

Electron Diffraction

1. Introduction

The 'wave-particle dualism' of matter plays a central role in modern physics. While on the deepest level any measurement device will only detect particles (cf. 'shot noise', 'collapse of the wave function'), the equations (Schrödinger/Dirac-Equation) that determine the probability to find these particles at a certain position in time and space allow for superposition and interference which are phenomena attributed to waves. The particle nature of the electron can be inferred from diffraction experiments with light (Compton-Effect). On the other hand, its wave-like propagation can be observed in diffraction and interference experiments. However the electron cannot be said to 'be' a wave in round terms.

In this experiment we aim to reveal the wave-like nature by diffracting an electron beam at the atom lattice of a graphite crystal. We determine the wavelength attributed to the electrons and several lattice plane spacings of graphite.

2. Theory

In this experiment, as in optical experiments (e.g. diffraction on a slit), we will observe interference that can only be explained when we meaningfully attribute wave-like quantities (frequency¹ ν , wavelength λ) to the electron, next to its particle quantities (mass m_e , charge e). The wavelength is related to the electrons energy and momentum as follows:

$$E = h \cdot \nu \quad (1)$$

$$E = p \cdot c \quad (2)$$

$$\Rightarrow p = \frac{h}{\lambda} \quad (3).$$

As later discussed in the description of the experimental setup, the electrons are accelerated between the cathode (which is at 0) and the anode (which is at potential U_a). Hereby the potential energy

$$E_{pot} = e \cdot U_a \quad (4)$$

is converted into kinetic energy completely. In general for a particle with mass m and velocity v we have:

$$E_{kin} = \frac{1}{2} m \cdot v^2, \quad m \cdot v = p. \quad (5).$$

With known anode potential U_a from (3)-(5) one can calculate the wavelength attributed to the electron:

$$\lambda = \frac{h}{\sqrt{2 \cdot m_e \cdot e \cdot U_a}} \quad (6).$$

m_e and e are the mass and the charge of the electron, respectively. The entity h is Planck's constant.

On impact of the electrons on the polycrystalline graphite film they are diffracted by the periodically stacked layers of carbon atoms in the crystallites. Constructive interference is observed under diffraction angles obeying the Bragg condition:

$$2 \cdot d \cdot \sin\theta = n \cdot \lambda \quad (7).$$

n is the diffraction order (the experimental setup only allows for the observation of the $n = 1$ reflex). Measurement of the radius r allows subsequent calculation of the diffraction angle θ and consequently calculation of the lattice constant:

$$d = \frac{2 \cdot R \cdot n \cdot \lambda}{r} \quad (8).$$

¹ ν is the Greek letter 'nu', not the Latin "v". The latter denotes the electron velocity in the following.

