Momentum Conservation

1 Motivation

We aim to validate momentum conservation by colliding two sliders of varying mass on an airoglide.

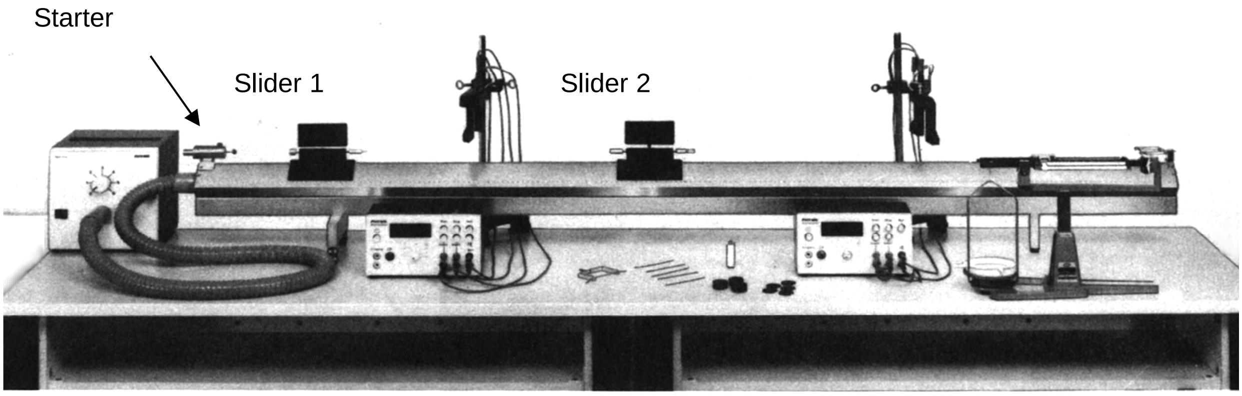

Fig. 1: Setup for the experiment on momentum conservation.

2 Conservation of momentum

Newtonian mechanics defines the momentum of a body as the product of a body's mass and its velocity. In a closed system, i.e., a system without external forces, momentum is conserved over time. This implies that the total momentum of a closed system does not change throughout any event.

The experiment aims to demonstrate this on controlled collisions. We restrict ourselves to central impacts, where the colliding objects move in the same line, such that the vectorial character of momentum is revealed only by its sign.

One distinguishes elastic and inelastic impacts:

In case of an **elastic impact** between two bodies of masses m_1 and m_2 both the kinetic energy and momentum are conserved:

Conservation of energy: $$\frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} = \frac{\vec{p}_1^{\,\prime 2}}{2m_1} + \frac{\vec{p}_2^{\,\prime 2}}{2m_2} \tag{1}$$

Conservation of Momentum: $$\vec{p}_1 + \vec{p}_2 = \vec{p}_1^{\,\prime} + \vec{p}_2^{\,\prime} \tag{2}$$

Here $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_1'$, $\vec{p}_2'$ are the momenta before and after the impact, respectively. If m_2 (Slider 2) is initially at rest ($\vec{p}_2 = 0$), energy and momentum conservation imply:

$$\vec{p}_1' = \frac{m_1 - m_2}{m_1 + m_2}\vec{p}_1 = -\frac{1 - \frac{m_1}{m_2}}{1 + \frac{m_1}{m_2}}\vec{p}_1 \tag{3}$$

$$\vec{p}_2' = \frac{2m_2}{m_1 + m_2}\vec{p}_1 = \frac{2}{1 + \frac{m_1}{m_2}}\vec{p}_1 \tag{4}$$

In case of an **inelastic impact** only momentum is conserved. The sliders join and some kinetic energy is lost in deformation. The sliders adopt the same velocity but different momenta:

$$\vec{v}_1' = \vec{v}_2' \;\Rightarrow\; \vec{p}_1' = \frac{m_1}{m_2}\vec{p}_2' \tag{5}$$

If again $\vec{p}_2 = 0$, momentum conservation and the equality of velocities after the impact imply:

$$\vec{p}_1' = \frac{1}{1+\frac{m_2}{m_1}}\vec{p}_1 \tag{6}$$

$$\vec{p}_2' = \frac{1}{1+\frac{m_1}{m_2}}\vec{p}_1 \tag{7}$$

In figures 2 to 5 the energies and momenta of the sliders after collision are compared for elastic (Fig. 2 and 3) and inelastic impact (Fig. 4 and 5) as a function of the mass ratio. The slider with mass m_2 is assumed to be at rest before collision.

