Inverse-square law and attenuation of gamma radiation

1. Introduction

Similar to the electrons in the atomic shell, the nucleons (i.e. protons or neutrons) in the atomic nucleus can be excited from their energy ground state to a higher state of energy. In this high energy state, they might emit a photon (quantized electromagnetic radiation) spontaneously to lower their energy like electrons do in the shell. Nevertheless the energies in the nucleus are a million times higher ($\sim$1 MeV vs. $\sim$1 eV) than in the shell, so that the photons coming out of the nucleus have much higher energy.

The described energy lowering process in the nucleus by spontaneous emission of high energy photons is called *gamma decay* and the electromagnetic radiation described by the photons is called *gamma radiation*.

2. ^{60}Co and its decay

For our measurements we will use the β^--decay of ^{60}*Co that provides gamma radiation* at a second reaction step at two different photon energies $E_{\gamma_1} = 1.333$ MeV and $E_{\gamma_2} = 1.173$ MeV:

1) ${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni}^* + e^- + \bar{\nu}_e$
2) ${}^{60}_{28}\text{Ni}^* \rightarrow {}^{60}_{28}\text{Ni} + \gamma_1 + (\gamma_2)$

The antineutrino $\bar{\nu}_e$ is a very weakly interacting particle, so we will neglect this part of the reaction completely.

The excited ${}^{60}_{28}Ni^*$ *emits two high energy photons*, called γ-quanta, that serve as the provided gamma radiation.

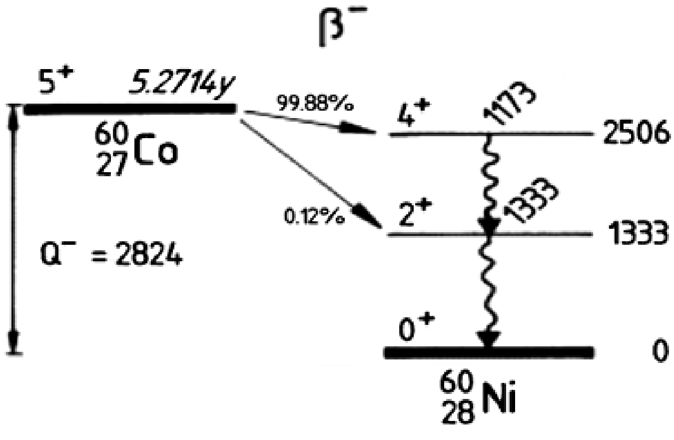

Figure 1: Decay of ^{60}Co. The energy differences between the states are given in keV. With probability of 99.88 %, the ^{60}Co decays to the 4^+-state of ^{60}Ni.*

3. Inverse-square law

Say our source emits a certain expanding total amount of γ-quanta per time $\dot{N}_{tot}$ through the detector surface A_D and say the γ-quanta can expand linearly in free space, i.e. without any interaction due to matter, then because of the conservation of γ-quanta the flux $\dot{N}_{tot} \cdot \mathrm{A_D}$ is a conserved quantity.

Since such an *expansion takes place on a spherical surface* $A_o = 4\pi r^2$ with r being the distance to the source, we can write

$$\dot{N}(r) = \dot{N}_{tot} \cdot \frac{A_D}{4\pi r^2}. \qquad (1)$$

One can see a similar effect by blowing up a balloon with dots on its surface. In this case the dots on the balloons surface are the conserved "quanta".

During the practical course we want to verify that $\dot{N}(r) \propto r^{-2}$. It is advantageous to do that by taking the logarithm of equation (1)

$$\ln\left[\dot{N}(r)\right] = -2 \cdot \ln(r) + C, \qquad (2)$$

so that one gets a linear equation. The experimental constant $C = \ln\left(\dot{N}_{tot} \cdot \frac{A_D}{4\pi}\right)$ shall not be analysed.

