

Magnetic Moment in Magnetic field

1 Goal and Concepts of the Experiment

In this experiment, we investigate the torque that is experienced by a current-carrying conductor loop, which is exposed to an approximately homogeneous magnetic field. The Lorentz force exerts a torque on the moving charges inside the loop as the effect of the magnetic field. We study the dependence of the torque on the size of the loop, the number of turns, the current intensity of the current flowing through the conductor loop, the orientation of the conductor loop relative to the magnetic field and the magnetic field strength. We also introduce the term magnetic moment.

We perform basic error calculation that is standard deviations of the measured values to evaluate the results. We show that how to determine the power of a variable by using power law definition and a log-log plot.

As an application, we investigate a simple electromotor. We illustrate the physical concepts of the electromotor without quantitative measurements.

2 Theory

2.1 Lorentz Force

The Lorentz force \vec{F}_L on an electric charge q (i.e. $q = -e$) with the velocity \vec{v} in a magnetic field \vec{B} is given by

$$\vec{F}_L = q \vec{v} \times \vec{B} = -e \vec{v} \times \vec{B} \quad (1)$$

The force is proportional to the charge, the velocity and the magnetic field. The cross product of the two vectors \vec{v} and \vec{B} provides a vector perpendicular to the both \vec{v} and \vec{B} (see Figure 1). Therefore, the charge experiences a force perpendicular to its direction of movement. Free electrons in a uniform magnetic field move on circular paths under the influence of the Lorentz force. The magnitude of the Lorentz force is given by

$$F_L = q \cdot v \cdot B \cdot \sin\phi = q \cdot v \cdot B \text{ when } \phi = 90^\circ \quad (2)$$

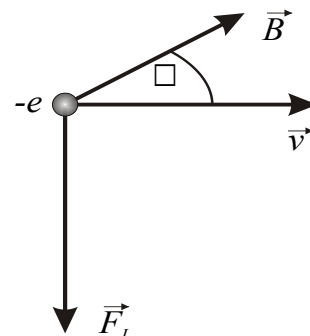


Figure 1: Lorentz force on a moving charge in a magnetic field (Left-Hand-Rule).

Therein, the angle j is the angle between two vectors \vec{v} and \vec{B} .

2.2 Force on a Straight Conductor

An electric current in a conductor is microscopically the electric charges (electrons, charge $q = -e$) move along the conductor with a velocity \vec{v} . Here, the relationship between current I and the velocity in a wire with a cross-section A and a carrier density n is given by

$$I = -n e v A \quad (3)$$

Here is important to have a linear relationship between the current intensity and the speed of the charge carrier. If we insert a current-carrying conductor in a magnetic field, the Lorentz force \vec{F}_L acts on the moving charges inside the conductor (see Figure 2). This applies a force

$$\vec{F} = I \cdot \vec{s} \times \vec{B} \quad (4)$$

on the entire conductor. Here \vec{s} is directed toward the length of the conductor. The force \vec{F} is perpendicular to the conductor and the magnetic field \vec{B} . The force disappears when the current flows parallel to the magnetic field.

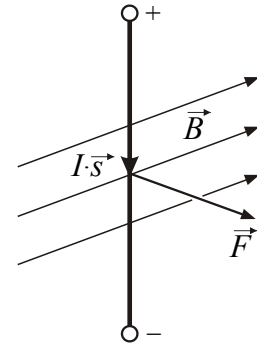


Figure 2: Force on a current-carrying conductor in a magnetic field

2.3 Torque on a Current-carrying Conductor Loop

Before we proceed with a general conductor loop, we consider the simple case of a rectangular loop. The rectangular loop is rotatable mounted in such a way that its axis is perpendicular to the magnetic field (see Figure 3).