The momenta of the sliders after the elastic impact are shown in **Figure 2**. The sum of momenta is constant and the total momentum therefore conserved. We further see that slider 1 is only repelled against its inital direction if $m_1/m_2 < 1$. For $m_1/m_2 = 1$ slider 1 is at rest after impact and all momentum has been transferred to slider 2. For $m_1/m_2 > 1$ both sliders move in the same direction after impact.

In **Figure 3** the energies of the sliders are depicted for elastic impacts. The total kinetic energy is constant and energy is conserved (which defines an elastic impact). Slider 1 loses energy with increasing mass ratio m_1/m_2 until the ratio reaches 1. Then slider 2 carries the complete kinetic energy.

Figure 4 shows the momenta of the sliders after an inelastic impact. Again, the total momentum is conserved. In this setup the velocities of the sliders are the same after impact. This can be seen for $m_1/m_2 = 1$ where we have $p_1 = p_2$.

Finally, the total kinetic energy of the sliders after an inelastic impact is shown in **Figure 5**. The kinetic energy is not conserved. From the initial energy (the kinetic energy of slider 1) we can determine the loss of kinetic energy. Since here the velocities of the sliders after the inelastic impact are the same, the above formulae imply that the loss of kinetic energy for $m_1/m_2 = 1$ is exactly equal to the total kinetic energy of the sliders after the collision.

3 Setup and Procedure

The setup is built according to Fig. 1. The sliders move on an airoglide so that friction can be neglected. The starter kicks off the slider. The spring can be set to three different predetermined positions to allow for reproducible runs. Always use the spring position delivering most kinetic energy.

To determine the momentum, read from the counter the duration the light barrier is intercepted by the screen on top of the slider.

The slider velocity is measured with a light barrier consisting of a photo sensor shaded by cover plates on the sliders. From the interception duration t, the length ℓ of the cover plate and the mass of the slider the momentum

$$p = m_{ges}\frac{\ell}{t}$$

is calculated. Since momentum is a vectorial quantity, a change of direction has to be accounted for by a changing sign.

When varying the mass ratio one has to make sure that masses are added symmetrically, i.e., on both ends of the slider. Before the experiments the rail has to be adjusted correctly – make sure it is leveled. The direction of the rail should only be changed in agreement with the tutor.

It is expected that you prepare a table at home in which the measured data can readily be noted down when conducting the experiment.

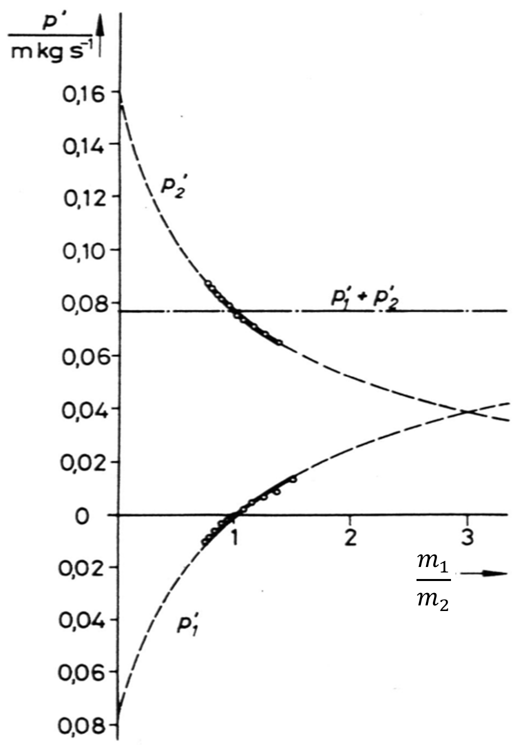

Fig. 2: Elastic impact: Momenta after impact as function of mass ratio
(- - - Theory, -.-.- Total Momentum)

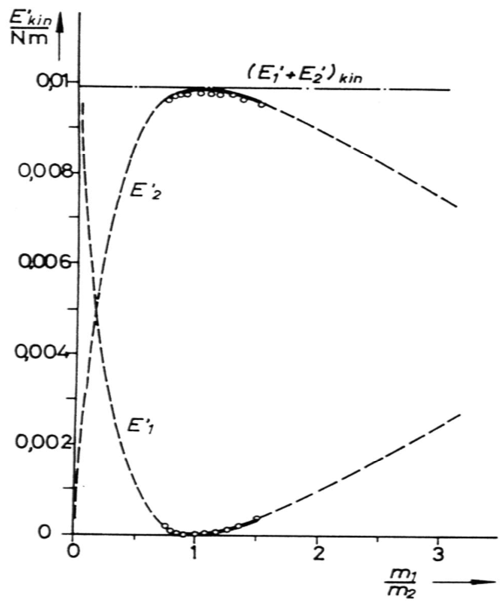

Fig. 3: Elastic impact: Energy after impact as function of mass ratio
(- - - Theory, -.-.- Total Energy)

