!! Regarding the information about figures (Fig.XY) please have a look at the german version !!

Spektroscopy

1. Introduction (under construction)

2. Prism and Grid Spectrometers – Basics of Construction and Operation

The experiment can be conducted with one of two types of spectrometer in which the central component is either a prism or grid, the properties of which will affect the selection of λ. These components undergo changes when exposed to continuous light in the direction of propagation in two cases; in the prism due to a two-stage opening and in the grid due to diffraction. It must be noted that the angle of diffraction σ is dependent on λ for both cases, as seen in the following figures.

2.1. Selection of λ in the Central Components of the Spectrometer

The function of prisms and grids are explained in the following three figures.

Fig. 1: Diffraction in a prism for various wavelengths. The primary beam shown is only representative of one of many rays emitted. Note that $\lambda_1<\lambda_2$ but $\sigma_1>\sigma_2$.

Fig. 2: In the grid, the primary beam may be split even when the wavelength is only represented by one value – but must not necessarily be depending on the values of (*). The **order of diffraction** n denotes the number of diffracted rays as shown. The primary direction (n=0) is also present after diffraction, unlike prisms where this is not always the case. The primary beam is again only representative of many beams. The **angle of deflection** δ is calculated and depicted from the numerical example. Note that σ is asymmetrical, light enters on a skewed angle because the grid is tilted by an **angle of incidence** α.

Fig. 3: Grid refraction from three different values for λ. A selection of λ for a spectrum of second order (n = -2) is shown. The faded refractions are for iterations of λ_1 when $n \neq -2$, be aware that the complete spectrum is many times more than what is depicted here and overlapping is also possible. The angle of deflection δ can be calculated and constructed from the numerical example. Note in the case of a prism the opposite arrangement of λ to δ values: $\lambda_1 > \lambda_2 > \lambda_3$ with $\delta_1 > \delta_2 > \delta_3$.

2.2. About the Surrounding of Central Components

The other components, which along with a prism or grid, are vital to the construction of a spectrometer. They are also the same in both types of spectrometer. We can already notice the importance of these components by observation of a wall illuminated with white light through only a grid, in which we see no colour phenomena. They can however be seen when in similar fashion a white line on a black background is illuminated (try it out). What the naked grid lacks is the appropriate type of illumination. This is where an upstream collimator comes into play. This can be understood when considering that one beam entering the grid from a group of parallel rays will continue to be parallel after leaving the medium, which also holds true for prisms. The downstream optics (telescope) can thus be set to ∞ and its optical axis of course must be aligned in the new direction of refracted light due to the prism or grid. This described arrangement of the collimated light field as it passes through the prism or grid (called the **telescopic beam path**) is the simplest possible case in order to realise an undisturbed picture of the entrance slit. The spectrometer has in both variants (prism, grid) three members in which the light successively traverses. The first comprises of the collimator, after which it goes through the main components before passing through the optics.

!! Regarding the information about figures (Fig.XY) please have a look at the german version !!

Note: The combination of the collimator and the telescope makes up an **imaging system** which then must be sharply focused on the entrance slit. Under the influence of the prism or grid, several mutually shifted images can be seen in the field of view of the telescope, as many as there are different wavelengths represented by the incoming light.

2.3 The Collimator

Fig. 4: Construction of the collimator. The assembly is designed to provide **planar light waves**, a term often referred to as 'parallel beams' or 'collimated light'. Correct adjustment requires that $f = s$.

The lenses are only in the basic case simple converging (positive) lenses as seen in Fig. 4. Generally they are more complicated lens systems which aim to reduce the principle in single lenses and therefore the inevitable aberration caused by combining multiple lenses. As aberrations are not caused by poor lens design or manufacturing errors, complicated lens systems have only been mentioned in passing. Aberrations cannot be completely rectified in all lens systems. Such errors are fundamental and are not production-technical in nature. In any case, the collimators of the spectrometers in this experiment are involved in the imaging of the entrance slit (see the note on page 3). The chromatic aberration is of particular interest in this experiment as different colours of light appear here due to the different values of the wavelength λ. It must also be taken into account that the focal length depends on λ in the case that the lens is not sufficiently corrected in a chromatic sense. What to do in this situation becomes clear when the operation of the collimator is properly understood. This will be explained in the next section:

So how does the lens operate? Basically, as any converging lens would, or like the opposite of a magnifying glass. The slit, when set to be narrow, defines an approximate centre point for the passing light, the smaller the gap the more accurate. The slit simply masks all incoming light that would not have otherwise landed on the respected centre point. From here the light continues as a divergent beam bundle in the direction of the lens in order to bypass the collimated light (reverse magnifying glass). It should be made clear that collimated light is only made possible when the slit is exactly the focal length away from the lens, otherwise the output of light may still be divergent or already convergent, depending on whether $s > f$ or $s < f$ (see Fig. 4). This means that s must equal f. Such an adequate calibration must already be achieved before any spectrometric measurements can be taken. How is this done? It must first be considered that in observations for every change in λ, the slit in the collimator must again be properly positioned, as f depends on λ. This would indeed be ideal, but making many adjustments to achieve this becomes quickly tedious and inconvenient. A way to overcome this is to set the slit in the collimator for at least one λ in the middle of the spectral range (green). For this purpose, the intense yellow-green mercury line is taken. The compromise of having an incomplete spectrum throughout the telescopic beam path (collimated illumination of prism or grid) is justifiable, as there are measurement errors which far outweigh the former. However, it must hold that:

Note: Since the lenses of the collimator and telescope contain significant chromatic errors, it is vital to the spectroscopic measurements to refocus the slit image for every new λ value, while the (non-ideal) collimator is left unchanged. We then settle with the knowledge that the telescopic beam path is only approximately given in the visible spectrum, except for a line of reference for the chosen green value of λ.

The collimator will only have lived up to its name if $s = f$. This is now understood. In such an ideal case, the effect of the collimator can be described as forming an entrance slit to infinity, as shown on the left in Fig. 4. So it makes sense to use a telescope further down the path of light at the end of the spectrometer.

2.4 The Telescope

Fig. 5: Construction of the observation telescope. The ocular tube with crosshairs can be moved into the main tube by mechanical adjustment in order to properly focus the telescope. A very fine tuning is possible. If the ocular tube is moved it takes the eyepiece with it, so that the crosshairs remain fixed to the eyepiece. The eyepiece is otherwise not able to be adjusted within the ocular tube in order to keep the crosshairs in focus. Both parts slide together with friction, so that they remain stable and in line with one another.

In the ideal case of this experiment, the intermediate image in the telescope lays at a distance of exactly the focal length behind the lens. The telescope can then 'see' to ∞. **Fig. 5** shows a small variation to the ideal case, as in this case $a > f_2$. In the ideal case $a = f_2$. The telescope in **Fig. 5** can see an object some distance away, but not infinitely far. The incident light is therefore slightly divergent, so slight that the real intermediate image lies somewhat further from the lens. The shift $a - f_2$ perceives at least 1 mm to 25 m away from the lens. **Fig. 5** does not show the best possible case for the experiment as that would be the telescopic beam path explained on page 3. The diagram helps however with all possible scenarios. In making adjustments in accordance with **Chapter 2.6** easy to accomplish, the following points are good to remember:

$b = f_3$ should be used for people with emmetropic (normal functioning) vision who can see sharply while looking to ∞ without squinting. People who are short-sighted when not wearing glasses should take $b < f_3$ and far-sighted people $b > f_3$. Working without glasses at the telescope in this experiment won't result in measuring errors. This method of working is even more practical, but be aware that when changing experimenters, the crosshairs may need to be refocused. This method is otherwise one of the foremost principles. Part of the correct operation of the spectrometer is to always have the real intermediate image simultaneously in focus with the crosshairs. Their positions then both fall on the optical axis.

Note: To begin with, the crosshairs are put into focus through the shifting of the lens in the lens tube. This is done without a fine adjustment screw, just the friction based movement of the two parts. This is a basic step in setting up the experiment and should be exercised carefully. It needs only be performed once if the experimenter does not change. Afterwards, the ocular tube including the eye piece is focused onto the respective spectral lines by fine tuning in such a way that the spectral line and crosshairs both appear sharply. The crosshairs can be illuminated by a lamp through a small hole in the side of the ocular tube. Without experience, none of these steps will be easy tasks.