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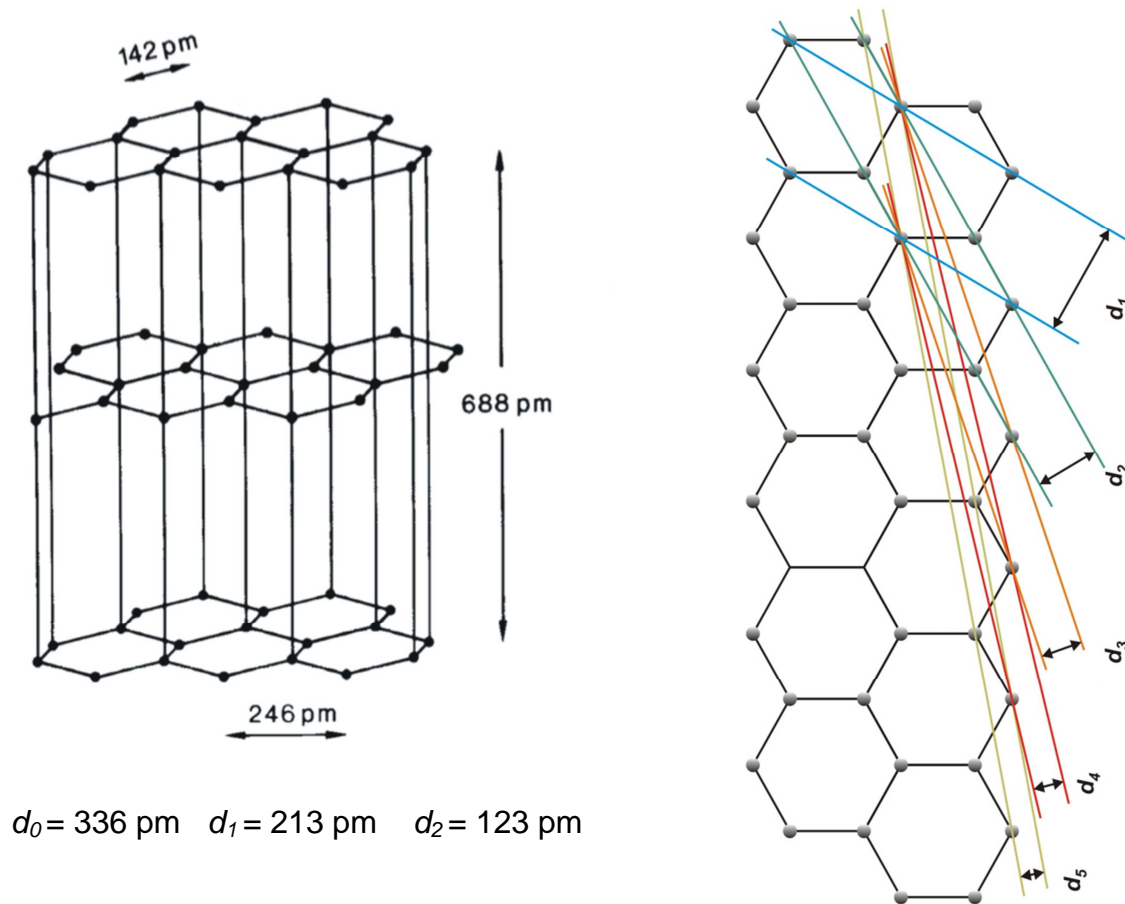


Figure 1: Graphite lattice and different lattice planes with spacings indicated.

The entities r and R are introduced in chapter 3 explaining the experimental setup. Convince yourself that for the angle α introduced in figure 2 the relation $\alpha=2\cdot\theta$ holds.

The experimental error is propagated from the measurement of the radius to the lattice constant²

$$\Delta d = \frac{2 \cdot R \cdot n \cdot \lambda}{r^2} \cdot \Delta r. \quad (9)$$

3. Experiment

Thermionic emission releases free electrons in an evacuated glass cylinder which are accelerated by a large potential U_a (of up to 10 kV). The target sample, a thin layer of graphite, is positioned at the end of the electron tube. The potentials G1 and G4 can be used to focus the electron beam but the electrons accumulate most (and for ease of calculation we assume all) of their kinetic energy while crossing G3. The wiring of the setup is depicted in figure 2.

The diffraction reflexes are casted onto the spherical fluorescent screen (Radius $R = 65 \text{ mm}$) that is positioned behind the sample. From Figure 2 we derive

$$\sin(2 \cdot \alpha) = \frac{r}{R} \quad (10)$$

We use the theorem $\sin(2\alpha) = 2\sin(\alpha) \cdot \cos(\alpha)$ and assume small angles α to obtain

$$\sin(2\alpha) \approx 2 \sin(\alpha). \quad (11)$$

² Check the script on error propagation. Formula (9) can be derived when the lattice constant d is treated as a function $d(r)$ of the radius.

