Hall effect

1 Goal

Characteristic data of a test semiconductor (Germanium) should be determined by measurements of the electric current and the Hall voltage. In particular, the current carrier density and energy gap are determined that way.
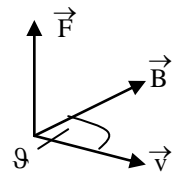
2 Basic data

2.1 Hall effect

On a moving charge q with speed $\vec{v}$ the Lorentz force $\vec{F}_L$ has an influence in the magnet field of flux-density $\vec{B}$:

$$\vec{F}_L = q \cdot (\vec{v} \times \vec{B})$$

If $\vec{v}$ and $\vec{B}$ are perpendicular to each other, it is applicable the total sum of Lorentz force:

$$F_L = |q| \cdot v \cdot B. \tag{1}$$

In the process of the current-carrying conductor, where a magnetic field is perpendicular to the current direction, it leads to the Hall effect. The electric current carriers are deviated by Lorentz force to the side and therefore come together on one of both conductor edges, fig. 1. The same size opposite charge appears on the other conductor edge in consequence of electron de-enrichment. As a result, an electric field $\vec{E}$ is established between the conductor edges, until balance of forces between electric force $\vec{F}_{el} = q \cdot \vec{E}$ and Lorentz force $\vec{F}_L$ prevails (stationary case, fig. 2). So, the voltage which appeared this way is the Hall voltage U_{Hall}.
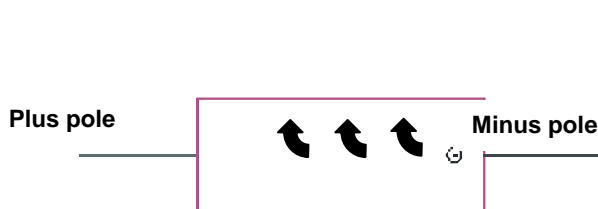

fig. 1: Direction change of charge carrier. Magnetic field perpendicular to the diagram.
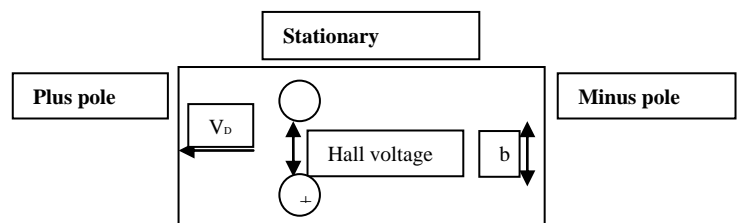

fig. 2: Hall voltage in a conductor. Magnetic field perpendicular to the plane.

In stationary case, the total sum of Lorentz force und electric force are of the same size,

$$F_{el} = |q| \cdot E = F_L = |q| \cdot v \cdot B$$

$$v \cdot B = E = \frac{U_{Hall}}{b}$$

$$U_{Hall} = v \cdot B \cdot b, \tag{2}$$

where b is the width of conductor. Except for the speed v of the electrons, all variables are directly measurable. There is the speed from the consideration of the current I in the conductor:

$$I = \frac{Q}{t} = \frac{e \cdot N}{t} = \frac{e \cdot nV}{t} = \frac{en \cdot bdl}{t} = enbd \cdot v$$

$$v = \frac{I}{enbd} \tag{3}$$

Q: total charge
N: number of electrons
e: elementary charge

$n = \frac{N}{V}$ Charge carrier concentration, $V=bdl$ volumes
b,d,l: width, thickness and length of the conductor

The Hall voltage results from (2) and (3):

$$\boxed{U_{Hall} = \frac{1}{e \cdot n} \cdot \frac{I \cdot B}{d} = R_H \cdot \frac{I \cdot B}{d}} \tag{4}$$

$\frac{1}{e \cdot n}$ it summarizes to the material- and temperature-dependent value R_H. R_H is called as Hall constant. $R_H = 5 \cdot 10^{-3}$ m^3/As is by experiment in the used n-doped Germanium at room temperature.

2.2 Band structure

Band structures periodicity appear in solid bodies by lattice in the energy spectrum. In that basis those states, which are occupied by electrons in the band structure of solid bodies can be distinguished in insulators, metals and semiconductor components. Metals differ by the fact that the highest occupied energy band is occupied only partially. The electrons in this so-called conduction band can make without cut-off energy for electrical conduction, so that metals have high electrical conduction (fig. 3).
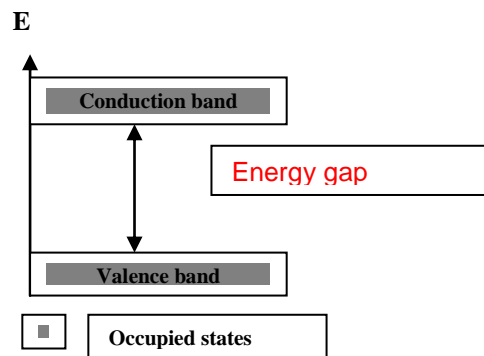

fig.3: Scheme of a metal. The electrons in only partially conduction band can make without cut-off energy for electrical conduction.