2.5 Surroundings of the Central Components (Diagram)

Fig. 6: In the construction of both types of spectrometer of this experiment: the diagram depicts the actual main components around the technical surroundings. The duty of the experimenter is to apply appropriate lighting, observation of the spectrum and to measure the angle of deflection δ for a given wavelength λ. A single λ is visible in the telescope as a spectral line. The actual main component, i.e. the prism or diffraction grid, is not shown in the picture, but their designated place is in the middle of the table. This, together with the large scale, forms the combined centre where the intercept of the optical axis of the collimator and the telescope is also located. The vernier scale refines the measurement of the angle. It is important to know that the verniers rotate alongside a pivot of the telescope. This coupling is represented by the 'Arm' element in the diagram, but be aware that symbols of the elements have been used for explanation purposes and are not to be found in reality. The verniers and telescope are in any case found together against the pivoting support of the collimator. The main scale will remain in place relative to the collimator when the support, including the telescope and verniers, is rotated. There is the possibility to allow the scale to rotate separately to all other parts by loosening a thumb screw in the case that a particular default position in desired. Don't forget to lock it back into place afterwards though.

2.6 Other Features and Adjustment Provisions

The surroundings of the main components (prism or grid) with all the elements and assemblies of **Fig. 6**, as well as provisions not yet mentioned, are supported and held together by means of a substructure. This foundation will be denoted as the **spectral apparatus** in the following text. The spectral apparatus is actually a special, very precise goniometer for measuring the angle of deflection δ. Preservation of the collimated state by the deflected element is required, otherwise the angle measurement will be unsuccessful.

Telescope slewing – coarse and fine.
The angle measurement in the spectral apparatus of this experiment is so precise that it made sense to equip the machine with the ability to swing the telescope not only in large steps 'on a long journey', but also with a fine-tune screw for angular adjustment of the telescope. An angular resolution of 1' (angular minute) can be achieved. Large rotations are done directly by grabbing and sliding the apparatus and can only be done when the fine-tune regulator is not enabled. For the fine adjustment knob, an intermediate body has been installed in the spectral apparatus, firmly connected to the chassis by means of a locking screw. Otherwise, the intermediate body, along with the other parts, may be pivoted about the vertical axis of rotation. When the intermediate body is locked in place, the telescope can only be adjusted using the fine-tune screw, in which case large rotations cannot be made. Put simply, the fine-tune screw is enabled in locking and large rotations are done when the device is not locked in place.

Vertical adjustment of the telescope.
The adjustment screws under the telescope and collimator cannot be seen in **Fig. 6**, each controlling the tilt of their vertical axes. With these screws, the optical axes can be brought into position exactly perpendicular to the vertical axis of rotation. This adjustment is essential and is achieved as part of the automatic collimation process described in the following section.

Positioning of the ocular tube in the telescope.
The telescopic beam path was required for spectroscopic measurements (see page 3). This means that the real intermediate image of the entrance slit in the telescope as an image of an infinitely

distant object lies exactly the focal distance *f* from the lens (see **Fig. 5**, although this condition is not shown there). The entrance slit is imaged by the collimator on the exit-sided focal plane of the telescopic lens: so is the telescopic beam path. It is actually the collimator which defines the beam path, if and only if the entrance slit sits exactly in the entrance-sided focal plane of the collimator lens. Nevertheless, in this experiment the telescope of the spectral apparatus must first be set, as there exists no simple method for positioning the slit in the collimator. It cannot be assumed that the entrance slit will be imaged exactly in the focal plan of the telescope lens, since it is not always certain that the entrance slit is itself sitting in the focal plane of the collimator lens. There is a control procedure that can be used to test whether the slit image is sitting in the focal plane of the collimator lens. How the crosshairs can be used to do this will be explained in the next paragraph.

Suppose the crosshairs could be brought directly into the focal plane of the telescope lens, then it could be used as a point of reference when comparing it with the image of the entrance slit, and the slit in the collimator is then positioned so that the slit image and crosshairs are both sharp and in focus, giving us a telescopic beam path. The important thing here is having slit image and crosshairs in focus. This is an objective criterion, because the telescopic beam path is independent from the observer. The sharp vision of both elements assumes of course a pre-positioned eyepiece within the ocular tube. The positioning is dependent on the characteristics of the eye of the observer. The inspection technique which establishes the desired position of the crosshairs in the focal length is the following automatic collimation method, which requires a mirror as an auxiliary aid.

