Phase and group velocity

of ultrasonic waves in liquids

1 Goal

In this experiment we want to measure phase and group velocity of sound waves in liquids. Consequently, we measure time and space propagation of ultrasonic waves (frequencies above 20 kHz) in water, saline and glycerin.

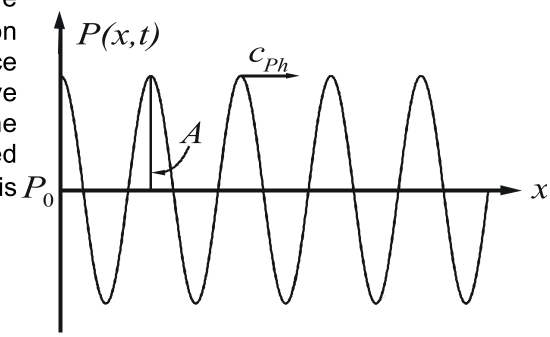
2 Theory

2.1 Phase Velocity

Sound is a temporal and special periodic sequence of positive and negative pressures. Since the changes in the pressure occur parallel to the propagation direction, it is called a longitudinal wave. The pressure function P(x, t) can be described by a cosine function:

$$P(x,t) = A \cdot \cos(kx - \omega t) \text{ mit } k = \frac{2\pi}{\lambda} \text{ und } \omega = \frac{2\pi}{T} ,(1)$$

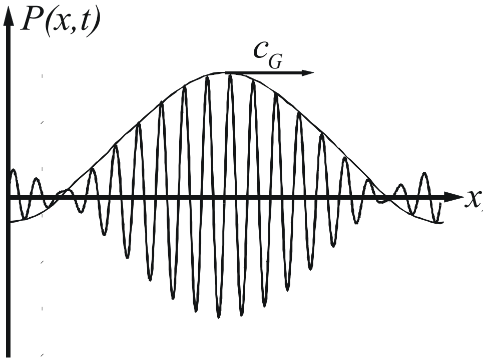
where A is the amplitude of the wave, and k is the wave number and λ is the wavelength. The propagation velocity C_{Ph} is expressed with the help of a reference point (e.g. the maximum).in the space. The wave propagates with the velocity of the points that have the same phase position in space. Therefore, C_{ph} is called phase velocity. Depending on the frequency ν it is defined as



$$c_{Ph} = \lambda \cdot \nu = \frac{\omega}{k} (2).$$

The phase velocity (speed of sound) is 331 m/s in the air. The phase velocity reaches around 1500 m/s in the water because of the incompressibility of liquids. It increases even to 6000 m/s in solids. During this process, energy and momentum is transported in a medium without requiring a mass transfer.

2.2 Group Velocity



A wave packet or wave group consists of a superposition of several waves with different frequencies, wavelengths and amplitudes (Not to be confused with interference that arises when superimposing waves with the same frequencies!). The longitudinal pressure P(x,t) of a wave packet, which is composed of i waves, can be written

$$P(x,t) = \sum_i A_i \cdot \cos(k_i x - \omega_i t). \qquad (3)$$

The envelope of the wave packet describes a pulse that propagates in space. While the phase velocities C_{PH} can be different for the individual partial waves, the group velocity of the total wave group C_G is clearly defined. It indicates the speed with which the maximum of the wave group moves in the propagation direction. Mathematically, the group velocity is defined as the derivative of the frequency of a partial wave in respect to wave number:

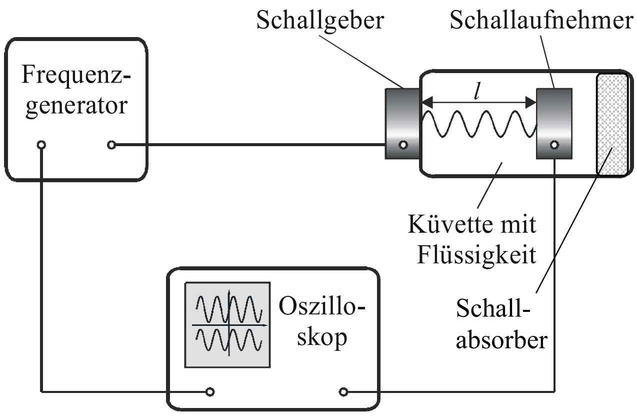
$$C_G = \frac{d\omega}{dk} \qquad (4)$$

In dispersive media, phase velocity is determined by angular frequency ω (see equation (2)). Thus, the different frequency components of the wave packet move at different speeds This results in the wave packet divergence. The relationship between phase and group velocity can be derived from the equations (2) and (4). It yields

$$C_G = C_{Ph} - \lambda \cdot \frac{dC_{Ph}}{d\lambda}. \qquad (5)$$

If there is no dispersion ($dC_{Ph}/d\lambda = 0$), then $C_G = C_{PH}$, and the wave packet retains its shape during the propagation.

