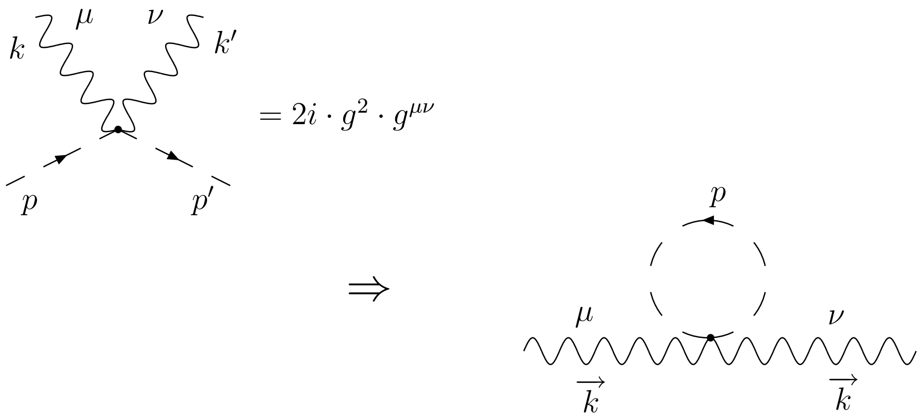

Figure A.2

The 1-loop corrections will split up between g and g' since the scalars acquire different KK-towers with masses

$$\begin{aligned} m_n^2 := m_n^2(h) &= k_n^2\ ,\quad k_n = \frac{n}{R} \\ {m'_n}^2 := m_n^2(h') &= k_n^2 + m_{h'}^2\ ,\quad m_{h'}^2 = \Phi_0^2 \quad . \end{aligned} \tag{A.1}$$

The VEV Φ_0 induces a change in the coupling g' relative to g. This change arises from the 1-loop vacuum polarization diagram built from two scalar-scalar-gauge boson vertices and is diagrammatically expressed as:

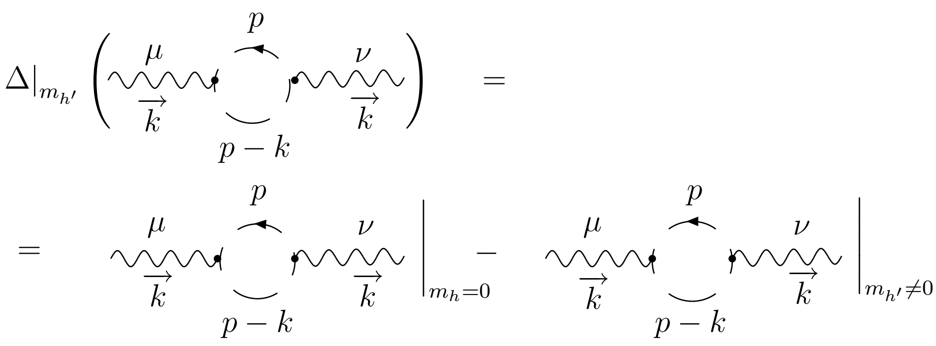

Figure A.3

Then to calculate the change the corresponding contribution of the 'bubble' vacuum polarization diagram of this scalar QED has to be added. The full 1-loop contribution of e.g. ϕ to the photon propagator can firstly be given as the dimensionally regulated sum of the KK-tower of the scalars running in the loop, which reads explicitly:

$$\begin{aligned} \Pi^{mn}_{(1)}\left(k^2\right) &= \left(g^{mn}\cdot k^2 - k^m k^n\right)\cdot \Pi_{(1)}\left(k^2\right) \\ \Pi_{(1)}\left(k^2\right)_h &= \frac{g^2}{(d-1)k^2}\sum_n \\ &\quad \mu^{4-d}\int_0^1 dx \int \frac{d^dq}{(2\pi)^d}\, \frac{(d-2)2q^2 - [1-4x-2(d-2)x^2]k^2 + m_n^2\cdot 2d}{\left[q^2+m_n^2+x(1-x)k^2\right]^2} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \Pi_{(1)}\left(k^2\right)_{h'} &= \frac{{g'}^2}{(d-1)k^2}\sum_n \\ &\quad \mu^{4-d}\int_0^1 dx \int \frac{d^dq}{(2\pi)^d}\, \frac{(d-2)2q^2 - [1-4x-2(d-2)x^2]k^2 + m_n^2\cdot 2d}{\left[q^2+{m'_n}^2+x(1-x)k^2\right]^2} \\ d &= 4-\epsilon \quad . \end{aligned} \tag{A.3}$$

Using now the standard results of dimensional regularization [120] (see also the standard text books on quantum field theory, e.g. [121]) these integrals can be evaluated to yield

$$\Pi_{(1)}\left(k^2\right)_h = -\frac{g^2}{d-1}\sum_n \int_0^1 dx\,[1-2(2-d)x+4(1-d)x^2]\cdot\frac{\Gamma(\epsilon/2)}{(4\pi)^{d/2}}\left(\frac{\mu^2}{\Delta}\right)^{\epsilon/2} \quad (A.4)$$

$$\Pi_{(1)}\left(k^2\right)_{h'} = -\frac{g'^2}{d-1}\sum_n \int_0^1 dx\,[1-2(2-d)x+4(1-d)x^2]\cdot\frac{\Gamma(\epsilon/2)}{(4\pi)^{d/2}}\left(\frac{\mu^2}{\Delta'}\right)^{\epsilon/2} \quad (A.5)$$

where Δ denotes $q^2+m_n^2$ and $\Delta' = q^2 + {m'_n}^2$. Expanding this about $\epsilon = 0$ yields

$$\Pi_{(1)}\left(k^2\right)_h = -\frac{g^2}{48\pi^2}\sum_n \int_0^1 dx\,[1+4x(1-3x)]\left[\frac{2}{\epsilon}-\gamma+\ln 4\pi-\ln\left(\frac{m_n^2-x(1-x)k^2}{\mu^2}\right)\right] \quad (A.6)$$

and the corresponding result for h'. As clearly visible, because of $m_{h'} \neq m_h = 0$ the 1-loop corrections to the photon propagators of A_M and A'_M split up to give a difference

$$\begin{aligned}\Delta\Pi_{(1)} &= \Pi_{(1)}\left(k^2\right)_h - \Pi_{(1)}\left(k^2\right)_{h'} \\ &= \frac{g'^2}{4\pi}b_h\cdot\sum_n \int_0^1 dx\,[1+4x(1-3x)]\cdot\ln\left(\frac{n^2M_c^2-x(1-x)k^2}{m_{h'}^2+n^2M_c^2-x(1-x)k^2}\right) \quad (A.7)\\ &= \frac{g'^2}{4\pi}b_h\cdot\left\{\frac{8}{3}-\ln\left(\frac{-k^2}{m_{h'}^2}\right)+f(k^2)\right.\\ &\quad \left.-2\sum_{n\geq 1}\int_0^1 dx\,[1+4x(1-3x)]\cdot\ln\left(\frac{m_{h'}^2+n^2M_c^2-x(1-x)k^2}{n^2M_c^2-x(1-x)k^2}\right)\right\}\ .\end{aligned}$$

Here $b_h = 1/12\pi$ denotes the 1-loop beta function coefficient of scalar QED. Note, that the splitting vanishes in the limit $k \to \infty$, implying for the function f that $\lim_{k\to 0} f(k^2) = 0$ and $\lim_{k\to\infty} f(k^2) = -8/3 + \ln(-k^2/m_{h'}^2)$.