Automatic collimation method.
The crosshairs become illuminated in order to send light through the lens, in the opposite direction to how a telescope would normally be used. The crosshairs, located in the focal plane of the telescope lens, will be imaged to infinity. Consider, for example, the divergent light bundle emanating from the axis point of the crosshairs passing through the lens and becoming a collimated beam, parallel to the telescope axis, and intersecting at infinity. Remember to follow this basic idea of automatic collimation.

A flat mirror is placed in front of the telescope lens in the light field originating from the illuminated crosshairs in order to reflect it straight back. This is the basic idea of auto collimation. The rays are collimated prior to reflection and continue to be so due to the flat mirror. The rays then pass a second time through the lens where they convert into a convergent bundle, meeting directly on the axis of the crosshairs where they originated.

The axis point of the crosshairs gets imaged onto itself, meaning the crosshairs are seated in the focal plane of the telescope lens. If the crosshairs lie inside the focal length, its image will be projected outside, and vice versa. It should now be understood how to achieve the desired position of the crosshairs: look through the pre-positioned lens to see if the crosshairs are sharp and in focus. A good quality flat mirror can be found in the centre of the spectral apparatus table and is orientated with its surface perpendicular to the optical axis, as exact as visual inspection can provide. The ocular tube must be finely driven in the main tube (see **Fig. 5**), while the crosshairs image is sought, making sure that the crosshairs and their image are both sharp. Now the crosshairs should be sitting in the focal plane of the telescopic objective as intended. Bringing the crosshairs into this position is the definition of automatic collimation.

Vertical adjustment of the telescope.
The prism, used as a mirror for the purpose of auto-collimation, is on the table along with focused crosshairs. At this stage of adjustment, the crosshairs and its image probably do not yet coincide. The aim now is to bring each to congruence in the vertical respect. The discrepancy is due to the

missing vertical adjustment of the telescope: the telescope axis is not aligned perpendicular to the rotation axis of the spectral apparatus, but this would be important for accurate measurement of angles. It must be assumed that the prism surface is parallel to the axis of rotation of the spectral apparatus. So, it can be said that in coincidence of the image with the crosshairs in the field of view of the telescope, the telescope axis is perpendicular to the rotation axis of the spectral apparatus. This coincidence is achieved by tilting the telescope with the adjusting screw underneath. Remember to tighten it again afterwards.

The end of automatic collimation.
After making the aforementioned adjustments the telescope will be vertically aligned and the crosshairs will be in the focal plane of the telescope. In this state, the telescope serves as a point of reference for the alignment of the collimator. For this reason, the ocular tube may not be displaced within the main tube until the collimator is properly adjusted, after which the collimator will act as the new reference point for the telescopic beam path and the fine focus of the eyepiece can and should be used again.

Adjustment of the collimator.
The entrance slit is to be brought into the focal plane of the collimator objective. The procedure on page 5 regarding the intensive yellow-green light can be applied here. A prism or grid is to be utilised in order to produce light of a selected wavelength. The telescope is then used to find the mentioned mercury line, without changing its pre-aligned, auto-collimated position, as it will be used as a reference for the adjustment of the collimator. It is imperative here that the ocular tube within the main tube not be shifted to try and focus anything in the field of view of the telescope, only in the case of a change in observer. The telescope can and should be swung, namely on the desired spectral line.

If the green mercury line is found, it will most likely not be seen as a sharp image on the entrance slit, only the other mentioned adjustments still hold.

Now to focus the mercury line. Never set it with the telescope though, it is still our reference point. Only focus it through proper positioning of the entrance slit against the collimator lens. It is essential that the spectral line is also clearly visible along with the crosshair. This setting should be carried out with extreme care.

There is still something missing, the optical axis of the collimator is then aligned perpendicular to the rotation axis of the spectral apparatus. For this, the set screw for the tilting of the collimator, found under the collimator, is used. The collimator is tilted so that the image of the entrance slit is in the middle of the field of view.

End of the preparatory adjustments.
Upon completion of this subchapter 2.6, the described adjustment work is completed. The collimator is now adjusted so that the telescope is no longer needed as a reference for the establishment of the telescopic beam path. From now on, the telescopic beam path is safe, because the entrance slit is now in the focal plane of the collimator lens. The focusing of the slit image on the telescope can now be done, i.e. the eyepiece is positioned against the main tube by fine drive. If the principal spectroscopic measurements are performed, for which the apparatus is there, then the slit image of each spectral line needs to be newly focused (needed because of the chromatic aberration), namely the adjustment of the eyepiece using the fine drive.

