Determining the electron charge to mass ratio (e/m)

1 Aim of the experiment

The purpose of this experiment is to measure the ratio of charge to mass (e/m) for the electron.

An electron beam in an evacuated glass chamber is deflected on a circular path by means of a uniform external magnetic field. The speed of the electrons is determined by the accelerating potential applied to the anode of the electron gun and the radius of the circular path is given by the magnetic force on a moving charged particle. The electron charge to mass quotient can be determined from the measured values of the accelerating voltage, the magnetic field and the radius of the circular path.

Thomson first performed this experiment in 1897 and discovered the first subatomic particle, the electron. 15 years later Millikan measured the charge of the electron precisely. This value together with e/m enables the mass of the electron to be determined.

2 Basic Theory

The electrons are produced by thermionic emission from a hot wire cathode and are accelerated by the voltage V_B applied to the anode of the electron gun. In this process the electrons acquire a kinetic energy E_{kin} given by:

$$E_{kin} = \frac{1}{2} m \cdot v^2 = e \cdot U_B = E_{pot} \quad . \tag{1}$$

Hence, the velocity v of the electrons leaving the electron gun is given by:

$$v = \sqrt{\frac{2 \; e \cdot U_B}{m}} \quad . \tag{2}$$

When the electron beam enters the magnetic field it is deflected by the *Lorentz force*

$$\vec{F}_L = q \cdot \vec{v} \times \vec{B} \quad . \tag{3}$$

The magnetic field exerts a force on any moving charge q that is present in the field. The direction of the Lorentz force force is perpendicular to the velocity $\vec{v}$ and the magnetic field $\vec{B}$ and is always perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Hence, the speed of the electron remains constant in the magnetic field. For negative charges, the direction of the force is given by the left-hand rule.

If we use the component of the $\vec{B}$-field which is perpendicular to the direction of the velocity $\vec{v}$ we can simplify the equation and inserting e for the electron charge we have:

$$F_L = e \cdot v \cdot B \quad . \tag{4}$$

The electrons move under the influence of a constant-magnitude force that is always at right angles to the velocity of the particle. Under these circumstances the path of the electrons is a circle of radius r, traced out with constant speed v. The centripetal acceleration is v^2/r and the only force acting is the magnetic force so from Newton's second law,

$$F = \frac{m \cdot v^2}{r} = e \cdot v \cdot B \quad . \tag{5}$$

Inserting equation (2) for the speed of the electrons we obtain,

$$\frac{e}{m} = \frac{2 \ U_{B}}{B^2 \cdot r^2} \ .$$

(6)

The electron charge to mass quotient e/m can be determined by measuring the accelerating voltage U_B, the magnetic field B (which depends on the current I_{Sp} flowing in the magnetic field coils) and the radius r of the circular path of the electron beam.

3 Experimental Setup

The key component in this experiment is a special glass fine-beam electron tube (Fadenstrahlrohr LEIFI-Physik) also known as a "Teltron electron beam tube" (previously manufactured by Teltron Inc). The evacuated glass tube contains a small amount of an inert gas which lights up upon electron impact and reveals the position of the electron beam. In order to measure the radius of the circles produced by the external magnetic there are four fluorescent rods mounted at distances of 4 cm, 6 cm, 8 cm and 10 cm from the end of the electron gun.

The electrons released from the thermionic cathode are accelerated by the potential difference U_B between the cathode and the anode and this is the sum of the voltage between cathode and grid (-50... 0 V) and the anode voltage (0... + 250 V), between grid and anode.