Note further, that the $1/\epsilon$-poles have dropped out. The correction to each propagator in itself possesses this pole and sums over it. The number of KK modes circulating in the loops is given by the UV-cutoff scale M of the theory as M/M_c. For each single KK mode the $1/\epsilon$-pole denotes the appearance of logarithmical divergence $\sim \ln M/(nM_c)$. Therefore, the fact that one sums the $1/\epsilon$-poles over all KK modes up to M/M_c denotes the appearance of a linear divergence $\sim M$ resulting from the summation of the logs $\ln M/(nM_c)$. However, here it proves to be crucial that the soft breaking of the Z_2-symmetry that originally linked the couplings g and g' together ensures that the poles in the 1-loop corrections of both couplings get multiplied by the *same* dependence on the KK-towers. (The pole as a UV-effect cannot be influenced by the soft breaking as an effect in the IR.) Therefore the difference of the propagators, and thus later of the inverse gauge couplings, is independent of the UV physics.

Now, since the interest is in the splitting of the photon propagators in the low energy effective theory at accessible energies, one has to calculate at vanishing external momentum

$k = 0$. Thus, the splitting becomes

$$\Delta\Pi_{(1)}\big|_{k=0} \;=\; \frac{{g'}^2}{4\pi} b_h \cdot \left\{ \frac{8}{3} - \ln\left(\frac{-k^2}{m_{h'}^2}\right) + 2\sum_{n\geq 1} \ln\left(1 + \frac{m_{h'}^2}{n^2 M_c^2}\right)\right\} \; . \tag{A.8}$$

Using Eq. (2.9) (note: $N = m_{h'}/M_c$) the sum in this expression can be estimated to yield

$$\sum_{n\geq 1} \ln(1 + N^2/n^2) = \pi \frac{m_{h'}}{M_c} - \frac{1}{2}\ln\left(\frac{m_{h'}^2}{M_c^2}\right) - (1 + \ln 2) \; . \tag{A.9}$$

One arrives at the final expression for the propagator splitting given by

$$\Delta\Pi_{(1)}\big|_{k=0} = \alpha'_{4d} \cdot b_h \cdot \left[\ln\left(\frac{M_c^2}{-k^2}\right) + 2\pi \cdot \frac{m_{h'}}{M_c} + \mathcal{O}(1)\right] \; , \; \alpha'_{4d} = \frac{{g'}^2}{4\pi} \; . \tag{A.10}$$

At the scale $k = M_c$ therefore the correction to the propagators is purely power-like. The correction to the difference of the inverse gauge couplings follows directly since the 1-loop corrections just add to the inverse gauge couplings

$$\alpha_{4d}^{-1}(M_c) - {\alpha'_{4d}}^{-1}(M_c) = 2\pi b_h \cdot \frac{m_{h'}}{M_c} = \frac{1}{6} \cdot \frac{m_{h'}}{M_c} \; . \tag{A.11}$$

Finally, it can be shown that one could have arrived at the same result directly with a manifestly 5d calculation. One begins using the fact that in $2n+1$ odd dimensions dimensionally regulated loop integrals do not show the logarithmic divergences of the corresponding $2n$-dimensional loop integrals as power divergences but just the finite part of the loops. This, in turn, resides in the fact that the Γ-function arising in the integrals in $2n$ dimensions with $\epsilon = 2n - d$ is finite at the half-integer valued arguments in $2n + 1$ dimensions. Using now again Eq. (A.5), one sets $\epsilon = 4 - d = -1$ for $d = 5$ and deliberately omits all the factors of μ^{4-d} in the expressions, arriving at

$$\Pi_{(1)}\left(k^2\right)_{h'} = -\frac{{g'}^2}{4}\int_0^1 dx\,[1 + 6x(1 - 8x/3)] \cdot \frac{\Gamma(-1/2)}{(4\pi)^{5/2}}\left(\frac{1}{\Delta'}\right)^{-1/2} \tag{A.12}$$

where now $\Delta' = m_{h'}^2 - x(1-x)k^2$. Using $\Gamma(-1/2) = -2\Gamma(1/2) = -2\sqrt{\pi}$ this yields in the limit of vanishing external moment $k = 0$

$$\begin{aligned} \Delta\Pi_{(1)}\big|_{k=0} &= -\,\Pi_{(1),h'}\big|_{k=0} \\ &= \alpha'_{5d} \cdot b_h \cdot m_{h'} \; , \; \alpha'_{5d} = \frac{{g'}^2}{4\pi} \end{aligned} \tag{A.13}$$

and this yields for the inverse gauge coupling differences

$$\alpha_{5d}^{-1}(M_c) - {\alpha'_{5d}}^{-1}(M_c) = b_h \cdot m_{h'} = \frac{1}{12\pi} \cdot m_{h'} = \frac{1}{12\pi} \cdot |\Phi_0| \; . \tag{A.14}$$

After one integrates this result over the compact S^1 with length $2\pi R = 2\pi/M_c$, this is, as expected, in agreement with the KK-summation result Eq. (A.11).

Appendix B

Cubic order of the 5d prepotential

The KK zero mode of 5d SYM is given by 4d $N=2$ SYM of an $N=2$ vector field comprised of $(V, \Sigma = \phi + iA_5 + \ldots)$ in $N=1$ fields. Thus, the zero mode lagrangian Eq. (3.1) is controlled by a holomorphic prepotential [123, 124]:

$$\mathcal{F}_{\mathcal{G}}(\Sigma) = \sum_{m\geq 2} c_m \cdot d^{(m)}_{a_1\cdots a_m} \Sigma^{a_1} \cdot \ldots \cdot \Sigma^{a_m} \tag{B.1}$$

where $d^{(m)}_{a_1\cdots a_m}$ denotes the m^{th}-order fully symmetrized Casimir invariant of the gauge group $\mathcal{G}$. Gauge invariance and SUSY in 5d restrict $\mathcal{F}_{\mathcal{G}}(\Sigma)$ to be of cubic order. This can be seen as follows.

Firstly, note that the problem can be reduced to the case of a $U(1)$. This relies on the fact, that for any $d^{(m)}_{a_1\cdots a_m} \neq 0$ due to its symmetric properties one can find a $U(1) \subset \mathcal{G}$, denoted by i, for which $d^{(m)}_{i\cdots i} \neq 0$ (and subsequently this $U(1)$ can be rotated into the Cartan subalgebra of $\mathcal{G}$). The further argument can now be done on this $U(1)$ where one defines:

$$\begin{aligned} \Sigma &:= \Sigma^i \ , \qquad d^{(m)} := d^{(m)}_{i\cdots i} \\ \mathcal{F}(\Sigma) &:= \mathcal{F}_{\mathcal{G}}(\Sigma^i T^i) = \sum_{m\geq 2} c_m \cdot d^{(m)} \Sigma^m \\ &\quad\ \mathcal{F}(\Sigma): \ \text{4d } N=2 \ \ U(1) \text{ prepotential.} \end{aligned} \tag{B.2}$$

Next, look at those 5d bosonic operators whose 4d reduction is contained in the gauge kinetic part of Eq. (3.1)

$$\frac{\partial^2 \mathcal{F}(\Sigma)}{\partial \Sigma^2} W^\alpha W_\alpha \bigg|_{\theta^2} + \text{h.c.} \ . \tag{B.3}$$

The bosonic operators contained in Eq. (B.3) consist of two gauge fields A_μ, two derivatives and arbitrary powers of ϕ and A_5. More specifically, they contain either $F_{\mu\nu}F^{\mu\nu}$ or $\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$ once. The bosonic fields of the 5d vector are A_m and ϕ. Lorentz invariant bosonic operators with up to two derivatives can be formed only by using the two 5d invariant tensors η^{mn} and ϵ^{mnpqr}. Then invariants can be constructed with:

I Pairwise contractions of arbitrarily many A_m and up to two ∂_m with η^{mn}-factors.

II Powers of ϕ.