!! Regarding the information about figures (Fig.XY) please have a look at the german version !!

3. Theory of Deviation through a Prism

Refraction must first be distinguished from diffraction. For prisms, refraction has two stages as shown in **Fig. 1**. A light beam at normal incidence to the interface of a new medium, here we have air to glass, the beam direction is not changed, but for oblique incidences it is. What law describes this elementary occurrence on the first and second surface of the prism? What influence does the medium have? What material properties are important?

Refractive index and absorption index are the most important optical material properties, since they are the only ones that are ever usually needed, in this experiment only the refractive index. This is called n and characterises the influence the material has on the velocity c_{Medium} of light propagating in the medium. Light usually travels slower through a medium than in vacuum. The term c is reserved for the speed of light in vacuum, established in 1983 as:

$$c = 299{,}792{,}458 \text{ m/s}.$$

The product of the refractive index and the speed of light within a given medium always equals c:

$$n_{Medium} \cdot c_{Medium} = c$$

The equation explains that a material that has a 'strong' effect corresponds to a greatly reduced light speed and high refractive index. Such a medium is described as being optically dense, and one with a smaller refractive index is optically thin.

Note: The ratio of the refractive index between two media is equal to the inverse ratio of the corresponding light speeds, $n_2/n_1 = c_1/c_2$.

A few n values for λ = 546 nm:

$n < 1$ occurs in some special cases. Does $c_{Medium} > c$ fit with Einstein? This topic unfortunately won't be covered in this text, so back to the question at hand. Why does the direction of a beam at an oblique incidence to the surface of another medium change? Let α and β be the angles of the rays in the transition from medium 1 to medium 2 normal to the surface, and let n_1 and n_2 be the respective refractive indices of the media. The beam of light has a finite width. Using **Fig. 7**:

Fig. 7. Peripheral rays 'left' and 'right' forming a finitely wide bundle of rays coinciding with a skewed surface is shown. The left ray travels through a thin medium and its path is extended by s_1, the right goes through a thick medium and its path is shortened by s_2 when compared to the respective right angled peripheral rays. From the **Note** on this page, it holds that $s_1/s_2 = n_2/n_1 = c_1/c_2$ because it takes the same amount of time for light to travel s_1 as it does s_2.

The mathematical proof in the caption to **Fig. 7** is missing clarification on whether the light beam 'bends' to the left or right. We have seen the equation $s_1/s_2 = n_2/n_1$, which shows that $s_1/s_2 > 1$ as the assumption was that medium 2 was optically denser than medium 1, so $s_1 - s_2 > 0$. This is precisely what determines the path and direction of the peripheral rays. The other way is the outer. This is why the light deviates right in the case of **Fig. 7**, as it arrives from the bottom left of a lower refractive index. If it arrives under the same condition but from the top left, the light deviates left.

Note: A beam of light in transition from an optically thin medium to optically thick will always deviate towards the line perpendicular to the interface, from air to glass for example.

Note: The light path is reversible.

!! Regarding the information about figures (Fig.XY) please have a look at the german version !!

Note: A beam of light in transition from an optically thick medium to optically thin will always deviate away from the line perpendicular to the interface, like when exiting a prism.

By applying the sine function to the two small right-angled triangles in **Fig. 7** on both sides of the interface, where s_1 and s_2 are, it can be seen that Snell's law must apply:

$$\frac{\sin(\alpha)}{\sin(\beta)} = \frac{n_2}{n_1}$$

Snell's law of refraction
n_1, n_2 … Refractive index of mediums 1 and 2
α … Angle of incidence in medium 1
β … Angle of reflection in medium 2

With the approximation $n_{\text{Luft}} = n_{\text{Vacuum}} = 1$, the following holds:

$$\frac{\sin(\alpha)}{\sin(\beta)} = n$$

Snell's law when light is incident from air.
n … Refractive index of the mediums
α … Angle of incidence in air
β … Angle of reflection in the medium

To avoid misunderstandings, it should be stressed that the terms of incidence and reflection angles are defined as:

Note: The angles of incidence and reflection of a light beam at an interface are always measured from the line perpendicular to the interface. The plane of incidence and exit plane refer to the planes containing the incident ray and the emergent beam respectively.