If all states are occupied in an energy band, the electrons must be excited in the next higher energy band to jump to free places of states and to control the current. If the energy gap band is relatively large between these bands (ca. 8 eV) in relation to the thermal energy at room temperature (ca. 0.025 eV), no electrons can be excited, and the solid body behaves as an insulator (fig. 4).
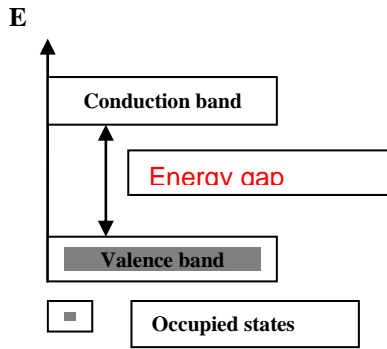
fig. 4: Scheme of an insulator or a semiconductor. With large forbidden band it is said about insulators, with small forbidden band (ca. 1 eV), which can be overcome by thermal or optical excitation, it is said about semiconductors.

If the forbidden energy band is smaller (ca. 1 eV), the solid body can be called "a semiconductor". The charge carrier can be excited by their thermal energy, or can be excited by the light in the conduction band. For low temperatures, where the thermal energy is not sufficient to excite charge carriers from the valence band into the conduction band, and the semiconductors are also insulators without excitation by the light. For room temperature the semiconductors still show a very low electrical conduction. In order to increase the electrical conduction of semiconductors, it is possible to dope them. In this case, foreign atoms with different numbers of valence electrons are incorporated into the crystal (see Addition). Therefore the donator or acceptor levels appear in the band structure. The electrons or holes of these foreign atoms need significantly low energy to be excited in the conduction or valence band and to contribute to electrical conduction (fig. 5).
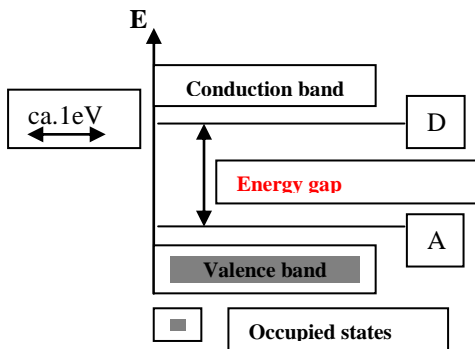

fig. 5: scheme of a dope semiconductor. A donator (D) and an acceptor level (A) appear by doping. Electrons or holes from this level can be already excited with energies, which are much smaller than the energy gap, into conduction and valence band.

The charge carriers of these foreign atoms dominate at low temperatures, semiconductor is situated in reserve of unionized impurity centers (field I in fig. 6). If the temperature rises, it comes in the so-called impurity exhaustion (field II in fig. 6), which means, all electrons or holes from the donator or acceptor levels are excited to the conduction and valence band. At higher temperatures, a lot of electrons have such great energy that they can overcome the forbidden energy band and can go from the valence to the conduction band. The same process can also be described for holes from the conduction band to the valence band. This is called the intrinsic region (field III in fig. 6).

We consider below only the specific case of an n-doped semiconductor crystal, as it is used in this experiment. A donator level under the conduction band appears by n-type doping. Here used germanium-crystal is situated at room temperature on the impurity exhaustion, i.e. all electrons from the donator level are in the conduction band. Heating (task 4) it brings the germanium crystal in the intrinsic region. From static thermodynamics, the charge carrier concentration in the conduction band appears in the intrinsic region (field III in fig. 6) to:

$$n = n_0 \cdot e^{-\frac{E_G}{2kT}}$$

n: charge carrier concentration, *T:* Kelvin temperature, $k = 8{,}617 \cdot 10^{-5}$ eV/K Boltzmann constant, E_G: forbidden band of semiconductor, n_0: charge carrier concentration for $T \to \infty$

We receive by logarithm:

$$\ln n = \ln n_0 - \frac{E_G}{2kT} = \ln n_0 - \frac{E_G}{2k} \cdot \frac{1}{T} \tag{5}$$