III Factors of $\epsilon^{mnpqr} A_m \partial_n A_p \partial_q A_r$.

Operators of type I and II do not mix with those of III. Now look at those bosonic operators of Eq. (B.3) containing the one factor $\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$. Such a 4d ϵ-symbol can arise only as the 4d reduction of a single 5d ϵ-symbol. Thus, the $\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$-containing operators of Eq. (B.3) can originate only from 5d bosonic terms containing III once and powers of ϕ (II):

$$\mathcal{O}^{\text{bosonic}}_{5\text{d},r} = \phi^r \cdot \epsilon^{mnpqr} A_m \partial_n A_p \partial_q A_r \quad . \tag{B.4}$$

Derivative-free factors I are absent since $\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$ being part of $W^\alpha W_\alpha|_{\theta^2}$ allows no further $A_\mu A^\mu$-factors from V. Comparison of Eq. (B.4) with Eq.s (B.2) and (B.3) shows that to each

$$\phi^{m-3} A_5 \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} \subset \frac{\partial^2 \mathcal{F}^{(m)}(\Sigma)}{\partial \Sigma^2} W^\alpha W_\alpha \Big|_{\theta^2} \qquad \text{with: } \mathcal{F}^{(m)}(\Sigma) := c_m \cdot d^{(m)} \Sigma^m \tag{B.5}$$

corresponds a

$$\mathcal{O}^{\text{bosonic}}_{5\text{d},m} = \phi^{m-3} \cdot \epsilon^{mnpqr} A_m \partial_n A_p \partial_q A_r \quad . \tag{B.6}$$

Its gauge variation is given by

$$\begin{aligned} &\delta A_m = \partial_m \Lambda \\ \Rightarrow \delta \mathcal{O}^{\text{bosonic}}_{5\text{d},m} &= \phi^{m-3} \cdot \epsilon^{mnpqr} \partial_m \Lambda \partial_n A_p \partial_q A_r \\ &= \phi^{m-3} \cdot \epsilon^{mnpqr} \partial_m \left(\Lambda \partial_n A_p \partial_q A_r \right) \end{aligned} \tag{B.7}$$

which ceases to be a total derivative for $m > 3$.

Thus, all terms $\mathcal{F}^{(m)}(\Sigma)$, $m > 3$ correspond to 5d operators, whose bosonic part is not gauge invariant, and consequently are forbidden in 5d SYM:

$$\mathcal{F}_{5\text{d},\mathcal{G}}(\Sigma) = c_2 \cdot d^{(2)}_{ab} \Sigma^a \Sigma^b + c_3 \cdot d^{(3)}_{abc} \Sigma^a \Sigma^b \Sigma^c \quad . \tag{B.8}$$

Appendix C

Power-like loop corrections and higher-dimension operators

Power-like loop corrections arise whenever a higher-dimensional GUT is spontaneously broken in the bulk by a VEV of a scalar field Φ. Then generically the gauge bosons of the broken direction acquire masses $\sim \Phi$. The surviving massless subgroups of the GUT then receive at 1-loop GUT non-universal corrections from the massive broken gauge bosons which are essentially given by the change the VEV-induced gauge boson masses induce in their 1-loop contribution:

$$\int^{\mu} \frac{d^d k}{(k^2+\Phi^2)^2} - \int^{\mu} \frac{d^d k}{(k^2)^2} \sim |\Phi| \ . \tag{C.1}$$

This finite 1-loop correction splits the unified gauge coupling among the surviving subgroups in a calculable way. However, the corresponding higher-dimension operator

$$\mathrm{tr}(|\Phi| \cdot F^2) \tag{C.2}$$

is non-analytic in Φ.

From the point of view of effective field theories (EFTs) this resembles a difficulty since despite the fact, that the competing tree-level analytic operator $\mathrm{tr}(\Phi \cdot F^2)$ can be easily forbidden by a Z_2-symmetry of the scalar potential $V(\Phi)$ under $\Phi \rightarrow -\Phi$, the generation of one non-analytic higher-dimension operator implies that all operators non-analytic in Φ have to be kept which are consistent with this symmetry.

In dealing with this problem note firstly, that the above non-analyticity is of global kind only. Writing

$$\mathrm{tr}(|\Phi| \cdot F^2) \tag{C.3}$$

as

$$\mathrm{tr}(\epsilon(\Phi) \cdot \Phi F^2) \tag{C.4}$$

where $\epsilon(x)$ denotes the sign function, the 1-loop term, in general, is seen to be locally analytic up to a finite number of points. The 1-loop term becomes analytic in Φ if the spinor matter running in the loops gets a large bulk mass.

As an illustrating example consider a massive Dirac ψ spinor gauged under a gauge group $\mathcal{G}$ in 4d, 5d and then 6d. Its lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \mathrm{tr}\left[i\bar{\psi}\Gamma^M D_M\psi - m\cdot\bar{\psi}\psi + \bar{\psi}\Phi\psi\right] \\ & \quad D_M = \partial_M + iA_M\end{aligned} \tag{C.5}$$

where m denotes its bulk mass. Let now Φ acquire a $\mathcal{G}$ non-universal VEV. Then difference in the contributions to the gauge coupling of broken directions of ψ which acquire additional mass through Φ and the unbroken directions is given by expressions of the form

$$\begin{aligned}&\int^{\mu}\frac{d^dk}{(k^2+(m+\Phi)^2)^2} - \int^{\mu}\frac{d^dk}{(k^2+m^2)^2} \sim \\ &\sim \begin{cases}\ln\left(\frac{\mu^2}{(m+\Phi)^2}\right) - \ln\left(\frac{\mu^2}{m^2}\right) \xrightarrow[m\to\infty]{} 0 & , d=4 \\ |m+\Phi| - |m| \xrightarrow[m\to\infty]{} \Phi & , d=5\end{cases} .\end{aligned} \tag{C.6}$$

Thus, heavy spinor matter effectively decouples from the loops in the sense, that it leaves as its only trace just a correction of the form of the analytic tree-level operator $\mathrm{tr}(\Phi\cdot F^2)$ already present from generation by massless spinor matter.

The situation in 6d for Ψ coupled to an arbitrary 6d bulk scalar VEV Φ in general is similar. However, it can be even better in case that the 5d scalar Φ arises from an A_6-Wilson line background after integrating out the 6-direction. In that situation in 6d with no other scalar VEV present, the lagrangian reads

$$\begin{aligned}\mathcal{L}_{6d} &= \mathrm{tr}\left[i\overline{\psi_{6d}}\Gamma^M D_M\psi_{6d} - m\cdot\overline{\psi_{6d}}\psi_{6d}\right] \ , \ D_M = \partial_M - iA_M \\ & \quad \psi_{6d} = \begin{pmatrix}\psi^1_{5d}\\ \psi^2_{5d}\end{pmatrix} \\ & \quad \Gamma^m = \begin{pmatrix}0 & \gamma^m\\ \gamma^m & 0\end{pmatrix} \text{ for } m=0,\ldots,3,5 \ ; \ \Gamma^6 = \begin{pmatrix}0 & \mathbf{1}\\ -\mathbf{1} & 0\end{pmatrix}\end{aligned} \tag{C.7}$$

which upon integrating out the 6-direction becomes

$$\begin{aligned}\mathcal{L}_{5d} &= \int_0^{2\pi R_6} dx^6\cdot\mathcal{L}_{6d} \\ &= \mathrm{tr}\left\{\sum_n\left[\left(\overline{\psi^{1,(n)}_{5d}}, \overline{\psi^{2,(n)}_{5d}}\right)(iD+M_n)\begin{pmatrix}\psi^{1,(n)}_{5d}\\ \psi^{2,(n)}_{5d}\end{pmatrix}\right]\right\} \\ & \quad \text{with} : D = \begin{pmatrix}\gamma^m D_m & 0\\ 0 & \gamma^m D_m\end{pmatrix} \ , \ M_n = \begin{pmatrix}\frac{n}{R_6}+A_6 & m\\ m & -(\frac{n}{R_6}+A_6)\end{pmatrix} .\end{aligned} \tag{C.8}$$