Fig. 8: Unlike in **Fig. 1**, a special case is shown here. The symmetrical beam path in a prism.
α … First angle of incidence
β … Last angle of reflection
Not labelled are the first angle of reflection and last angle of incidence which occur within the prism.
δ, γ … Deflection and refraction angles

Prisms create two refractions of light, as two interfaces are present. The two refraction stages are measured with a wide angle δ and are dependent on the **angle of refraction** γ, enclosed by the two surfaces. The corresponding prism edge is called the **refracting edge**. After a little geometry and application of sine laws and Snell's law, the most important equation of a prism spectrometer can be determined:

$$n = \frac{\sin\left(\frac{\delta + \gamma}{2}\right)}{\sin\left(\frac{\gamma}{2}\right)}$$

This law was created to presuppose the symmetrically extending beam path of the prism. To establish this, the prism is rotated relative to the incident beam in such a way that the first angle of incidence α and the last angle of reflection β (both measured from the line perpendicular to the interface) are equal. See **Fig. 8**. Such a symmetry exists because the three beams within and outside the prism after reflection on the angle bisector of the two prism surfaces, along with itself, constitute congruency. Then the prism and beam section are exactly perpendicular to the angle bisector of the concerned prism surfaces.

Note: Eq. (3) only holds for the symmetrical beam path in the prism. This case happens when the incident angle of the first prism surface is equal to the angle of deflection at the second. In this state, the angle of deflection is at a minimum.

Equation (3) has many advantages; it is mathematically easier due to its specialization and it is valid for a state that is experimentally convenient because it can be easily controlled. The criterion is just δ's minimum. In addition to Eq (3), the generalized equation for arbitrary orientation of the prism to the incident light includes a further measured variable, for example α, without its existence a solution could not be found. It is therefore emphasised again:

Note: In spectroscopic measurements, the symmetrical beam path in the prism is to be established. This is found using the minimum of δ. The telescope is swung onto a new spectral line so that the beam path is initially not aligned with the intended symmetry. For every new λ, a further adjustment should be made to minimise δ.

In considering a particular spectral line to which a steady telescope is already roughly aligned, notice that the spectral line moves relative to the crosshairs when carefully rotating the prism on the table, but after a certain point will reverse its motion. This is where δ is at a minimum, as desired.

4. Possible Application of a Prism Spectrometer.

4.1 Measuring Device for λ and its Calibration

The first application of a prism spectrometer is its use as a tool to measure λ, which will be explained in the following passage.

Source of a line spectrum.
Spectrometry was used in the creation of gas discharge lamps. Such lamps are part of a common family that, when in a working state, contain a gaseous element or a mixture of several elements. The gas particles participate in a gas discharge, where they are exposed to collisions with electrons and are stimulated with different energy levels. The individual excitation has a particular transition energy, but there are several possible that occur in the entirety of the small parts. The return their original state occurs spontaneously and releases energy portions, their possible values are discretely on the number line. Other values in intermediate positions are quantum-mechanically impossible. The individual transition to a lower energy state is the elementary process of emission by the giving up of a photon. The single photon is the elementary contribution to the emitted light field and has a suitable λ for the transitional energy. For every discrete value of the transition energy there is a discrete value λ, giving a line spectrum. The sun, light bulbs and all light emitting bodies send a continuous spectrum.

Calibration source.
Suppose that a source with a known line spectrum would is available, for example, the mercury vapour lamp (Hg lamp) in this experiment. Spectral lines would be seen as well as a known list of all their λ values.

Calibration.
Take a gas discharge lamp with a known line spectrum and a prism spectrometer with a prism whose type of glass need not necessarily be known. Then, δ is measured for each spectral line, all with known λ values. All corresponding λ and δ values are then put into a table so that the relationship λ

↔ δ is measured point-wise. After representation of the value pairs in a diagram and supplemented by a (smooth) compensation curve we get the continuous function:

$$\lambda = \lambda(\delta)$$

This function is called a calibration function and its associated curve a calibration curve. Each unknown λ, which is fixed in the form of a spectral line, can now be measured as δ is. The δ value is then put into the calibration curve in order to obtain λ.

Thus, the spectrometer is a λ-measuring instrument.