3

So if we bring the logarithm of the charge carrier concentration n to 1 / T, we receive from Eq. (5) in the intrinsic region (field III) as the slope $-E_G$ / 2k. In the field of reserve of unionized impurity centers (field I) we receive in accordance with the slope $-E_{Don}$ / 2k, here the electrons must be excited by thermal excitation from the donator levels in the conduction band and overcome E_{Don}. In field II the charge carrier concentration changes only slightly with temperature. Here all donators are ionized, but the thermal energy is not enough to excite electrons from the valence band into the conduction band.
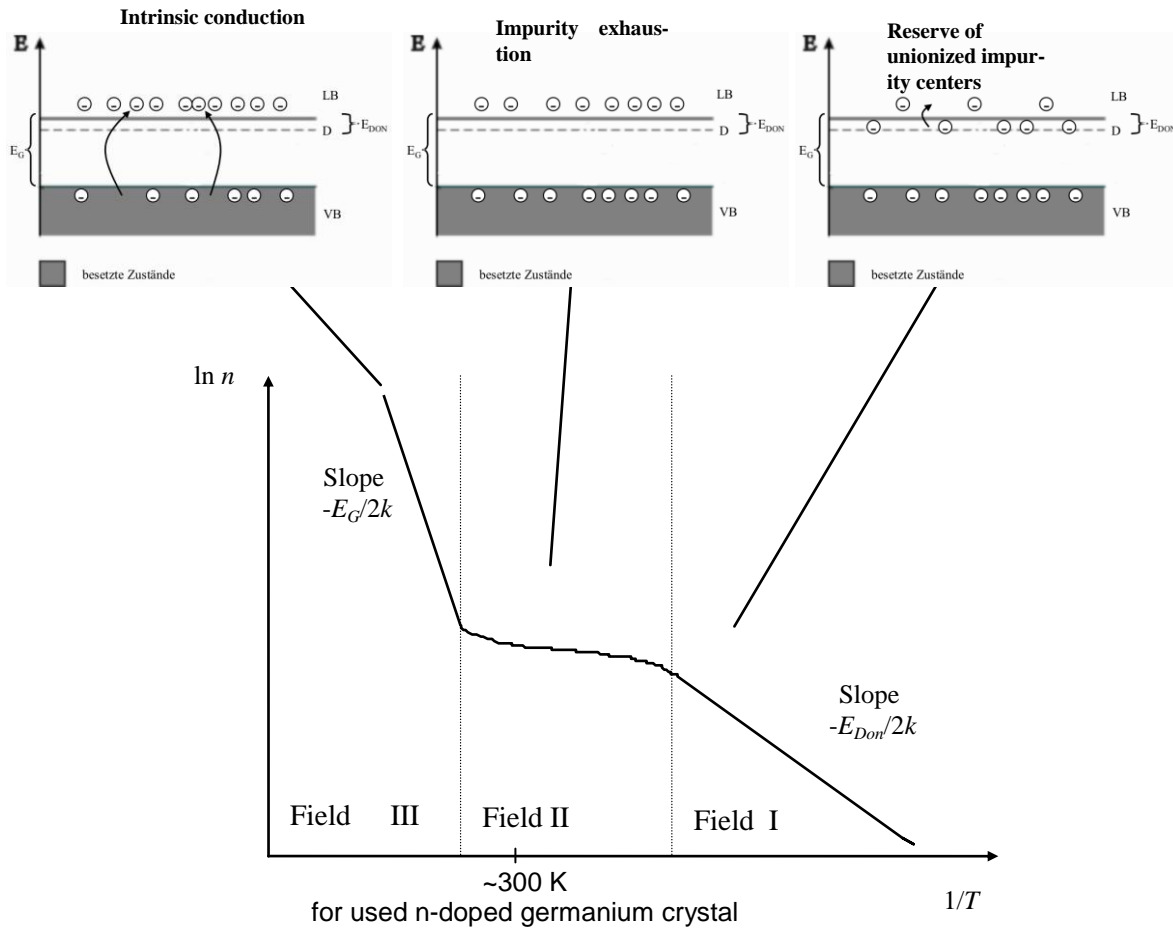
fig. 6: charge carrier concentration logarithmically brought to 1/T. From the slope in the intrinsic region we receive the forbidden band of semiconductor E_G, from the slope in the field of reserve of unionized impurity centers – the depth of the donator levels E_{Don}.