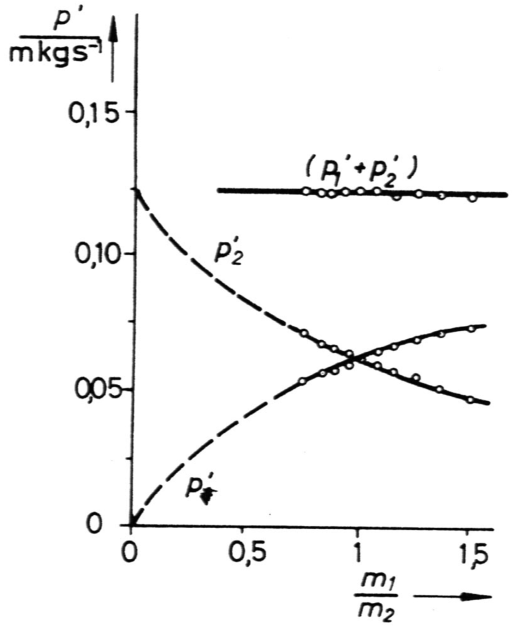

Fig. 4: Inelastic impact: Momenta after impact as function of mass ratio
(- - - Theory, -.-.- Total Momentum)

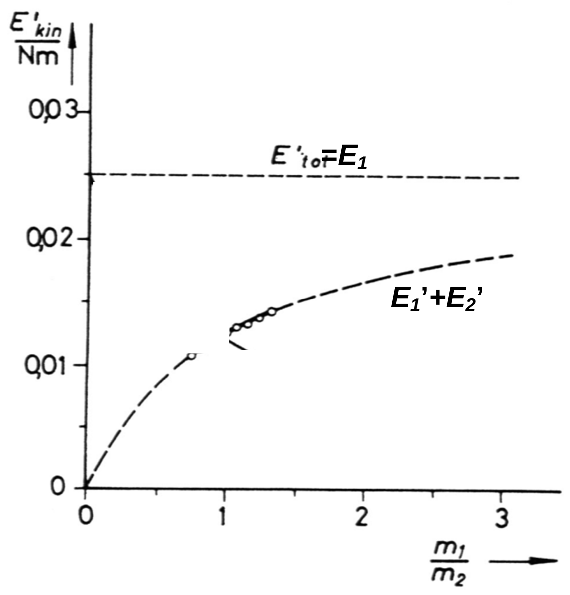

Fig. 5: Inelastic impact: Energy after impact as function of mass ratio (**- - -** Theory)

4 Measurements

For both kinds of impact:

- Measure the interception duration of the light barrier before the impact. Load 100 g extra weight onto slider 1.
- Keep slider 1 loaded with 100 g extra weight. Load slider 2 with extra masses of 0, 20, 40, …, 140 and 240 g. Measure the interception duration three times for both sliders after the impact in each configuration.
- Measure all other quantities needed to calculate the momentum.

5 Evaluation

- Describe the experimental setup.
- From the data for elastic and inelastic impacts determine the initial momentum. To do this build the arithmetic mean of the three time measurements and their standard deviation. Calculate the momentum and its propagated error.
- From the data calculate for each mass ratio the momenta of both sliders after impact and their sum. Also determine the kinetic energies of both sliders and their sum after impact. All results need an error estimation.
- For the elastic and inelastic collisions collect the individual momenta of the sliders and the total momentum after impact in a diagram. Have the mass ratio m_1/m_2 on the x-axis and the momenta on the y-axis. Do the same for the kinetic energy and the total kinetic energy.

  Comment on the curves: What do you see? Are there trends? Are there characteristic points (e.g. mass ratio of 1) where a certain result is expected?
- Use formulae (3) and (4) to calculate the theoretical curves for the elastic impact and (6),(7) for the inelastic impact. Add the theoretical curves to the experimental data and discuss differences.
- What kinds of errors enter the experiment? How could this be improved?