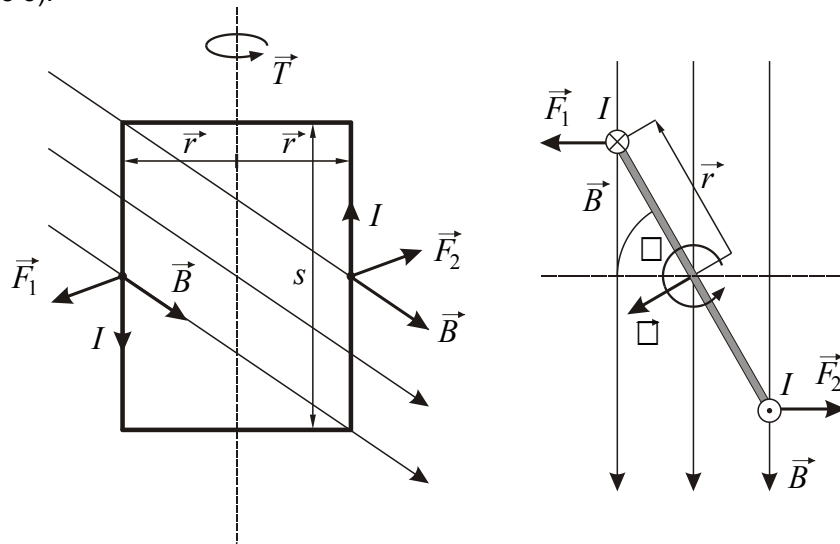


Figure 3: A current-carrying conductor loop in a magnetic field. The couple of forces \vec{F}_1 and \vec{F}_2 causes torque \vec{T} .

The conductor loop can be divided into four separated parts. According to the equation (4), each part is affected by Lorentz force differently. The forces \vec{F}_1 and \vec{F}_2 are perpendicular to the axis of rotation and form couple forces, which exert a torque on the lever arm \vec{r} ;

$$\vec{T} = \vec{r} \times \vec{F} \quad (5)$$

For the magnitude of the torque, we have

$$\begin{aligned}
T &= r \cdot F \cdot \sin \alpha \quad \text{with } F = 2 \cdot s \cdot I \cdot B \quad (\text{equation (2)}) \\
&= 2 \cdot r \cdot s \cdot I \cdot B \cdot \sin \alpha \\
&= A \cdot I \cdot B \cdot \sin \alpha
\end{aligned} \tag{6}$$

The area of the loop is $A = 2 \cdot r \cdot s$, and α is the angle between \vec{F} and \vec{r} . Equation (6) states that for a given magnetic field B, the torque depends on the conductor loop area A and the current I. Also, the torque is independent of the geometric form of the conductor loop. The product of A and I is the magnetic moment

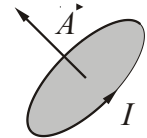
$$\vec{\mu} = I \cdot \vec{A} \tag{7}$$

\vec{A} is a vector with value A which is the surface area and its direction is the surface normal vector. Thus, the torque of a magnetic moment in a magnetic field is generally given by

$$\vec{T} = \vec{\mu} \times \vec{B} \tag{8}$$

In this experiment, we want to check this relationship between torque, magnetic moment and magnetic field. The magnetic moment of a circular conductor loop with N turns and radius r, which is carrying a current I_{LS} is given by

$$\mu = N \cdot \pi \cdot r^2 \cdot I_{LS} \tag{9}$$



For the torque on a circular conductor loop we have

$$T = N \cdot \pi \cdot r^2 \cdot I_{LS} \cdot B \cdot \sin \alpha \tag{10}$$

Two current-carrying Helmholtz coils generate the magnetic field B with $B = c \cdot I_{HH}$. There, c is a proportional factor. In this experiment we want to obtain this equation:

$$T = c \cdot I_{HH} \cdot N \cdot \pi \cdot r^2 \cdot I_{LS} \cdot \sin \alpha \tag{11}$$

3 Set-up and Experiment Procedure

A pair of Helmholtz coils produces the magnetic field. The magnetic field is homogeneous in between the coils. In the middle of the coils, a rotary conductor loop is connected to a rotary force gauge. The force exerted on the conductor loop is determined as follows:

- Turn off the Helmholtz coil current / conductor loop current, so that the conductor loop is not exposed to the torque
- Bring the conductor loop to the desired angular position
- Set the torque meter in such a way that the upper conductor loop strap is balanced with the underlying markings. The same person should check the balance during a series of measurements to keep reading errors (parallax) constant. Write down the value that the torque meter is shown.
- Switch on the current of Helmholtz coil / conductor loop (the conductor loop will adjust according to the acting force)
- Reset the torque meter again in such a way that the upper conductor loop strap is balanced with the underlying markings. Record the value that the torque meter is shown and then subtract it from the previous one.

