Simple Pendulum and Spring Pedulum

1 Exercise Aims

This exercise aims to investigate the physical properties of two different kinds of pendulums. Therefore the exercise is divided into two parts. In the first part the oscillation period T of a mathematical pendulum is measured, which allows the determination of the gravitational constant g. In the second part, the spring constant D as well as the mass of a spring pendulum m_F are determined. This too, allows the calculation of the gravity acceleration. The results are compared to each other and the literature value.

2 Theory

2.1 Simple Pedulum

A mass m is mounted to a cord of length l und swings under the influence of gravity with the oscillation period T. For the theoretical description the setup is approximated by a so called *simple pendulum*, which has the following restrictive properties compared to the real pendulum:

- Massless cord
- Frictionless pivot of the cord
- Punctiform mass of the pendulum body (no air resistance)

To investigate the oscillation of the pendulum, we have to find the equation of motion of the mass m, which is done in the following paragraphs.

The mass is moving on a circular arc with radius l. The acting forces are the weight $G=m*g$ and the tension on the cord. In figure 1 you can see, that the tangential component of the resulting force is

$$\left|\vec{F}_T\right| = -G \cdot \sin\phi = -m \cdot g \cdot \sin\phi \qquad (1)$$

This is the force acting against the displacement of the mass.

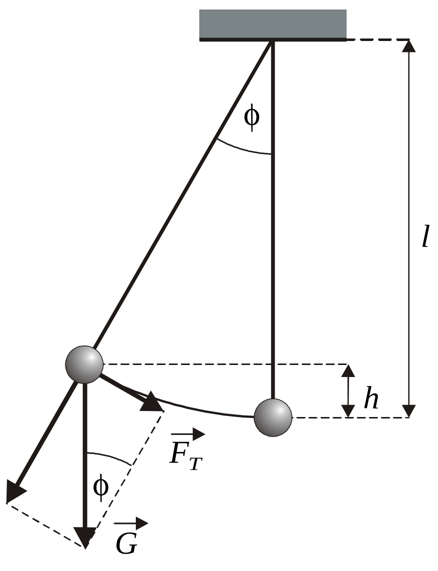

Figure 1: Set-up of a swing pedulum

By using $F=m*a$, we see that the following must be true as well.

$$\vec{F}_T = m \cdot \vec{a}_T . \tag{2}$$

The tangential acceleration a_T depends on the angle acceleration α and the radius l and can be written as:

$$a_T = l\alpha = l\frac{d^2\phi}{dt^2} . \tag{3}$$

By equating equations (2) and (3) we get the differential equation for the tangential movement:

$$ml\frac{d^2\phi}{dt^2} = -mg\sin\phi$$

or

$$\frac{d^2\phi}{dt^2} + \frac{g}{l}\sin\phi = 0 . \tag{4}$$

The solution to this differential equation is the function $\phi(t)$, which describes the movement of the mass m. If the angle ϕ is small, which is true for small oscillation amplitudes, we can use the small-angle approximation $\sin\phi \approx \phi$, which leads to the following simplification of equation (4):

$$\frac{d^2\phi}{dt^2} + \frac{g}{l}\phi = 0 . \tag{5}$$

This can be solved by using the harmonic approach:

$$\phi(t) = \phi_0 \sin\omega t \tag{6}$$

With $\omega^2 = g/l$ and the oscillation amplitude ϕ_0. From this the oscillation period in first approximation can be derived:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}} . \tag{7}$$

Thus the oscillation period for small amplitudes is independent from the starting angle ϕ_0. For larger angles the second approximation can be derived as:

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\phi_0}{2}\right) . \tag{8}$$

In this approximation the oscillation period therefore is a function of the oscillation amplitude

2.2 Spring pendulum

The differential equation for a spring pendulum as seen in figure 2 can be derived analogous to equations (1)-(4) from the restoring force

$$\vec{F}_R = -D\vec{x} \tag{9}$$

as

$$m\frac{d^2x}{dt^2} = -Dx \tag{10}$$

If the spring pendulum is displaced around x_0 at t=0 and allowed to swing, the solution to the equation of motion is:

$$x(t) = x_0 \cos(\omega t)\,, \tag{11}$$

with

$$\omega = \sqrt{\frac{D}{m}}\,. \tag{12}$$

Which analogous to (7) leads to:

$$T = 2\pi\sqrt{\frac{m}{D}}\,. \tag{13}$$

This relation allows the calculation of the spring constant D by measuring the oscillation period T as a function of the mass m.