4. Attenuation of gamma rays

4.1. Three independent absorption effects

The attenuation of gamma rays due to matter is described by three different and independent effects:

a) *Compton scattering*: This effect is named after Arthur Compton (1892-1962). He observed that γ-quanta are scattered by quasi-free electrons sitting on the outer shell of an atom. During the scattering process they transfer a part of their energy to the electrons.

b) *Photoelectric effect*: Albert Einstein explained in 1905 that a γ-quantum can transfer its total energy to a bound electron in the atom. If the energy transferred to the electron is high enough, the atom will be ionised. Such an ionisation process was first observed by Heinrich Hertz 1887.

c) *Pair production*: The production of e^+-e^--pairs was discovered by Carl Anderson in 1932. When the γ-quantum propagates in a strong Coulomb field, e.g. close to an atomic nucleus, it can decay into an e^+-e^--pair. For reasons of energy conservation the energy E_γ has to be higher than twice the rest energy of an electron/positron: $E_\gamma > 2m_e c^2 = 1.022$ MeV. The existence of the close atomic nucleus ensures the momentum conservation and should be also considered for a precise energy conservation analysis.

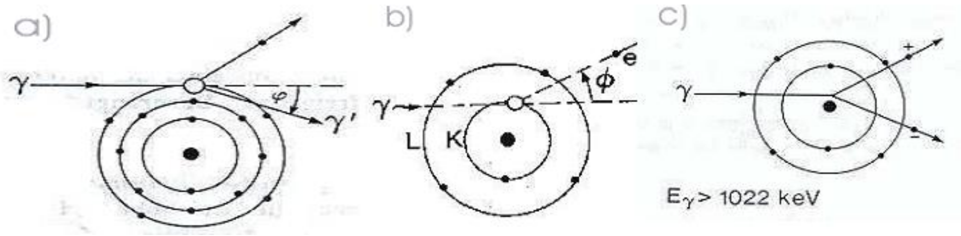

Figure 2: Sketchesof the three independet effects of gamma ray attenuation: a) Compton scattering on quasi-free electrons, b) Photoelectric effect on bound electrons, c) Pair production close to an atomic nucleus.

4.2. Characteristic attenuation properties of a material

The attenuation of a certain material can be described by a simplified but very useful *exponential attenuation equation*

$$\dot{N}(d) = \dot{N}(0) \cdot \mathrm{e}^{-\mu d}. \quad (3)$$

This equation relates the amount of counted quanta per time $\dot{N}$ to the thickness d of the attenuating material parallel to the axis of the Geiger counter (detector) by using only one parameter μ, called the *attenuation coefficient*. It contains all information about the interaction between the gamma rays at a certain energy and the material with its properties. Concerning the three effects named above, the attenuation coefficient is capable of being totalled

$$\mu = \sigma + \tau + \kappa, \quad (4)$$

where σ is the attenuation coefficient of the Compton scattering, τ the one of the photoelectric effect and κ stands for attenuation due the pair production process. In our case, i.e. 0.5 MeV$< E_\gamma < 5$ MeV, the Compton scattering is dominating.

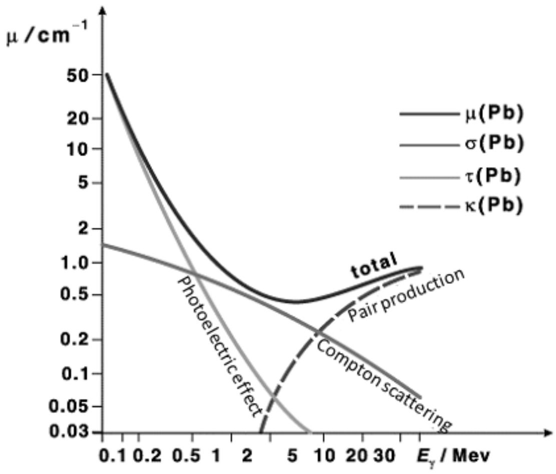

Figure 3: Plot of the attenuation coefficient μ of Pb over the energy of the incoming gamma radiation E_γ. The total attenuation coefficient is the sum of all three effects: Compton scattering σ, Photoelectric effect τ and Pair production κ.

During the practical course we want to measure the attenuation coefficient μ. Therefore we again take the logarithm of the above equation (3) and get

$$\ln[\dot{N}(d)] = -\mu d + \ln[\dot{N}(0)]. \tag{5}$$

From this linear equation we can extract μ after measuring $\dot{N}(d)$ for a given material. With μ we can directly evaluate the *half value thickness* (HVT)

$$\mathrm{d}_{1/2} = \frac{\ln 2}{\mu}. \tag{6}$$

So we can predict how thick the inserted material should be to reduce the counted quanta per time by one half.