4.2 Measuring Instrument for the Refractive Index *n* and the Dispersion *n* = *n*(λ)

Using one or more known gas discharge lamps and a prism spectrometer, the prism being used as a material sample, the refractive index *n* of a substance is measured. Here, δ is measured for every observable spectral line whose λ value is known. For each measured δ, Eq. (3) gives an *n*-value.

Thus, the spectrometer is an *n*-measuring instrument.

Dispersion is the phenomenon that *n* depends on λ. If the measurement method described was repeated with the same prism for various known spectral lines, the applicable function for the material is found.

$$n = n(\lambda)$$

With this the dispersion of the material of the prism may be examined. It is also possible to investigate the dispersion of liquids in a hollow prism.

5. Theory of Deviation through a Grid. Diffraction.

The type of grid used in this experiment can be easily imagined. There is a series of straight, parallel and equidistant ridges in a plane with very simple optical properties. The ridges absorb the light in the light field so that the light in the first half-space are caught but not emitted again in the second half-space. The gaps between the ridges do, however, permit light to continue into the second half-space just as it arrived. It could also be said that the grid, within its plane, makes gaps in the light field. The gap width and their mutual distances are microscopic. Specifically, the grid period *a* is within 1 to 100 µm. For comparison, the width of the collimated light field in front of and behind the grid is 200 to 20,000 times as large (about 20 mm). λ is still less than 1 µm.

According to these explanations, the collimated beam should continue when a grid blocks the way just the same as if there were no grid. It continues, although somewhat weaker, because light is absorbed. Using the spectrometer when δ = 0, the image of the entrance slit will be found. This is the 0^{th} order of diffraction, as mentioned above in **Fig. 1** and **Fig. 2**. The 0^{th} order of diffraction still exists even without any diffraction, it is only named as such for the sake of how it is spoken. The main interests here are the higher (real) diffraction orders. How they arise is explained in the next section.

Firstly, it is because the grid, within its plane, removes light from a light field that light exists in other places where there was otherwise none. In wave optics, it is a property of diffraction orders that ‘less is more’. Refraction occurs due to the Huygen’s principle. In a modern sense, it means the following. Imagine an arbitrary plane within a light field and every point in the plane is the centre of a spherical wave. These waves are called Huysgen’s elementary waves. The light does not land on the source side half-space at each point of the plane, but simply anywhere on the plane and with a certain amplitude and phase. Both state variables are set for their excitation in the centre of the corresponding elementary waves, just as the light arrived there.

Keeping this concept in mind, the core statement of Huygen's principle is as follows. In the half-space, on the opposite side of the plane from the source, the light field is equal to the superposition of all Huygen elementary waves. 'Superposition' in a mathematical sense means the 'addition of wave functions'. If the imaginary plane is not comprised of any matter, is truly only in the mind, then the mathematical representation of the superposition actually leads to the undisturbed propagation of the light field. A collimated beam will continue to travel straight and without any deviations. This results in the calculation, just as it corresponds to real life experience. Something else the elementary waves can do when a diffraction grid in a light field causes a disturbance, will be explained in the next section.

The effect of a diffraction grating in this experiment is understood most easily when the gaps between the ridges are taken to be very narrow when compared to λ, then the gaps act as point sources, each propagating an elementary wave into the observation room, towards the telescope.

Fig. 9 illustrates that for a fixed direction in the observation room, according to the terms of the telescopic beam path in the spectrometer (see page 3) in the case of a deviation (δ ≠ 0), light can be observed.

The Huygens elementary waves, whose light paths represent a collimated beam, interfere constructively. This means that all rays in the bundle conjoin within the telescope in the plane of the intermediate image so that their crests or troughs in each case all merge at the same time. Extinction would take place if, for example, mountains and valleys were paired together. Why are the mountains and valleys being paired? Answer: As **Fig. 9** shows, the elementary waves have different distances to travel. The path difference of each beam to the next, in the case of **Fig. 9**, amounts to exactly λ. If there is no path difference (for δ = 0 and *n* = 0) then its values will be whole multiples of λ (for δ ≠ 0 and *n* = 1, 2, 3 etc.).

Fig. 9: Diffraction at a grid with perpendicular lighting. *a* is the grid period and δ the angle of deflection. The constructive interference in the diffraction order *n* = 1 will be demonstrated. Adjacent beams have a path difference of exactly λ in the observation room. A and B are both faces of the same phase.