Upon diagonalization of mass matrix one arrives at two massive 5d Dirac eigenstates whose KK zero modes, e.g., receive a mass

$$M_0^2 = A_6^2 + m^2 \quad . \tag{C.9}$$

Their contribution to Eq. (C.6) reads

$$\begin{aligned} \Delta\alpha_{4D}^{-1}(M_C) \quad &\sim \quad \sqrt{A_6^2 + m^2} - |m| \\ &\underset{m\to\infty}{\longrightarrow} \quad \frac{A_6^2}{2m} + \mathcal{O}(A_6^4/m^3) \longrightarrow 0 \end{aligned} \tag{C.10}$$

which just shows that massive gauged 6d spinors decouple *completely* from the A_6-induced 1-loop correction to the gauge coupling. The dangerous higher-dimension operators of the type of Eq. (C.2) thus do not appear in this 6d case as long as all symmetry breaking in the bulk is done by the Wilson line.

Two important inputs have been used in this conclusion. The first one is the structure of the A_6-Wilson line induced 1-loop correction to the gauge coupling. A derivation along the lines of [45] can be found in Appendix D. Secondly it was used that there is no mass term for gauged 6d spinors besides the one written down in Eq. (C.7). This is shown in Appendix E using results of [122]. The immediate use of these results becomes clear in case of higher-dimensional supersymmetry, since by its virtue the decoupling found above then carries over to all sectors of the theory.

Appendix D

6d Wilson line corrections

In going from 5d to 6d one first has to ask where the gauge coupling splitting 1-loop corrections of the 5d vector multiplet do come from in 6d. The answer is given by the change of the N=1 vector when going from 5d to 6d: the real scalar ϕ of the 5d vector becomes A_6, the 6-component of the gauge field in the 6d vector multiplet. Thus, the background VEV $\langle\phi\rangle$ of ϕ in 5d now becomes a background value $A_6^{(0)}$ in 6d which is induced by an A_6-Wilson line around the S^1-compactified 6-direction of space-time.

Such a non-trivial A_6-background induces masses for degrees of freedom in the vector that do not commute with the $A_6^{(0)}$-direction in group space. Therefore, the 1-loop corrections to gauge coupling unification in 6d will be given by the change induced in the 1-loop corrections of the gauge couplings by the $A_6^{(0)}$-broken massive directions of the vector.

In order to facilitate comparison with the results from the 5d situation these loop corrections will be calculated on $\mathcal{M}^5 \times S^1$ incorporating the 6-direction via its KK-reduction on an S^1 (otherwise also the definition of a Wilson line 'around' the 6-direction becomes unclear). Therefore, firstly one has to determine how an A_6-Wilson line modifies the KK-spectrum of the gauge and gauged fields. In [45] it was shown that the Wilson line background gauge field of the compactified extra dimension can be locally gauged away by applying a gauge transformation $U(x_6) = exp(-ix_6A_6)$ to the gauge fields as well as the gauged fields. This, however, then leads to a modification of the periodicity conditions of the fields on the S^1 given by

$$\begin{aligned} A_m^i(x^m, x^6 + 2\pi R_6) &= A_m^i(x, x^5, x^6) \\ A_m^\alpha(x^m, x^6 + 2\pi R_6) &= e^{-i\cdot 2\pi R_6 A_6^i \alpha_i} A_m^\alpha(x^m, x^6) \\ \Phi_\lambda(x^m, x^6 + 2\pi R_6) &= e^{-i\cdot 2\pi R_6 A_6^i \lambda_i} \Phi_\lambda(x^m, x^6) \ , \end{aligned} \tag{D.1}$$

where α denote the roots and λ the weights of the representation Φ transforms in and

$$A_M(x^m, x^6) \;=\; A_M^i(x^m, x^6)T_i + A_M^\alpha(x^m, x^6)E_\alpha \tag{D.2}$$
$$E_\alpha \;:\; \text{generator corresponding to the root } \alpha\;.$$

This leads to a linear shift in the KK-tower of the fields yielding KK-masses of

$$\begin{aligned} m_n^2(\alpha) &= (k_n + c'_\alpha)^2 \\ m_n^2(\lambda) &= (k_n + c'_\lambda)^2 \\ &\quad \text{where} : k_n = \frac{n}{R_6} \;,\; c'_\alpha = A_6^{(0)^i}\alpha_i \;,\; c'_\lambda = A_6^{(0)^i}\lambda_i \;. \end{aligned} \tag{D.3}$$

Next comes the calculation of the 1-loop corrections to the gauge couplings induced by these shifts. For that purpose the $U(1) \times U(1)'$ theory of Eq. (1.18) in App. A is useful again. Imagine now A_6 as acquiring a background value by a Wilson line instead of giving a VEV to the scalar Φ. Then the 1-loop corrections to the gauge couplings arising from the scalar loops will split up between g and g' since the scalars acquire different KK-towers with masses

$$\begin{aligned} m_n^2 := m_n^2(h) &= (k_n + c')^2 \;,\; c' = qA_6^{(0)} \\ {m'_n}^2 := m_n^2(h') &= k_n^2 \end{aligned} \tag{D.4}$$

where the scalars h, h' coupled to the two U(1)s now have charge q. The contribution to the 'VEV' c' induced change in the coupling g relative to g' which arises from the 1-loop vacuum polarization diagram built from two scalar-scalar-gauge boson vertices is diagrammatically expressed as:

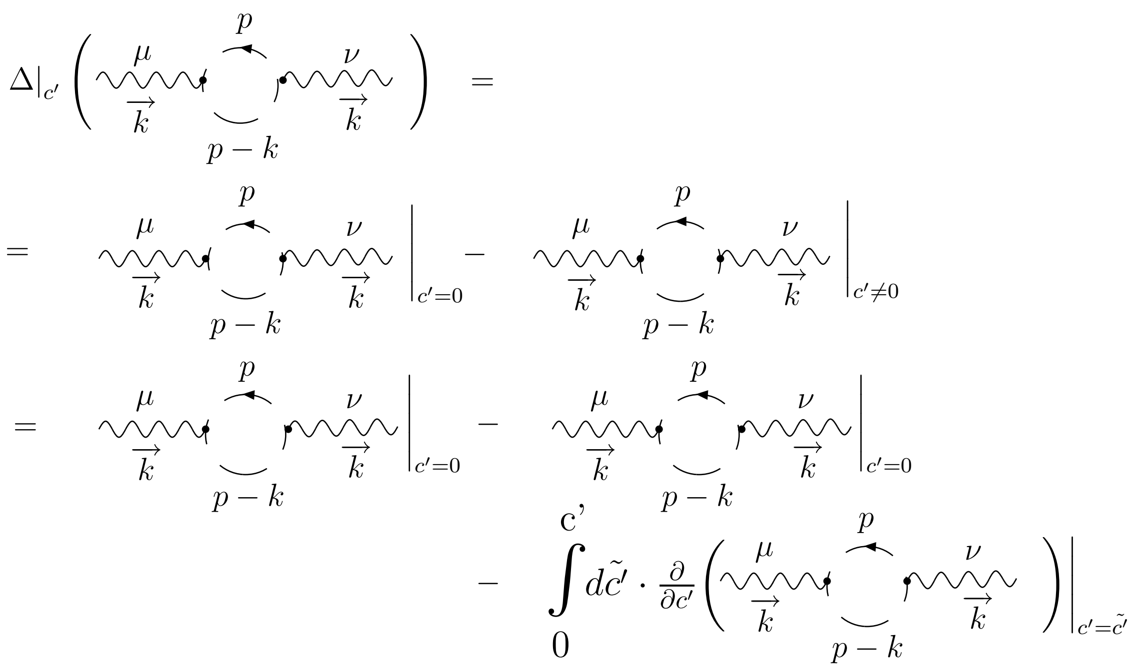

Figure D.1

Then to calculate the change the corresponding contribution of the 'bubble' vacuum polarization diagram of this scalar QED has to be added. The full 1-loop contribution of e.g. h to the photon propagator in DR one can take from Eq. (A.2) where, however, one now has to plug in:

$$m_n = \frac{n}{R_6} + c' \ \ , \ d = 5 - \epsilon \ \ . \tag{D.5}$$

From the above diagrammatical argument the induced splitting of the gauge couplings arises as the difference between the $U(1)$- and $U(1)'$-photon propagators at vanishing external momentum k in presence of the Wilson line. Using Eq.s (A.12) and (A.13) to calculate the loop integrals, this splitting is therefore given as

$$\begin{aligned}
\Delta\Pi_{(1)}\left(k=0\right) &= \int_0^{c'} d\tilde{c}' \sum_n \frac{\partial}{\partial c'} \Pi_{(1)}\left(k^2=0\right)\big|_{c'=\tilde{c}'} \\
&= g^2 \frac{q^2}{48\pi^2} \frac{1}{R_6} \int_0^c d\tilde{c} \sum_n \frac{n+\tilde{c}}{|n+\tilde{c}|^{1+\epsilon}} \ \ , \ c = R_6 c' \\
&\xrightarrow[\epsilon\to 0]{} \frac{g^2}{48\pi^2} \frac{1}{R_6} \int_0^c d\tilde{c} \left(\frac{\tilde{c}}{|\tilde{c}|} - 2\tilde{c} \right) \\
&= g^2 \frac{q^2}{48\pi^2} \frac{1}{R_6} |c|(1-|c|) \ \ .
\end{aligned} \tag{D.6}$$

This result is just a special case for the case of one compact dimension of the more general threshold formulae in presence of a Wilson line valid on a general S^1 or T^2 that have been derived in [45]. If the general 6d expressions given there are reduced to the case of the 6-direction compactified on an S^1 and the above $U(1) \times U(1)'$-'GUT' one finds the above result.

From Eq. (D.6) one derives the 4d gauge coupling at the scale R_5^{-1} as

$$\alpha_{4d}^{-1}(R_5^{-1}) = \alpha_{4d}^{-1} - \frac{q^2}{6} \frac{R_5}{R_6} |c|(1-|c|) \ \ . \tag{D.7}$$

In the limit of $R_5^{-1} << A_6^{(0)} << R_6^{-1}$ which is $c << 1$ this reduces to

$$\alpha_{4d}^{-1}(R_5^{-1})\big|_{c<<1} = \alpha_{4d}^{-1} - \frac{q^2}{6} R_5 \left| A_6^{(0)} \right| \tag{D.8}$$

which - noting the $A_6^{(0)} = \phi$ in the 5d theory - is precisely the 5d result.

The same $U(1)$-result could have been obtained directly from Eq. (3.8). One just has to replace m by a properly DR regulated sum over the KK masses

$$
\begin{aligned}
\delta\left(\frac{1}{g_5^2}\right) &= -\frac{q^2}{8\pi^2}\sum_n \left|\frac{n}{R_6}+c'\right|^{1-\epsilon} \\
&= -\frac{q^2}{8\pi^2}\int_0^{c'} d\tilde{c}'\sum_n \frac{\partial}{\partial c'}\left|\frac{n}{R_6}+c'\right|^{1-\epsilon}\Bigg|_{c'=\tilde{c}'} - \underbrace{\frac{q^2}{8\pi^2}\sum_n\left|\frac{n}{R_6}\right|^{1-\epsilon}}_{\text{universal infinite const.}} \\
&= -\frac{q^2}{8\pi^2}\frac{1}{R_6}\int_0^{c} d\tilde{c}\sum_n \frac{n+\tilde{c}}{|n+\tilde{c}|^{1+\epsilon}} \\
&= -\frac{q^2}{8\pi^2}\frac{1}{R_6}\int_0^{c} d\tilde{c}\left\{\epsilon(\tilde{c})+\sum_{n=1}^{\infty} n^{-\epsilon}\left[(1+\tilde{c}/n)^{-\epsilon}-(1-\tilde{c}/n)^{-\epsilon}\right]\right\} \\
&= -\frac{q^2}{8\pi^2}\frac{1}{R_6}|c|(1-|c|) \qquad \text{(D.9)}
\end{aligned}
$$

where $\epsilon(x)$ is the sign function. The infinite constant is universal and has therefore been left out as a renormalization constant. In both, Eq.s (D.6) and (D.9), properties of the ζ-function [125] - the familiar relation

$$
\lim_{\epsilon\to 0}\epsilon\sum_{n=1}^{\infty} n^{-1-\epsilon} = 1\,, \qquad \text{(D.10)}
$$

have been used to evaluate the sum and a function $f(\epsilon) = 1 + \mathcal{O}(\epsilon)$ has been neglected which, however, does not change the result since the sum $\sum_n (n+\tilde{c})/\left|n+\tilde{c}\right|^{1+\epsilon}$ is convergent anyway. The difference in the numerical prefactor between the Eq.s (3.8) and (D.6) arises because Eq. (3.8) is derived for a 5d hypermultiplet of charge q. Compared to the single complex scalar of charge q used to derive Eq. (D.6) this gives an additional factor $6 = 8_{\text{5d fermion}} - 2_{\text{2 compl. scalars}}$.

Therefore, the non-abelian result in 6d can be obtained by just taking the full 5d result Eq. (3.7) and then replacing

$$
\left|\phi^i\right| \longrightarrow \frac{1}{R_6}\left|R_6 A_6^{(0),i}\right| \longrightarrow \frac{1}{R_6}\left|R_6 A_6^{(0),i}\right|\left(1-\left|R_6 A_6^{(0),i}\right|\right) \quad . \qquad \text{(D.11)}
$$

Note, however, the absence of a Chern-Simons term and bulk mass terms in 6d. In 5d, bulk mass terms can be present for gauged matter. In 6d, however, as this is shown in Appendix E, the structure of spinor bilinears leads to the absence of any form of gauged matter besides $\mathcal{N}=4$ which due to its vanishing beta function does not contribute to the gauge couplings at any loop order. One arrives immediately at

$$\begin{aligned}
\Delta\left(\frac{1}{g_{5,i}^2}\right) &= \frac{1}{8\pi^2 R_6}\left[\sum_\alpha \alpha_i^2 \left|R_6\alpha_j A_6^{(0),j}\right|\left(1-\left|R_6\alpha_j A_6^{(0),j}\right|\right)\right. \\
&\quad \left. -\sum_f\sum_\lambda \lambda_i^2 \left|R_6\lambda_j A_6^{(0),j}\right|\left(1-\left|R_6\lambda_j A_6^{(0),j}\right|\right)\right] \\
&= \frac{1}{8\pi^2 R_6}\left[\sum_\alpha \alpha_i^2 \left|\alpha_j c^j\right|\left(1-\left|\alpha_j c^j\right|\right)-\sum_f\sum_\lambda \lambda_i^2\left|\lambda_j c^j\right|\left(1-\left|\lambda_j c^j\right|\right)\right] \\
&\quad \text{with: } c^j = R_6 A_6^{(0),j}\ . \qquad\qquad \text{(D.12)}
\end{aligned}$$

Thus, the corrections at the scale R_5^{-1} to the 4d coupling of the i^{th} subgroup $\alpha_{4d,i}^{-1}$ with the 5^{th} dimension compactified on an S^1/Z_2 is given by

$$\Delta\alpha_{4d,i}^{-1}\left(R_5^{-1}\right) = \frac{R_5}{2R_6}\left[\sum_\alpha \alpha_i^2\left|\alpha_j c^j\right|\left(1-\left|\alpha_j c^j\right|\right)-\sum_f\sum_\lambda \lambda_i^2\left|\lambda_j c^j\right|\left(1-\left|\lambda_j c^j\right|\right)\right] \quad \text{(D.13)}$$

This result describes the complete A_6-induced 1-loop correction to the gauge couplings in a 6d supersymmetric GUT once the massless sector of the theory is specified.