If a mass m is applied to the spring, the static displacement x is described by:

$$mg = Dx \tag{14}$$

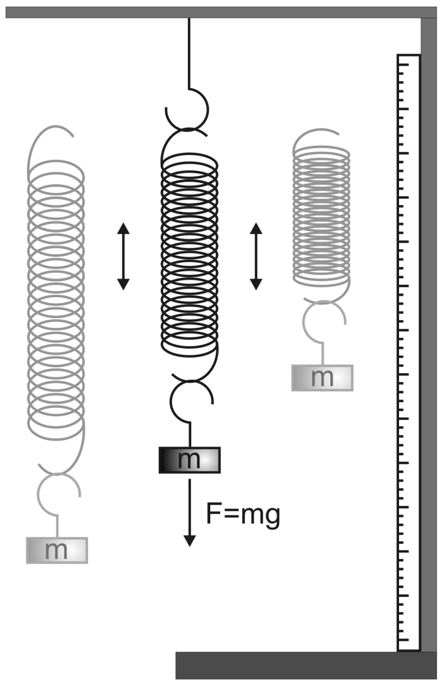

Abb. 2: Set-up of a spring pendulum

Since the spring constant D is already known from the previous measurements this allows the calculation of the gravity acceleration g.

2.3 Measurements and Analysis

2.4 Simple Pendulum

In the following exercises the oscillation period T shall be measured in dependence of the oscillation amplitude ϕ_0 and the cord length l.

2.4.1 Checking the Set-Up

The electronic clock with a light barrier must be set up to measure to whole oscillation period. Check if the photo diode is triggered by the cord of the pendulum (not the bob or the mounting ring). Measure the cord length l and don't forget to consider the bob radius (R=16 mm) and determine the maximum error.

2.4.2 Measuring the oscillation period as a function of the oscillation amplitude

Measure the oscillation period T as a function of the oscillation amplitude ϕ_0 at constant cord length for an as large as possible angle range. Average over a few oscillations respectively. Plot the oscillation period against the oscillation amplitude as well as against $\sin^2(\phi_0/2)$. Determine the regression line and compare the regression coefficients (slope and axis intercept) to the expected values from the second approximation. You can use the literature value of g=9,81 m/s² to calculate the coefficients.

2.4.3 Measuring the oscillation period as a function of cord length

Measure the oscillation period T as a function of the cord length l for small oscillation amplitudes. Plot the measurements so that you get a linear dependence by modifying equation (7). Determine the gravity acceleration g with error σ_g rom the slope of the linear regression and compare this value to the literature one g = 9,81 m/s^2.

2.5 Vertical spring pendulum

2.5.1 Choosing the measuring method

Load the spring with a mass without overextending the spring and measure 3 times 10 oscillations to determine the oscillation period by using a stop watch. Start and stop the measurements at the maximum displacement of the pendulum. Then do a second set of 3 times 10 measurements. This time starting and stopping the measurements a zero displacement of the pendulum. Calculate the average oscillation period as well as the standard deviation of the average value for both sets of measurements. Use the more precise method for all following measurements.

2.5.2 Determination of the spring constant

Now measure the oscillation period as a function of the mass m by applying at least five different masses. Be careful not to overextend the spring. Measure 3 times 10 oscillations for each mass, so you get a more accurate value. Plot T^2 against the mass m and determine the spring constant D from the slope of the linear regression (with offset! see next exercise) and equation (13).

2.5.3 Determination of the spring mass

The line in 3.2.1 is no line through origin. This is due to the previously neglected mass of the spring m_F, which shifts the axis intercept by $-m_F/3$. Use equation

$$T = 2\pi\sqrt{\frac{m + \frac{1}{3}m_F}{D}} \tag{15}$$

instead of equation (12) to calculate the spring mass from the null of the regression line. Weigh the spring mass and note the maximum error to be able to do a comparison to the calculated value.

2.5.4 Determination of the gravitation acceleration

To determine to gravitation acceleration, the static displacement x of the spring pendulum is measured as a function of the mass. For this the pendulum is weighted with at least five different masses and the resulting displacement is noted. The displacement is then plotted against the mass. Solving equation (14) for x, yields a linear relation between x and m. By doing a linear regression and using the calculated spring constant from exercise 3.2.2 the gravitation acceleration can be calculated. Compare this value to the one obtained in exercise 3.1.3. and the literature value.