Fig. 9 leads directly to the equation for the relationship (λ, *a*) ↔ δ by application of the sine function on the small triangle in the figure.

$$\lambda = a \cdot \sin(\delta)$$

Eq. (4) applies only to perpendicular lighting and only in the first diffraction order. For an arbitrary *n* it will be made clear (see also **Fig. 9**):

$$n \cdot \lambda = a \cdot \sin\big(\delta(n)\big)$$

Eq. (5) only applies to perpendicular lighting, but otherwise also generally. This equation is not enough for this experiment, as it follows the generalization of oblique illumination, which is illustrated in the following figure:

Fig. 10: Similar to the previous diagram, however here the general case of oblique illumination is shown, where *a* and *b* are the angles of incidence and reflection, and s_1 and s_2 are the path differences of neighbouring rays in the illumination or observation room which add up. The path difference of neighbouring rays on the way from A to B total s_1 + s_2. The condition for constructive

interference is therefore $s_1 + s_2 = n \cdot \lambda$, where *n* is the order of diffraction. The corresponding angle δ is then required. Incidentally, δ = α + β.

The question of the deflection angle δ with oblique illumination for different *n* values (diffraction) is given in **Fig. 10**, on one hand with reference to the equation

$$n \cdot \lambda = s_1 + s_2$$

which is the condition for constructive interference in the direction of the n^{th} diffraction order. On the other hand, the following relationships can be seen in **Fig. 10,**

$$s_1 = a \cdot \sin(\alpha)$$

and

$$s_2 = \ a \cdot \sin(\delta - \alpha)$$

which again, through the application of the sine function and some geometry on the two small triangles in the diagram, may be determined. It follows from equations (6) and (7) and δ = α + β that

$$n \cdot \lambda = a \cdot (\sin(\alpha) + \sin(\delta - \alpha))$$

Eq. (8) describes the relationship between the deflection angle δ and the angle of incidence for α for the given values of *a*, λ and *n*. Eq. (8) thus provides the function δ = δ(*a*). This function is of particular interest, as the angular position of the grid in the centre of the table of the spectrometer induces the angle of incidence α and apparently affects the deflection δ. Practically speaking, if the grid on the central table with a stationary telescope were turned, then the spectral lines migrate upon the crosshairs, because δ changes with α. It turns out though, that this migration of the spectral lines reverses after a certain point: δ assumes a minimum!

Theoretically speaking, when given *a*, λ and *n*, the brackets in Eq. (8) are constant:

$$K(\alpha, \delta) \coloneqq \sin(\alpha) + \sin(\delta - \alpha) = \text{const.}$$

Under this condition, α and δ may vary according to:

$$\frac{\frac{\partial K}{\partial \alpha}}{\frac{\partial K}{\partial \delta}} = -\frac{\mathrm{d}\delta}{\mathrm{d}\alpha}$$

Using Eq. (9) it follows that:

$$\frac{\cos(\alpha) - \cos(\delta - \alpha)}{\cos(\delta - \alpha)} = -\frac{\mathrm{d}\delta}{\mathrm{d}\alpha}$$

This differential quotient is set to zero, as the extrema for δ(α) are to be found. Doing this, Eq. (11) becomes:

$$\cos(\alpha) = \cos(\delta - \alpha)$$

The solution to this is α = δ/2 = β as δ = α + β. **Fig. 10** explains this relationship, showing that α must be equal to β, so we have symmetry.

Note: The symmetrical beam path through the grid is the one in which the deflection angle δ assumes a minimum - similar to the prism. This case is established in the process of the experiment for good reason: it is easy to control. Again, new spectral lines require renewed setting of the symmetry case.

Those belonging to the symmetrical beam path need a specialised form of Eq. (8), formed by inserting $\alpha = \delta/2$:

$$n \cdot \lambda = 2 \cdot a \cdot \sin\left(\frac{\delta}{2}\right)$$

Note: Eq. (13) applies only in the case of symmetry. It is the main equation for the grid spectrometer. It describes the relationship $\delta \leftrightarrow \lambda$ for a given *a* and *n*.

Note: A grid spectrometer does not require calibration: If the grid is known, everything is there in order to measure λ. This is different with prism spectrometers. A grid spectrometer may also be used to calibrate a prism spectrometer.

6. Homework

Will be announced on the day of experimentation.