

# Ideal Gas Equation of State

## 1. Purpose of the experiment

In this experiment, the thermodynamic behavior of an approximately ideal gas will be investigated. The state of the gas is represented by three quantities, which are pressure  $p$ , temperature  $T$ , and volume  $V$ .

As we have three different variables at the lab, we study the physical relationships of the variables in three independent conditions. In each condition, we keep one of the variables constant, and we measure the second one as a function of the third variable. Regarding to these three different conditions we can determine the expansion coefficient  $\alpha$ , the pressure coefficient  $\beta$ , and the compressibility  $\kappa$  of the gas.

## 2. Theory

To illustrate the basic concepts of thermodynamics, we examine the model of an ideal gas. In this model, the gas particles are assumed as point particles of classical mechanics, and they have only elastic collisions and there is no interaction between the particles. Elastic collisions have to be considered in this conceptual model; otherwise it does not set a thermal equilibrium. We assumed a simple approximative model of a real gas, however the particles have an intrinsic volume and also they interact with each other inelastically. This approximation is valid while we are far from the condensation point of the gas.

We can predict the state functions pressure  $p$ , temperature  $T$  and volume  $V$  of the gas by assuming this simple approximation.

If we consider the temperature of an enclosed gas is constant, the product of pressure and volume will be constant and leads to **Boyle-Mariotte's law**:

$$p \cdot V \Big|_{T=\text{const}} = \text{const}_1$$

The volume of an enclosed gas is proportional to the absolute temperature, as long as the pressure does not change which shows **Gay-Lussac's 1st law**.

$$\frac{V}{T} \Big|_{p=\text{const}} = \text{const}_2$$

When the pressure of a closed gas is proportional to the absolute temperature, as long as the volume is not changed, we have **Gay-Lussac 2<sup>nd</sup> law** or **Amontons' law**.

$$\frac{p}{T} \Big|_{V=\text{const}} = \text{const}_3$$

The mentioned laws can be summarized to the equation  $p \cdot V = \text{const}_4 \cdot T$ . It turns out that this constant is proportional to the number of particles, which results in the **ideal gas equation of state** or **Ideal gas law**:

$$p \cdot V = \nu \cdot R \cdot T$$

This equation is independent of the chemical function of the gas of interest.  $R = 8.314 \text{ J / (K.mol)}$  is referred to the universal gas constant, and  $v$  is the mass quantity in mole. For further characterization, the following three quantities are defined:

**expansion coefficient**  $\alpha = \frac{1}{V_0} \cdot \frac{\partial V}{\partial T} \Big|_{p=\text{const}}$  with  $V_0 = V(T = 0^\circ\text{C})$

**pressure coefficient**  $\beta = \frac{1}{p_0} \cdot \frac{\partial p}{\partial T} \Big|_{V=\text{const}}$  with  $p_0 = p(T = 0^\circ\text{C})$

**compressibility**  $\kappa = -\frac{1}{V_0} \cdot \frac{\partial V}{\partial p} \Big|_{T=\text{const}}$

The compressibility  $\kappa$  (kappa) can be expressed by the coefficients  $\alpha$  and  $\beta$ . Just we need the mathematical insertion in the formula.

$f(x, y)$  is a function of two variables. The operation  $\frac{\partial f}{\partial x} \Big|_{y=\text{const}}$  is a partial derivative, meaning that  $f(x, y)$  is derived by  $x$  while  $y$  is assumed to be constant. The **total differential** of the function  $f(x, y)$  is defined as:

$$df = \frac{\partial f}{\partial x} \Big|_{y=\text{const}} \cdot dx + \frac{\partial f}{\partial y} \Big|_{x=\text{const}} \cdot dy.$$

This represents the change of the function  $f(x, y)$  conducting by a change in  $x$  and  $y$ , shown by  $dx$  and  $dy$ .