For further information about the band models, please see the Addition "binding relation and band models".
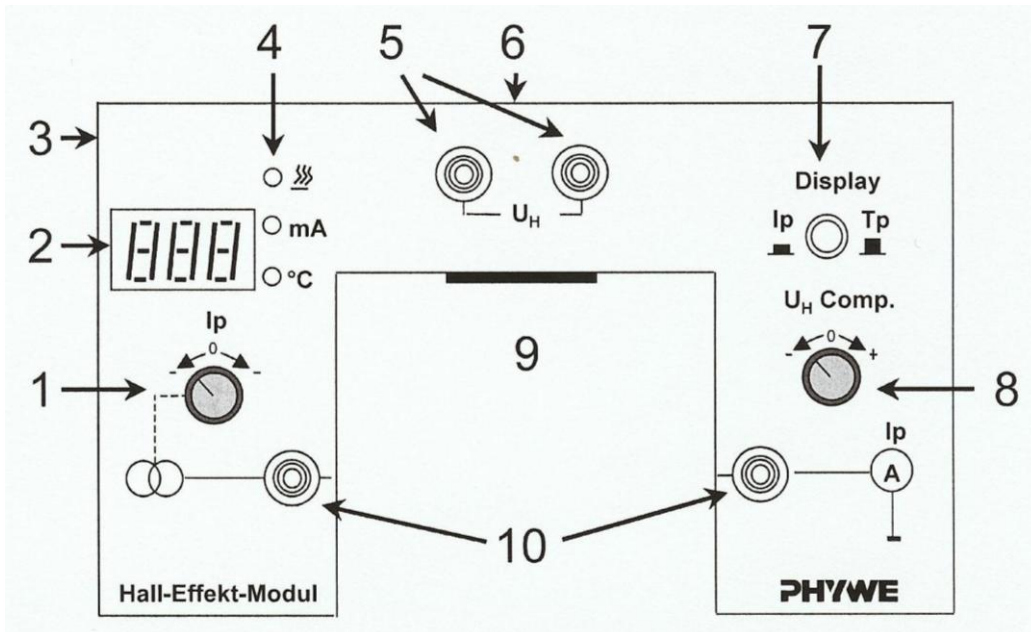
3 Building

Fig. 7: Forward side of Hall effect-measurement device

The measurement device is supplied with voltage with the help of the sockets on the backside of it. The Hall element is situated on the charging chute 9. The set of the control current is over the potentiometer 1 and the sockets 10, the longitudinal voltage can be measured with the Hall element. The Hall voltage is measured in the sockets 5. False voltage appears by possible small offset of the voltage taps, in addition to the measured Hall voltage, which can be compensated with the potentiometer 8.

Among the test, there is a heating spiral, which is turned on the backside. If the LED 4 shines, the heating works. We can select using the button 7: the test current or the test temperature should be shown on the display 2.

The Hall device is fixed with measurement device on mount and hangs between the pole shoes of an electromagnet.

ATTENTION!

The Hall element is very sensitive the same as all semiconductors. It breaks easily or separates from the carrier plate. It suffers in the heat. Therefore, the temperature should not be above 150 ° C.

The carrier board with the test semiconductor can become very hot during work. There is a risk to burn the hands. The board may be touched only in "on" position after an appropriate cool time.

4 Task formulation:

1. (Take sonde from electromagnets!) In the absence of a magnet field the voltage U_{Hall} is calibrated to zero at a test current $I_P=0$. Then the false voltage U_{Fehl} is determined with $I_P=+50$ or -50 mA. The longitudinal voltage U_S is noted for both installations. The control system element 8 will not be changed any more. We prove with diagrams that $U_{Fehl} \sim I_P$ is as $U_{Fehl} \sim U_S$! For further correction of accounts a) should be provided the quotient $\Delta R := U_{Fehl}/I_P$ (for task 2), and b) the quotient $U_{Fehl}/U_S = \Delta\ell/\ell$ (for task 4). $\Delta\ell$ is the offset distance between the Hall contacts; ℓ is the total distance between the current contacts. Please keep in mind: while the distances are naturally not independent of temperature, also not the voltages U_{Fehl}, or U_S.

2. At constant magnetic field (current through the spools of the electromagnet $I_M = 2$ A), the Hall voltage should be measured depending on test current I_P. The measured and correcting quantities are entered into a diagram.

3. With high test current ($I_P = 50$ mA), the Hall voltage should be measured depending on flowing current I_M by the magnet. The magnetic flux density B can be calculated with equation (4) and the Hall constants R_H for room temperature.
   It should be represented U_{Hall} and B depending on the current by the magnet in diagrams (take into consideration the correction!). It varies I_M from -2 to $+2$ A without direction change, stops at 2 A, then back from $+2$ to -2 A. Both branches should be marked in the same diagram to show the hysteresis.