Before closing, a comment on the regularization independence of the above result is in place. In fact, returning to the level of actual loop integrations, the calculation can be performed without ever introducing a regularization. It is clear that the desired A_6 dependence of the gauge coupling can be extracted from the c dependence of a sum of 5d one-loop integrals, conveniently written as the integral of a sum, of the form

$$\begin{aligned}
I(c) &= \int \frac{d^5k}{(2\pi)^5}\sum_{n=-\infty}^{+\infty}\frac{1}{\left[k^2+(n+c)^2\right]^2} = \int\frac{d^5k}{(2\pi)^5}\sum_{n=-\infty}^{+\infty}\left\{-\frac{\partial}{\partial k^2}\left(\frac{1}{k^2+(n+c)^2}\right)\right\} \quad \text{(D.14)} \\
&= \int\frac{d^5k}{(2\pi)^5}\left\{-\frac{\partial}{\partial|k|^2}\left(\frac{1}{|k|}\cdot\frac{\pi\,\sinh(2\pi|k|)}{\cosh(2\pi|k|)-\cos(2\pi c)}\right)\right\} = -\frac{c(1-c)}{16\pi^2}+\{c\text{-indep.}\}\,.
\end{aligned}$$

Here the focus was on the simplest scalar integral appearing in the detailed calculation, rescaling the 5-momentum according to $k \to k/R_6$ and again c' to $c = R_6 c'$, and suppressing an overall A_6-independent factor. Thus, all one needs is $I'(c)$, which is finite simply because the first derivative with respect to c of the integrand in Eq. (D.14) falls exponentially for $|k| \to \infty$. Here *Mathematica* [126] has been used for evaluating the sum and the integral. Note also that dropping all higher KK modes (i.e., restricting the sum to $n = 0$) corresponds to the replacement $c(1-c) \to c$ in the final answer.

Performing the sum before the (in general divergent) 5d loop integration is crucial because in this way one is sure to respect the non-locality of the Wilson line effect in the 6d theory. This non-locality is the reason for finiteness. Regularization is just useful for finding the explicit result in a somewhat simpler way, not necessary at the conceptual level. Of course, it has to respect the non-local structure of the Wilson line wrapping the S^1. Clearly, dimensional regularization in the 5 non-compact dimensions satisfies this requirement. All relevant loop corrections could have been reduced to the form of Eq. (D.14) to arrive at the result. However, in this thesis the emphasis was on the dimensional regularization approach as the simplest way to obtain the answer directly from the known 5d formula.

Appendix E

Mass terms in 6d

E.1 spinor bilinears and gauge invariance in 6d

This Appendix will analyze the structure of mass terms of 6d gauged spinors. For that purpose it is necessary to give a short review of spinor bilinears in 6d along the lines of [122]. Some basic notions about the Dirac algebra in d dimensions are in order to begin with. Since besides the usual $\Gamma^m,\ m = 0 \ldots d-1$ also $(\Gamma^m)^*$, $(\Gamma^m)^T$ and $(\Gamma^m)^\dagger$ satisfy the Dirac algebra, one can find matrices B_1, C_1 and D_1 such that

$$\begin{aligned} (\Gamma^m)^* &= -B_1 \Gamma^m B_1^{-1} \\ (\Gamma^m)^T &= -C_1 \Gamma^m B_1^{-1} \\ (\Gamma^m)^\dagger &= -D_1 \Gamma^m B_1^{-1} \end{aligned} \tag{E.1}$$

which for d even and

$$\eta^{mn} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{pmatrix} \tag{E.2}$$

have the properties

$$\begin{aligned} C_1 &= B_1 D_1\ ,\quad D_1 = \Gamma^0 \\ C_1 &= \delta_1 C_1^T\ ,\quad B_1^* B_1 = -\delta_1 \\ \delta_1 &= \begin{cases} +1\ , & d = 6, 8 \bmod 8 \\ -1\ , & d = 2, 4 \bmod 8 \end{cases} \quad . \end{aligned} \tag{E.3}$$

Next one defines charge conjugation as

$$\psi^c := B_1^{-1} \psi^* =: \mathcal{C}\psi \quad . \tag{E.4}$$

ψ^c transforms the same way as ψ under Lorentz transformations. For the case $d = 6$ Eq. (E.3) implies, firstly, that a γ^5-like matrix

$$\bar{\Gamma} = \det(\eta^{mn}) \cdot \Gamma^0 \cdots \Gamma^{d-1} \tag{E.5}$$

with $\bar{\Gamma}^2 = 1$ and thus Weyl spinors $\psi_\pm$: $\bar{\Gamma}\psi_\pm = \pm\psi_\pm$ exist. Secondly, $B_1^* B_1 = -1$ which implies via $\mathcal{C}^2 \neq 1$ the non-existence of Majorana spinors. And thirdly, Weyl spinors in 6d are self-conjugate:

$$\left[\mathcal{C}, \bar{\Gamma}\right] = 0 \ . \tag{E.6}$$

From here one can now analyze the properties of a general bilinear of two 6d Dirac spinors and later reduce it to the case of Weyl spinors. Such a bilinear in spinors may be written as

$$\chi^T A \psi \tag{E.7}$$

with A a $d \times d$ complex matrix. Demanding this to be a Lorentz invariant yields a constraint on A. Under infinitesimal Lorentz transformations

$$\delta\psi = i\epsilon_{mn}\Sigma^{mn} \cdot \psi \ , \quad \Sigma^{mn} = -\frac{1}{4}\left[\Gamma^m, \Gamma^n\right] \tag{E.8}$$

the variation of the bilinear to $\mathcal{O}(\epsilon^2)$ is given as

$$\delta\left(\chi^T A\psi\right) = i\epsilon_{mn}\chi^T \cdot \left[\left(\Sigma^{mn}\right)^T A + A\Sigma^{mn}\right] \cdot \psi \tag{E.9}$$

which for invariance has to vanish. Using Eq. (E.1) this yields the constraint on A as

$$A^{-1}C_1\Sigma^{mn}C_1^{-1}A = \Sigma^{mn} \ . \tag{E.10}$$

One obvious solution is $A = C_1$. The general solution then can be written as $A = C_1 A'$. Next, the totally anti-symmetrized gamma matrices

$$\tilde{\Gamma}^m := \Gamma^{[n_1}\Gamma^{n_2} \cdots \Gamma^{n_m]} \tag{E.11}$$

form a basis of the $d \times d$ complex matrices. Thus, every A' can be written as

$$A' = \sum_{m=0}^{d} c_m \tilde{\Gamma}^m \ . \tag{E.12}$$

Now, there are just two invariant tensors that can be used for contractions of these Lorentz tensors:

$$\eta^{mn} \ , \quad \epsilon^{m_1 \cdots m_d} \ . \tag{E.13}$$

Contraction with η^{mn} yields zero due to the antisymmetry of the $\tilde{\Gamma}^m$. Then Lorentz scalars can be formed only with $\tilde{\Gamma}^0 = 1$ and $\epsilon^{m_1 \cdots m_d}\tilde{\Gamma}^d = \bar{\Gamma}$. As the result, just one scalar and one pseudo-scalar invariant

$$\mathcal{L}_s = \chi^T C_1 \psi \ , \quad \mathcal{L}_{ps} = \chi^T C_1 \bar{\Gamma}\psi \tag{E.14}$$

can be formed from χ and ψ. $\mathcal{L}_{ps}$, however, is not really independent, since on 6d Weyl spinors $\bar{\Gamma}$ just takes the values ± 1 which reduces thus $\mathcal{L}_{ps}$ to $\mathcal{L}_s$ if they are expressed in Weyl spinors. For each Weyl spinor now there exists its charge conjugate with the same chirality. Thus in terms of the two spinors of opposite chirality $\psi_\pm$ within ψ one can now write down four invariants

$$\begin{aligned} &\psi_+^T C_1 \psi_+ \quad , \quad (\psi_+^c)^T C_1 \psi_+ \\ &\psi_+^T C_1 \psi_- \quad , \quad (\psi_+^c)^T C_1 \psi_- \ . \end{aligned} \tag{E.15}$$

From Eq.s (E.1), (E.6) and $\psi_\pm = 1/2 \cdot (1 \pm \bar{\Gamma})\psi$ the first line evaluates to

$$\begin{aligned} \psi_+^T C_1 \psi_+ &= \psi^T C_1 \frac{1-\bar{\Gamma}}{2}\frac{1+\bar{\Gamma}}{2}\psi = 0 \\ (\psi_+^c)^T C_1 \psi_+ &= (\psi^c)^T C_1 \frac{1-\bar{\Gamma}}{2}\frac{1+\bar{\Gamma}}{2}\psi = 0 \end{aligned} \tag{E.16}$$

which implies that non-vanishing invariants in 6d can only be formed by Weyl spinors of opposite chirality (second line above). These can be conveniently rewritten as

$$\psi_+^T C_1 \psi_- = \overline{\psi_+^c}\psi_- \ , \ (\psi_+^c)^T C_1 \psi_- = \overline{\psi_+}\psi_- \ . \tag{E.17}$$

Thus, the most general complex lagrangian of bilinears of two 6d Weyl spinors of opposite chirality

$$\chi_+ = \frac{1+\bar{\Gamma}}{2}\chi \ , \ \psi_- = \frac{1-\bar{\Gamma}}{2}\psi \tag{E.18}$$

is given by

$$\mathcal{L}_m = a \cdot \overline{\chi_+}\psi_- + b \cdot \overline{\chi_+^c}\psi_- + c \cdot \overline{\chi_+}\psi_-^c + d \cdot \overline{\chi_+^c}\psi_-^c \ . \tag{E.19}$$

In 6d, Eq.s (E.1) and (E.3) imply

$$(\overline{\psi}\chi)^\dagger = (\overline{\chi}\psi)^* = \overline{\chi^c}\psi^c \tag{E.20}$$

which means that $\mathcal{L}_m$ can be made real by the choice $d = \bar{a}$ and $c = \bar{b}$. A subsequent redefinition of the phases of the spinors then allows one to rewrite this as

$$\mathcal{L}_m = m \cdot \overline{\chi_+}\psi_- + m' \cdot \overline{\chi_+^c}\psi_- + \text{h.c. with } m, m' \in \mathbb{R} \ . \tag{E.21}$$

This is the most general real mass term lagrangian for 6d Weyl spinors. Now in the last step think of both Weyl spinors as being gauged under some $U(1)$. Without loss of generality, choose them to be equally charged under the $U(1)$. Then $q(\overline{\chi_+}\psi_-) = 0$ while $q(\overline{\chi_+^c}\psi_-) \neq 0$ which implies that for gauged Weyl spinors just one mass term remains

$$\mathcal{L}_m = m \cdot \overline{\chi_+}\psi_- + \text{h.c.} \ . \tag{E.22}$$

E.2 Mass terms in 6d SUSY

The next step will be to study possible mass terms in 6d SYM allowed by Eq. (E.22). Begin with the action of free massless right- and left-handed hypermultiplets (H_R, H_R^c) and (H_L, H_L^c) in 6d [62]:

$$\begin{aligned}
\mathcal{L}_{matter} &= \left(\overline{H_L^c} H_L^c + \overline{H_L} H_L + \overline{H_R^c} H_R^c + \overline{H_R} H_R\right)\Big|_{\theta^2\bar{\theta}^2} \\
&\quad + \left[\left(H_L^c \partial H_L + H_R^c \bar{\partial} H_R\right)\Big|_{\theta^2} + \text{h.c.}\right] \\
&\quad \text{with: } \partial = \partial_z = \partial_5 - i\partial_6 \ , \quad z = x^5 + ix^6
\end{aligned} \tag{E.23}$$

$SO(2) \subset SO(5,1)$-transformations of z: $z \longrightarrow e^{i\theta} z$ have the 56-derivative transforming as $\partial \longrightarrow e^{-i\theta}\partial$. Now think of gauging these hypermultiplets under a $U(1)$ with an, e.g., right-handed vector multiplet (V_R, Σ_R). Under a gauge transformation it transforms as

$$\begin{aligned}
V_R &\longrightarrow V_R + \Lambda_R + \overline{\Lambda_R} \\
\Sigma_R &\longrightarrow \Sigma_R + \sqrt{2}\partial\Lambda_R \ .
\end{aligned} \tag{E.24}$$

The transformation properties of Σ_R and ∂ require Σ_R to transform under the above $SO(2)$ the same way as ∂. Thus, Σ_R can form a covariant derivative only with ∂: $\partial \rightarrow D_R = \partial - \Sigma/\sqrt{2}$, but not with $\bar{\partial}$. In conclusion, a 6d vector of a given handedness can gauge only hypermultiplets of the opposite handedness.

The spinors contained in 6d vector and hypermultiplets are 6d Weyl spinors. Eq. (E.22) now dictates that possible mass terms must connect multiplets of opposite handedness among the gauged fields. Given a situation with a right-handed vector and a number of ungauged right- and gauged left-handed hypermultiplets this implies that mass terms can only link the left-handed hypermultiplets with the right-handed vector. This resides on the fact that mass terms linking hypermultiplets of different handedness are not gauge invariant because only the left-handed ones are gauged by the vector.

Now call the right-handed gaugino λ_R and the spinors of n left-handed hypermultiplets $\chi_{L,j}$. Then the mass term linking the spinors according to Eq. (E.22) has to be:

$$\begin{aligned}
\mathcal{L}_{6d,mass} &= \overline{\Phi} M \Phi \\
\Phi &= \begin{pmatrix} \lambda_R \\ \chi_{L,1} \\ \vdots \\ \chi_{L,n} \end{pmatrix} \ , \quad M = \begin{pmatrix} 0 & m_1 & \cdots & m_n \\ m_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ m_n & 0 & \cdots & 0 \end{pmatrix}
\end{aligned} \tag{E.25}$$

Diagonalization of this mass matrix yields two eigenvalues $\pm m$ and the other $n-1$ eigenvalues to be zero. Thus, the only possible mass term for gauged matter in 6d gives mass to the whole vector and exactly one (adjoint in the non-abelian case) gauged hypermultiplet with equal values, which, however, means that effectively a whole $\mathcal{N} = 4$ vector multiplet becomes massive. As a result, no 6d chiral gauged massive matter exists, the only massive gauged matter in 6d is $\mathcal{N} = 4$.