The equation of state can also be written as follows:

$$V = \frac{v \cdot R \cdot T}{p}$$

The volume  $V$  is a function of  $p$  and  $T$  ( $v$  is constant). The total differential of  $V$  is:

$$dV = \frac{\partial V}{\partial T} \Big|_{p=\text{const}} \cdot dT + \frac{\partial V}{\partial p} \Big|_{T=\text{const}} \cdot dp$$

For constant volume, that is,  $dV = 0$ , it follows:

$$\frac{\frac{\partial V}{\partial T} \Big|_{p=\text{const}}}{\frac{\partial V}{\partial p} \Big|_{T=\text{const}}} = -\frac{dp}{dT} = -\frac{\partial p}{\partial T} \Big|_{V=\text{const}}$$

The right side of the equality signs holds because the pressure  $p$  depends only on  $T$  at constant  $V$ . With the above definitions for the thermal coefficients  $\alpha$  and  $\beta$ ,  $\kappa$  can be calculated by the following relation:

$$p_0 \cdot \beta = \frac{\alpha}{\kappa}$$

### 3. Experimental setup

The design of the experiment is schematically shown in the Figure 1.

Our ideal gas is the air, which is trapped in an enclosed mercury glass bulb. The temperature is adjusted with a thermostat-controlled water flow surrounding the measurement volume. It can be read on a thermometer. Compensation vessel can change the pressure exerted on the gas by a vertically adjusting in the height.

The pressure  $p$  of the gas is the sum of external air pressure  $P_a$  and the pressure  $p$ , which is generated by the different height of mercury column.

$$p = p_a + \Delta p$$

The height difference between the two mercury surfaces in mm  $\Delta h$  is exactly the pressure  $p$  in the unit mmHg (millimeters of mercury). A barometer is used to determine the external air pressure  $P_a$ . The measurement volume is derived from the read of the height of the air column  $\Delta l$  crossed to the known cross-sectional area  $A$  of the pipe.

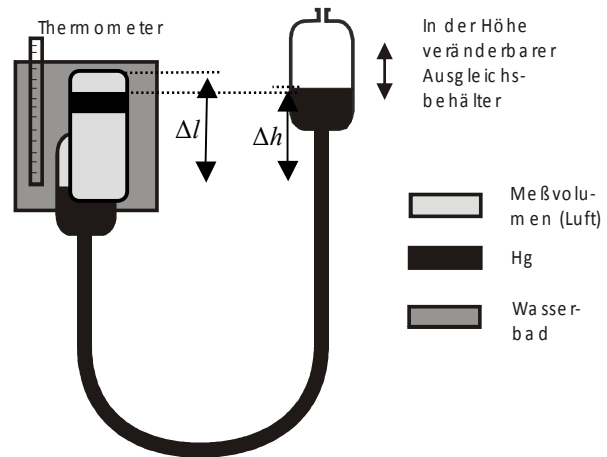


Fig. 1: schematic of experimental setup

### 4. Task

Show that the air behaves as an ideal gas in the adjustable temperature and pressure range by

a) the volume as a function of temperature at constant pressure (1<sup>th</sup> law of Gay-Lussac)

b) the volume as a function of pressure at constant temperature (Boyle's law)

c) the pressure as a function of temperature at constant volume (2<sup>nd</sup> law of Gay-Lussac)

d) Determine the expansion coefficient  $\alpha$ , the pressure coefficient  $\beta$  and the compressibility  $\kappa$ .

Note: The expansion tank is closed to reduce harmful mercury vapors with a rubber stopper. During the measurement it must be removed otherwise keep the vessel always closed!

Theoretical values:  $\alpha = 1/273.15 \text{ K} = 3,661 \times 10^{-3} \text{ K}^{-1}$   
 $\beta = 1/273.15 \text{ K} = 3,661 \times 10^{-3} \text{ K}^{-1}$   
 $\kappa = 1/p_0$  (The exact value depends on the molar amount of the air which is unknown in this experiment.)

<sup>i</sup> 1 mmHg = 1 Torr = 1.33322 mbar and 1 mbar = 1 hPa

<sup>ii</sup> Cross sectional area of the rohr:  $A = 1.02 \times 10^{-4} \text{ m}^2$