The glass electron tube is mounted between the *Helmholtz coils* which produce the magnetic field. The coils are separated by a distance equal to the coil radius and the magnetic field in the central region is very uniform. The radius of the coils is $R = 0.2$ m and each coil has $n = 154$ windings.
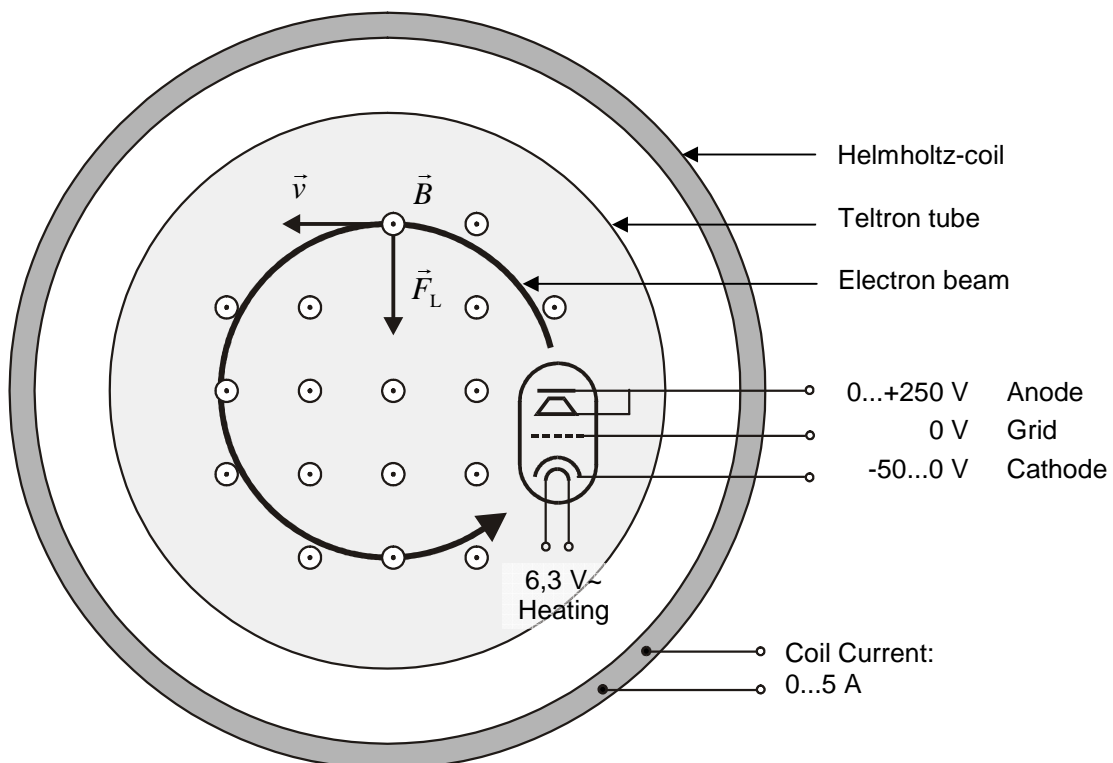

Fig. 1: Schematic of the experimental setup

The strength of magnetic field can be varied by adjusting the coil current (0… 5 A) provided by an external DC power supply. The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a conducting of radius a, carrying a current I_{Sp}. The field depends on the distance along the axis from the center of the loop. If there are N loops, the field is multiplied by N.

$$d\vec{B} = \frac{\mu_0}{4\pi} N \cdot I_{Sp} \cdot \frac{d\vec{l} \times \vec{r}}{r^3}$$

(7)

The magnetic field B in the center on the axis between the Helmholtz coils is given by:

$$B = \mu_0 \cdot \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{I_{\text{Sp}} \cdot N}{R} = I_{\text{Sp}} \cdot 6{,}926 \cdot 10^{-4} \ \frac{\text{T}}{\text{A}} \ . \tag{8}$$

$\mu_0 = 4\pi \cdot 10^{-7}$ T·m/A.

The coil current and the accelerating voltage are measured using multimeters. The circular path of the electron beam can be observed in a darkened room, provided the direction of the magnetic field is correct.

4 Uncertainty of Measurement Results

The objective of the error analysis is to assess of the accuracy of the result and to assess the impact of various types of errors. In this experiment it is simply not possible to determine the values of U_{B}, I_{Sp} and r with arbitrarily high precision using the equipment provided. Hence, it is important to know how a measurement error or the statistical distribution of multiple individual measurements modifies the overall result. Error propagation can provide an answer: The error $\Delta f(x,y,z)$ of a function $f(x, y, z)$, which depends on the values of x, y and z (which all have errors) is, (see script error calculation)

$$\Delta f(x,y,z) = \sqrt{\left(\frac{\partial f}{\partial x} \cdot \Delta x\right)^2 + \left(\frac{\partial f}{\partial y} \cdot \Delta y\right)^2 + \left(\frac{\partial f}{\partial z} \cdot \Delta z\right)^2} \ . \tag{9}$$

Where $\partial f/\partial x$ is the partial derivative of the function $f(x, y, z)$ with respect to x, with all other variables constant. The derivative describes how much the function f changes for a small change in x. In this sense, the differential coefficients in equation (10) can be understood as weighting factors, which make it possible to evaluate the individual errors Δx, Δy, Δz of the variables x, y, z depending on how strongly they influence the final result.