Furthermore, we want to calculate the *mass attenuation coefficient* μ_m. Therefore we measure the density ρ of the material and divide μ by that quantity:

$$\mu_m = \mu/\rho. \tag{7}$$

The mass attenuation coefficient is often used in literature to compare different materials. The value is independent of how compressed the analysed material will be and it is similar for many materials.

5. Experimental setup

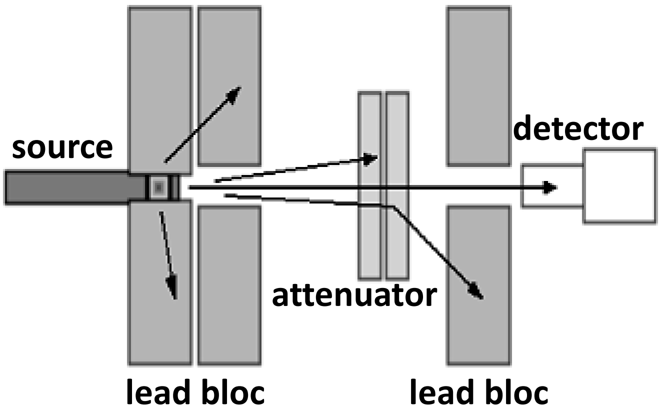

Figure 4: sketch of the experimental setup.

The source of ^{60}Co is bolted at a lead bloc. To delimit the gamma rays there are other lead blocs all around the setup. The detector is inserted to another lead bloc at the end of the setup and connected to a digital counter. Between the source and the detector one can dispose the attenuation material on a pedestal.
The time for one measurement, i.e. to get 1 000 counts on the digital counter, can be measured by a stopwatch.
In order to obtain the density of a given material, there are a scale and a calliper gauge on hand.

6. Exercises

At first, you have to measure the *background rate* $\dot{N}_0$ over a time of 3 minutes (180 seconds). All further rates during the whole experiment should be subtracted by $\dot{N}_0$.

6.1. Inverse-square law

The rate $\dot{N}(r) = N(r)/t$ is measured by *varying the distance* r between the source and the detector. Hence, the source bloc is shifted in 2 cm steps from 20 cm up to 40 cm. At every distance the time for *at least 1 000 counts* is measured.

Attention: Always note the exact number of counts corresponding to your time measurement!

Plot the logarithm $\ln[\dot{N}(r)]$ over $\ln(r)$ and extract the slope value by linear regression.

Do you get a value of -2? How is the error of this value? Did you verify the inverse-square law, i.e. equation (1)?

6.2. Attenuation of gamma rays

At least two given materials should be fully characterised during the practical course. For every material you should determine the attenuation coefficient μ, the HVT $d_{1/2}$ and the mass attenuation coefficient μ_m.

For all attenuation measurements $r = 20$ cm. Here the rate $\dot{N}(d) = N(d)/t$ is measured by *varying the thickness* d of the attenuating material parallel to the axis of the detector standing on the pedestal. For at least 5 different values of d the time for at least 1 000 counts is measured.

Corresponding to equation (5) the logarithm $\ln[\dot{N}(d)]$ is plotted over d. By using the linear regression technique one can extract μ and its error. With equation (6) you can calculate $d_{1/2}$ and its error.

In order to get μ_m you first the density $\rho = \frac{\text{mass}}{\text{volume}}$ of the given material. The volume V can be sized by using the calliper gauge and the mass is weighed by employing the scale. *You should estimate your maximum error to all of the measurements and propagate them to the density!*

Once you get the density, you will obtain μ_m with equation (7). How is the error of this value? Compare the results of the two given materials!

Security advice:

The experiment is dealing with radioactive material and specific safety instructions are valid. You will get special radiation protection instructions by the experiment adviser. Every student has to confirm that he/she got the instructions before starting any measurements.

This experiment is not allowed for certain groups of people. Eating and Drinking is not allowed in the experiments room.