3 Set-up of the Experiment



The sound generator is externally attached to a liquid-filled cuvette. A sound sensor is immersed in the liquid with a distance *l* to the sound generator. A frequency generator with alternating voltage powers the sound generator. We can monitor output of the frequency generator and its signal by an oscilloscope. The signal of the sound receiver can also be measured with the oscilloscope.

For a better connection, the sound-emitting surface of the sound generator is coated with glycerol and then mounted to the wall of the cuvette. The opposite wall of the cuvette is covered with sound-absorbing material (for example, with crumpled paper). Thus, we avoid sound reflections, the possible developing multiple echoes and standing waves.
Sound transmitters and receivers consist of piezoelectric crystals, which have the property to produce an electric voltage under the influence of mechanical deformation. Vice versa, they deform when a voltage is applied to them. Hence, if we use the frequency generator to apply an alternating voltage to the sound generator's crystal, it will deform periodically and emit a sound. The resulting deformations of the sound receiver's crystal can be detected as an alternating voltage by the oscilloscope.

4 Implementation and Evaluation

4.1 Determination of the generator frequency

The oscillation period of the generator signal is displayed as large as possible on the oscilloscope. Then the period T can be read easier. In order to keep the reading errors on the oscilloscope as small as possible, it is preferable to take several oscillation periods on the oscilloscope. This leads to a better evaluation.

4.2 Determination of the phase velocity

The sound generator is driven with the sinusoidal signal of the frequency generator. The signal of the sound pickup and the monitor signal of the frequency generator are displayed on the oscilloscope by internal trigger. We can display the sound pickup and the monitor's signal in phase by moving the sound pickup longitudinally on the screen. From this position, we move the sound receiver to find the new in phase passages. Then we record the displacement of the receiver as Δl and the number of this in phase passages as n.

4.3 Determination of the group velocity

The frequency generator is operated in the pulse mode. The ascending slope of the input signal triggers the oscilloscope. The signal of the sound sensor should be displayed on the oscilloscope in such a way that you can see the shift of running time pulses between sound generator and sound receiver. Now the zero point for the pulse's running time must be defined at the sound receiver. From a starting position near the sound generator, one moves the ultrasonic transducer and determines the change in the pulse duration Δt from the shift of the pulse's slope in the oscilloscope.

4.4 Measurements

The measurements for determining the phase and group velocity are performed in the liquid water, saline, and glycerol.

5 Implementation and Evaluation

- Introduction (motivation, the goal of the experiment)
- Theoretical background (introducing all relevant measures and derivation of all formulas which are used, the answers to the 5 questions in the questionnaire must be found in the text)
- Implementation (description of the structure, sketch, description of tasks)
- Results (measured value tables, graphs)
- Summary / Conclusion (Review your results and compare them with literature values or settings, check possible sources of error)

6 Tips for evaluation

- Determine the frequency generator and indicate the relevant error!
- Determine the phase velocities by a linear equation which includes wavelength as the slope: $\Delta l = \lambda \cdot n$
- Use linear regression WITHOUT intercept method (see error calculation script) for determining the wavelength error

- Calculate the phase velocity, frequency and wavelength errors (error propagation)
- Determine group velocity with a linear equation $\Delta l = C_G \cdot \Delta t$ in which the group velocity is the slope
- Use linear regression WITHOUT intercept method (see error calculation script) to determine the error of the group velocity
-

Literature:

H. Stocker: Taschenbuch der Physik. Verlag Harry Deutsch,, 1993

Water $c_{20\,°C}$ = 1480 m / s
Seawater $c_{20\,°C}$ = 1470 m / s } phase velocities
Glycerin $c_{20\,°C}$ = 1920 m / s

D.R.Lide: Handbook of chemistry and physics. CRC press, inc. 1995

Water $c_{25°C}$ = 1496 m/s
Seawater $c_{25°C}$ = 1531 m/s } phase velocities
Glycerin $c_{25°C}$ = 1904 m/s

values for C_{Ph} and C_G at 25°C, measured with the experimental set-up:

	c_{Ph} (m/s)	c_G (m/s)
Wasser	1500	1506
NaCl-solution (saturated)	1800	1800
Glycerol*)	1880	1900

*)

Because glycerol is hygroscopic (attracts water), a slower sound velocity is often measured for glycerol comparing what is indicated, here.