4. At high test current (50 mA) and constant magnetic field ($I_M = 2$ A), the Hall voltage U_{Hall} and the voltage drop U_S should be measured in the sonde depending on the temperature. The charge carrier concentration n can be determined for each temperature from the Hall voltage with equation (4) and $B(I)$-diagram from task 3 (consider false voltage!). It carries $\ln(n)$ to $1/T$ (Kelvin T) in a diagram and determines from the slope in the intrinsic region the forbidden band E_G of germanium (comparison with theory). In addition, it carries R_H and U_{Hall} in a diagram against the temperature.

All diagrams can be discussed (comparison with theory).

5 Addition:

Binding relations and band models.

Most important raw materials for the production of semiconductor - building elements (ex. Ge and Si) are tetravalent, i.e., the outermost electron shell contains their atoms: 4 valence electrons each. They form regular crystals in such a way that each atom is surrounded by 4 neighboring atoms at equal intervals; fig. 8 shows a simplified two-dimensional model. The binding of atoms in the crystalline combination comes from such a fact that the neighboring atoms exchange constantly their valence electrons ("exchange interaction", "electron pair binding", "covalent binding"). At very low temperatures ($T \approx 0$ K) all electrons have got an energy state, which links them to the atoms. Therefore, a semiconductor contains at temperature T=0 K no free charge carriers; its resistibility is equivalent to an insulator ($R \rightarrow \infty$). By increasing the temperature the crystal lattice gives more energy each time, so that a growing proportion of the electrons separates from bindings and is freely movable in the crystal. The electrical conduction of the semiconductor increases accordingly with increasing temperature strongly (exponentially), its resistibility decreases accordingly.

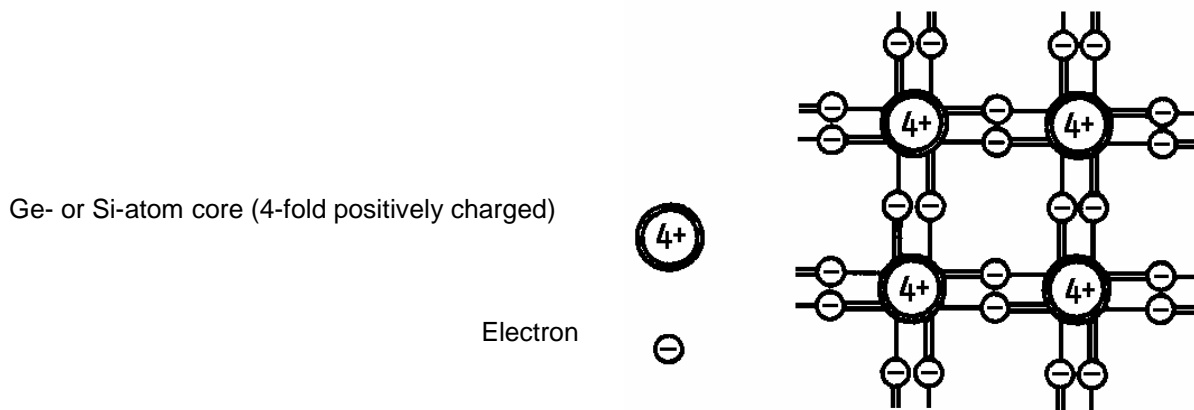Ge- or Si-atom core (4-fold positively charged)

Electron

fig. 8: electron pair binding in an undoped crystalline semiconductor (highly simplified level representation)

The electrical conduction of the semiconductor is, however, not determined by the free electrons: Everywhere where an electron leaves its atomic binding, a positive electric residue remains there (defect electron, hole). These positively charged holes may also keep moving in the crystal lattice, while they are filled with an electron from a neighboring binding, though then a new hole appears in the neighborhood. By consecutive repeating of this process a hole can also move through the crystal lattice as a freed electron from its binding (hole conduction), so it behaves as a positive free charge carrier.

Free electrons and holes always appear in an undoped semiconductor in pairs (pairing), so that both bring about the electrical conduction. The electrical conduction of a semiconductor therefore depends not only on the density of free electrons but also on the density of free holes in the interior. In a great measure independent of the temperature (field II in fig. 6) the density of the free electrons can be increased by building into 5-valent foreign atoms (eg. P, As, Sb) (n-type doping), the density of the free holes - by the building into 3-valent foreign atoms (eg. B, Al, In) (p-type doping) in the crystal lattice. Not only semiconductor crystals, whose electrical conduction is mainly carried by electrons (n-conductors), but also those, whose electrical conduction is mainly based on holes (p-conductor), can therefore be produced by appropriate doping. The electrical conduction of the doped crystals grows with the proportion of the brought foreign atoms; they can be changed by different strong doping over several decimal exponents.