In this experiment the function is:

$$f(x,y,z) = \frac{e}{m} = f(U_{\text{B}}, B, r) = \frac{2 \ U_{\text{B}}}{B^2 \cdot r^2} \ . \tag{10}$$

The error propagation is:

$$\Delta\left(\frac{e}{m}\right) = \sqrt{\left(\frac{\partial\left(\frac{e}{m}\right)}{\partial U_{\text{B}}} \cdot \Delta U_{\text{B}}\right)^2 + \left(\frac{\partial\left(\frac{e}{m}\right)}{\partial B} \cdot \Delta B\right)^2 + \left(\frac{\partial\left(\frac{e}{m}\right)}{\partial r} \cdot \Delta r\right)^2} \ , \tag{11}$$

$$= \sqrt{\left(\frac{2}{B^2 \cdot r^2} \cdot \Delta U_{\text{B}}\right)^2 + \left(\frac{4U_{\text{B}}}{B^3 \cdot r^2} \cdot \Delta B\right)^2 + \left(\frac{4U_{\text{B}}}{B^2 \cdot r^3} \cdot \Delta r\right)^2} \ , \tag{12}$$

$$= 2\frac{e}{m} \cdot \sqrt{\left(\frac{\Delta U_{\text{B}}}{2U_{\text{B}}}\right)^2 + \left(\frac{\Delta B}{B}\right)^2 + \left(\frac{\Delta r}{r}\right)^2} \ . \tag{13}$$

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It should be noted that *B* is not measured directly since we only measure the coil current I_{SP}. A similar error analysis has to be carried out to determine the error in B from the error in I_{SP}.

Equation (13) can be used to calculate the error in e/m resulting from the uncertainties in the values of U_B, *B,* and *r*.

5 Tasks to be performed

The fine beam electron tube and the Helmholtz coils should be connected as indicated on page 2 with measuring instruments for voltage and current at appropriate places.

First set the accelerating voltage U_B to a constant value and then vary the magnetic field B until the radius of the orbit of the electron beam corresponds to the distance defined by one of the fluorescent rods. The rods are located at distances of 2×*r* = 4 cm, 6 cm, 8 cm and 10 cm from the electron source. For each of the four rods, you should record a series of measurements varying the anode voltage in 11 steps from 300 V and 200 V (in 10 V steps) and adjusting the coil current accordingly. The ratio of e/m for the electron should then be calculated each of the sets of values of U_B, I_{SP} and *r*. Finally the average value of e/m should be calculated.

When comparing your result to values in the literature, it is important to understand the measurement errors in your data. Only then it is possible to ascertain whether systematic errors have occurred or whether the differences are due to unavoidable random measurement inaccuracies.

The individual measurement errors in ΔU_B, ΔI_{SP} and Δr must be estimated reasonably.

For each individual result for *e/m* the error $\Delta(e/m)$ must be calculated using eq. (13). Finally, using all of the individual values of *e/m* the average value and the standard deviation must be calculated.

The results should be compared to values from the literature and the possible causes of differences should be discussed.

The 44 values obtained for *e/m* should be plotted against the accelerating voltage so that they can be distinguished by the corresponding radii of the orbits. In addition, the average value and the current most accurate value for e/m should be plotted. It should be noted that the electrons can have quite high speeds so the electron mass can increase (relativistic effect). Systematic differences in the average values of e/m as a function of the radius of the electron orbits should be discussed.

Reference:

National Institute for Standards and Technology NIST,
http://physics.nist.gov/cuu/Constants/index.html:

$$\frac{e}{m} = \left(-1{,}758\,820\,088 \pm 0{,}000\,000\,039\right) \cdot 10^{11} \frac{C}{kg}$$