The following dependencies should be identified:

- Strength of the magnetic field or current I_{HH} through Helmholtz coils 0 - 4A, (apply $I_{HH} > 3A$ only for a short time)
- Current I_{LS} through the conductor loop 0-3A, ($I_{LS} > 3A$ only for a few seconds. After reading of the force value, turn off the power immediately)
- Radius r of the conductor loop (small, medium, large)

- N is the number of the turns of the conductor loop ($N = 1,2,3$)
- The angle α is between the surface normal vector of the conductor loop and the magnetic field (0 to 90°).

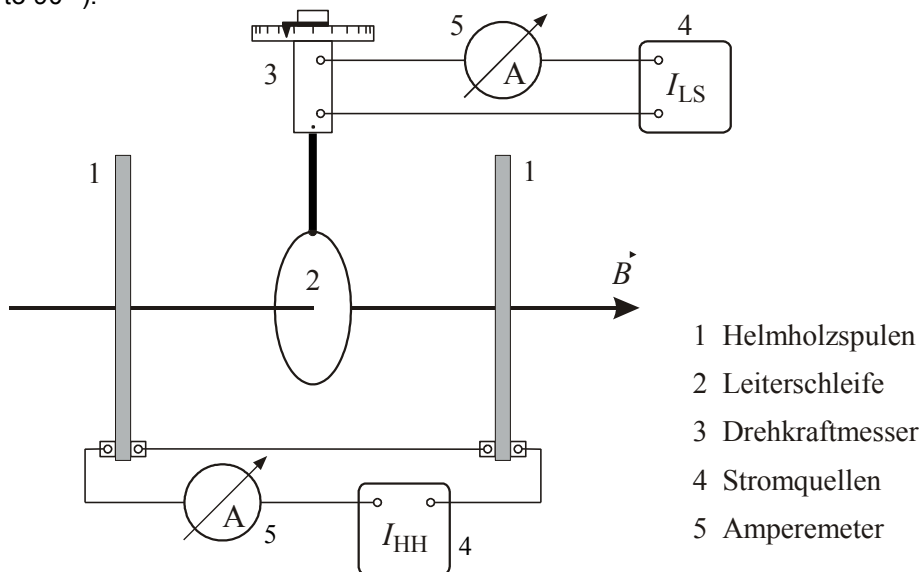


Figure 4: Figure of the experiment

What kind of dependency do you expect after the implementation of the theory? To check relation (11), we change one of the parameters in each part of the experiment, while the others are held constant.

Choose the constant parameters for each series of measurements and follow this guideline:

- The current of Helmholtz coils must not exceed 3A in any continuous procedure.
- Let the power supply wires hang (mechanically) stress-free on the coil carrier.
- Control that the torsion dynamometer to be at zero point before starting the measurement and adjust it if it is necessary.
- Very small torque occurs during the torque measurement depending on the current of Helmholtz coils. Therefore, it is suggested to use the conductor loop with three turns and an increasing coil current for a short time to the max. 4A. Adjust the angle of the coil carrier in steps of 15° by using the dents of the coil carrier.

For each series of measurements, the torque is to be measured for almost 6 different settings of the variable.

4 Evaluation of the Experiment

Determine the relationship between the torque T and the different measured variables. Use power-law

$$x, x \in \{I_{HH}, I_{LS}, r, N, \sin\theta\}$$

$$T = a \cdot x^n \quad (\rightarrow \log T = n \cdot \log x + \log a) \quad (12)$$

and determine the corresponding power n. There is a linear dependence to the variable when $n = 1$. For this purpose, we need to plot the data double-logarithmically. Then draw a straight line through the data points. The slope of the line is the power n. Now, Compare the experimentally measured powers with the expected ones from the theory.

The errors for the determined powers are calculated by the linear regression. The method of the linear regression is based on the fact that we have a straight line through a set of measurement values (x_i, y_i) .

$$y = a x + b \quad (13)$$

The line is placed in such a way that the sum of the squared distance of the measuring points from the determined straight line is minimum.

The quality of the linear regression is calculated by the correlation coefficient r , ($r \leq 1$). The correlation coefficient $r = 1$ means that the data are exactly located on a straight line. Oppositely if $r = 0$, then a straight line cannot describe the data. In the present case, the variables (x_i, y_i) are the logarithms of the measured values, and they make a straight line in a double logarithmic representation. The powers are the slope of the line.