Appendix F

Moduli fields of Calabi-Yau compactifications to 4d

The geometric compactification moduli arise as follows: write the 10d metric in terms of the product geometry of a general Calabi-Yau compactification to flat 4d Minkowski space-time

$$ds_{10}^2 = \eta_{\mu\nu} dx^\mu dx^\nu + g_{\alpha\beta}(y) \cdot dy^\alpha dy^\beta \quad . \tag{F.1}$$

Here $g_{\alpha\beta}$ denotes the metric of the Calabi-Yau threefold and $\mu, \nu = 0 \ldots 3$ and $\alpha, \beta = 4 \ldots 9$ the 4d and compact 6d coordinate indices, respectively ($M, N = 0 \ldots 9$ are the 10d indices then).

Let the Calabi-Yau metric now fluctuate. Since a Calabi-Yau as a Kähler manifold carries a complex structure it is now convenient to change to complex internal coordinates $m, n = 1 \ldots 3$ and $\bar{m}, \bar{n} = 1 \ldots 3$. It is a well known fact [100] that the Ricci-flatness of Calabi-Yau threefolds (every Calabi-Yau, being Kähler, admits a hermitian metric $g_{m\bar{n}}$ for which its Ricci-tensor R^p_{mpn} vanishes) admits two classes of metric fluctuations which are non-trivial in the sense, that written in p-form language they are not exact: fluctuations corresponding to harmonic $(1,1)$-forms which preserve the hermitian form of the metric (therefore called Kähler moduli)

$$\delta g_{m\bar{n}} = v_a \omega^a_{m\bar{n}} \ , \ a = 1 \ldots h^{1,1} \ , \ v_a \in \mathbb{R} \tag{F.2}$$

and harmonic $(2,1)$-form fluctuations (which leave the hermiticity of the metric and thus change the complex structure of the Calabi-Yau, therefore called complex structure moduli)

$$\delta g_{\bar{m}\bar{n}} = z_A \eta^A_{\bar{m}\bar{n}} \ , \ A = 1 \ldots h^{1,2} \ , \ z_A \in \mathbb{C} \quad . \tag{F.3}$$

Here the $\eta^A_{\bar{m}\bar{n}}$ are in one to one correspondence with the harmonic $(2,1)$-forms χ^A on the Calabi-Yau

$$\eta^A_{\bar{m}\bar{n}} = \chi^A_{pq\bar{n}} \frac{\Omega^{pq}_{\bar{m}}}{||\Omega||^2} \tag{F.4}$$

Ω denotes the unique harmonic and holomorphic 3-form characteristic for each Calabi-Yau. The number of independent $(1,1)$- and $(2,1)$-forms (which equals the dimension of the corresponding Dolbeault cohomologies $H^{1,1}_{\bar\partial}$ and $H^{2,1}_{\bar\partial} = H^{1,2}_{\bar\partial}$, respectively) is given by the Hodge numbers $h^{1,1}$ and $h^{2,1} = h^{1,2}$, respectively. The Hodge dual in the compact directions induces the split into imaginary self-dual (ISD) forms $*_6\omega^+ = i\omega^+$ and anti-ISD forms $*_6\omega^- = -i\omega^-$. The $(1,2)$-form $\bar\chi^{\bar A}$ and the holomorphic $(3,0)$-form Ω are anti-ISD and their $(2,1)$- and $(0,3)$-form partners χ^A and $\bar\Omega$ are ISD [16].

Next, apply the factorization of the metric to the 10d Ricci scalar

$$
\begin{aligned}
R &= g^{MN} R_{MN} = \eta^{\mu\nu} R^P_{\mu P\nu} + g^{mn} R^P_{mPn} \\
&= \underbrace{\eta^{\mu\nu} R^\rho_{\mu\rho\nu}}_{R_4} + \eta^{\mu\nu} R^m_{\mu m\nu} + g^{mn}\left(R^\mu_{m\mu n} + R^p_{mpn}\right)
\end{aligned} \tag{F.5}
$$

and consider terms

$$
\Gamma^S_{m\mu}\Gamma^\mu_{Sn} = \Gamma^\sigma_{m\mu}\Gamma^\mu_{\sigma n} + \Gamma^p_{m\mu}\Gamma^\mu_{pn} \subset R^\mu_{m\mu n} \quad . \tag{F.6}
$$

By virtue of the product structure of the metric this simplifies using

$$
\Gamma^p_{m\mu} = \frac{g^{pq}}{2}\,\partial_\mu g_{mq} \quad , \quad \Gamma^\mu_{pn} = -\frac{g^{\mu\sigma}}{2}\,\partial_\rho g_{np} \tag{F.7}
$$

into $R^\mu_{m\mu n}$ containing terms

$$
R^\mu_{m\mu n} \supset -\frac{g^{pq}}{4}\,\partial_\mu g_{mq}\partial^\mu g_{np} \quad . \tag{F.8}
$$

Note, that the Riemann tensor consists of terms $\Gamma^\cdot_{\cdot\cdot}\Gamma^\cdot_{\cdot\cdot}$ and $\partial_\cdot\Gamma^\cdot_{\cdot\cdot}$ which all contain two derivatives. Now the first term of Eq. (F.5) vanishes since the 4d metric is Minkowski space-time. The next two terms contain besides the pieces of Eq. (F.8) only pieces containing either $\partial_\nu g_{\mu m}$ or $\partial_m \eta_{\mu\nu}$ which both vanish because of the product structure of the metric and $\eta_{\mu\nu} = const.$ Thus, only the last term of Eq. (F.5) could possibly contribute terms in g_{mn}. However this term $R^p_{mpn} = R_{mn}$ is the Ricci tensor of the compact manifold which on a Calabi-Yau vanishes, too. This implies that Eq. (F.8) contains all terms in g_{mn} including possible fluctuations since only fluctuations of the compact metric which respect the Ricci flatness of the Calabi-Yau will be studied [100].

Now write the compact part of the metric as

$$
g_{mn} = g^{(0)}_{mn} + \delta g_{mn} \quad , \tag{F.9}
$$

plug this into Eq. (F.8), use the product structure of the metric and expand the result to $\mathcal{O}(\omega^2, \eta^2)$. One arrives at an expression

$$
\begin{aligned}
g^{mn} R^\mu_{m\mu n} &= -\frac{C}{4}\, g^{(0)mn} g^{(0)\bar p\bar q} \omega^a_{m\bar q}\omega^b_{n\bar p}\partial_\mu v_a \partial^\mu v_b \\
&\quad -\frac{C}{4}\, g^{(0)m\bar n} g^{(0)p\bar q} \eta^A_{mq}\bar\eta^{\bar B}_{\bar n\bar p}\partial_\mu z_A \partial^\mu \bar z_{\bar B} \quad .
\end{aligned} \tag{F.10}
$$

C is a constant that accounts for the fact that there are several terms in $R^{\mu}_{m\mu n}$ which have the structure of Eq. (F.8). This implies that the Einstein-Hilbert term of the 10d action upon Calabi-Yau compactification is

$$\begin{aligned} R &= R_4 - \frac{C}{4}\, g^{(0)mn} g^{(0)\bar{p}\bar{q}} \omega^a_{m\bar{q}} \omega^b_{n\bar{p}} \partial_\mu v_a \partial^\mu v_b \\ &\quad - \frac{C}{4}\, g^{(0)m\bar{n}} g^{(0)p\bar{q}} \eta^A_{mq} \bar{\eta}^{\bar{B}}_{\bar{n}\bar{p}} \partial_\mu z_A \partial^\mu \bar{z}_{\bar{B}} \end{aligned} \tag{F.11}$$

that is, the geometric fluctuations of the Calabi-Yau compactification, Kähler and complex structure moduli, become free massless scalar fields in the 4d low energy effective theory, which is why they are called moduli fields.

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