According to the condition that the sum of the distance squares should to be minimized, we calculate some parameters of the linear regression to check the correlation coefficient. Here, we have only the result of the calculation. But first, we define some useful variables:

$$\bar{x} = \frac{1}{m} \sum_{i=1}^m x_i, \quad \bar{y} = \frac{1}{m} \sum_{i=1}^m y_i, \quad \bar{X} = \frac{1}{m} \sum_{i=1}^m x_i^2, \quad \bar{Y} = \frac{1}{m} \sum_{i=1}^m y_i^2, \quad \bar{u} = \frac{1}{m} \sum_{i=1}^m x_i y_i \quad (14)$$

In (13), each m is the number of the measured values. We use them to calculate the intercept b and the slope

$$\bar{y} - a\bar{x}$$

$$a = \frac{\bar{u} - \bar{x}\bar{y}}{\bar{X} - \bar{x}^2} \quad \text{und} \quad b = \bar{y} - a\bar{x} \quad (15)$$

The correlation coefficient is defined by

$$r^2 = \frac{(\bar{u} - \bar{x}\bar{y})^2}{(\bar{X} - \bar{x}^2) \cdot (\bar{Y} - \bar{y}^2)} \quad (16)$$

Discuss the obtained results and compare them with the pertaining theory. In the discussion, take into account the statistical and systematic errors. Where have the biggest error occurred and how can we improve the experiment?

5 Electromotor

We do not make any quantitative measurement in this last part of the experiment. Instead, we explore an application of the physics we have learned - an electromotor.

Imagine in the present experiment, if we replace the fixed conductor loop at the torsion dynamometer with a conductor loop, which can freely rotate around its axis, we will have an oscillation around its equilibrium position. This leads to an overshoot. At this point, change the direction of the current through the conductor loop (or the external magnetic field). This will change the direction of the torque and the conductor loop will turn to 180° into equilibrium position. Now, we have a rudimentary electromotor.

Examine and explain the present model of an electromotor. Here, the polarity is not reversed by hand but by a so-called commutator to ensure us that the current of the coil is changed to the right directions.

Appendix: Magnetic Moment of a General Conductor Loop

In section 2.3, the magnetic moment of a rectangular loop with the area A has been discussed. This is given by

$$\vec{\mu} = I \cdot \vec{A}, \quad (17)$$

while the conductor loop is traversed by a current of strength I . The magnetic moment of any conductor loop can be made up of the magnetic moments of rectangular conductor loops (see Figure 5). Each planar conductor loop can be filled in by rectangular conductor loops in the described manner is shown. The anti-parallel currents, which extend inner currents, cancel each other out. So, there is a real original conductor loop equivalent to the image. According the superposition principle, we can add the torques on the different parts of the conductor loops. This results in the magnetic moment of the larger conductor loop

$$\vec{\mu} = \sum_i \vec{\mu}_i = I \sum_i \vec{A}_i = I \cdot \vec{A}. \quad (18)$$

This derivation is kept rather qualitative. But one can also derive the magnetic moment of a general conductor loop from the forces on each infinitesimal conductor pieces to have a formal integration. We refer you to the relevant elementary textbooks.

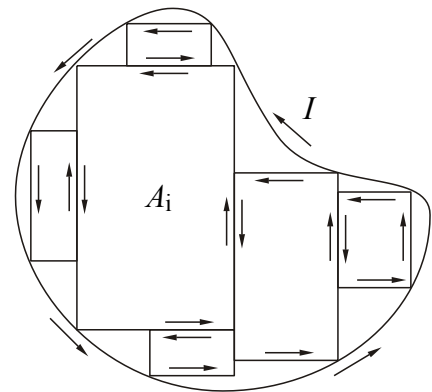


Figure 5: Magnetic moment of an arbitrary conductor loop

Preparation for the experiment

Before you perform the experiment, the trial supervisor will check your knowledge. It helps you to know what you are doing and also to make you ready for the comprehensive questions, which will be asked. All questions can be answered by studying this instruction manual. In addition, you should know by heart how to do the experiment. If you cannot answer these questions, you will have a N.V. (Unprepared) status. Here, there are some examples of (but not all) questions:

What is the measured value?

What does determine that value?

How do you determine these dependencies?

What is the setup of the experiment?

What causes this effect?

What is the application of this effect?

Explain the operation of the application!

Why is a double logarithmic representation used in this evaluation?

Is there any difference between plotting versus the measured force or the torque in this illustration?

What can be expected given which parameters?

What do r and n express?

etc....