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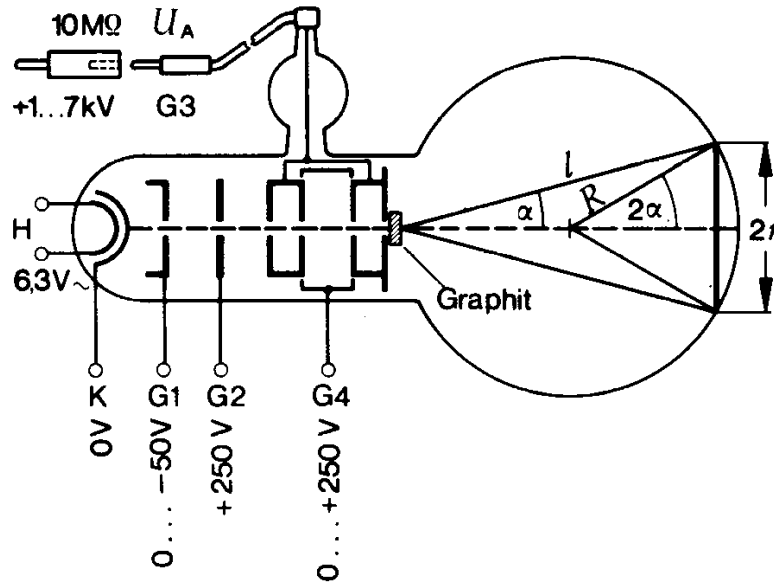


Figure 2: Setup of an electron tube with the four potentials G1 – G4, the graphite target and the fluorescent screen

For the measurement data analysis one needs the acceleration potential U_a as well as the radii of the diffraction rings r . The graphite sample is polycrystalline, i.e., it consists of a great number of small crystallites with random orientation. Therefore, the Bragg condition is always met in some of the crystallites and their random orientation does not favour a particular direction, resulting in a ring of diffraction reflexes. This sets the so-called Debye-Scherrer method apart from other methods using single-crystalline samples, in which point-like diffraction reflexes are observed. The diffraction rings are called Debye-Scherrer rings.

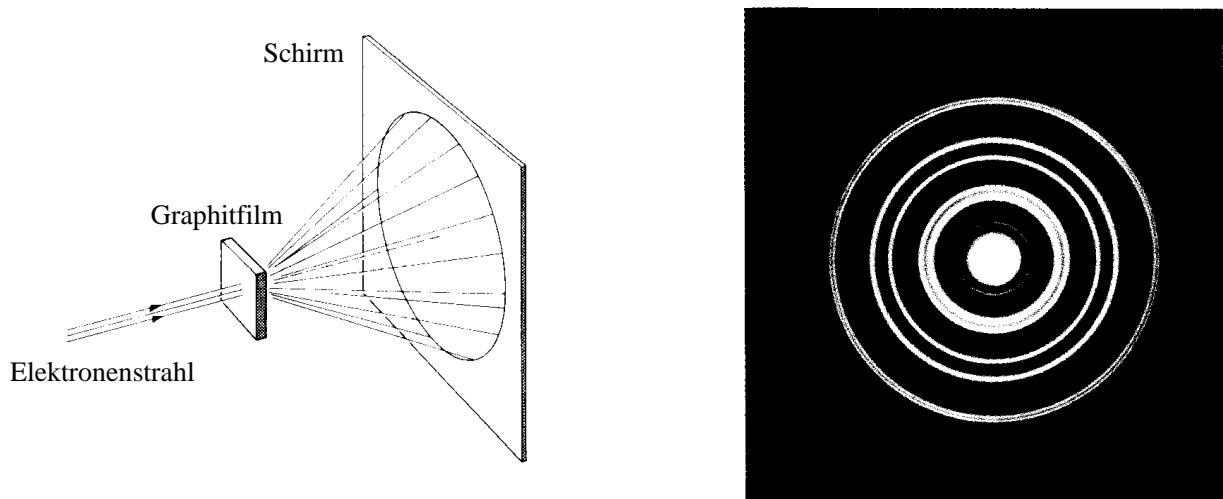


Figure 3: Diffraction pattern observed on the screen of the electron tube.

4. Questions

- (1) How does an electron tube operate? What is the purpose of the potentials G_1, \dots, G_4 ? What is a Wehnelt-Cylinder?
- (2) How can diffraction and interference be explained?
- (3) Understand the derivations of formulae (6) and (8)!
- (4) Look up the numerical values of m_e , e and h in formula (6)!

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- (5) How can Bragg diffraction be explained (consider the sketch)? Why is $\alpha = 2\theta$?
- (6) What kind of diffraction pattern would you expect for a single crystal?
- (7) Clarify error propagation calculation (mean value, standard deviation, relative/absolute error)!

5. Setup

Figure 4 depicts the wiring between the power sources and the electron tube.

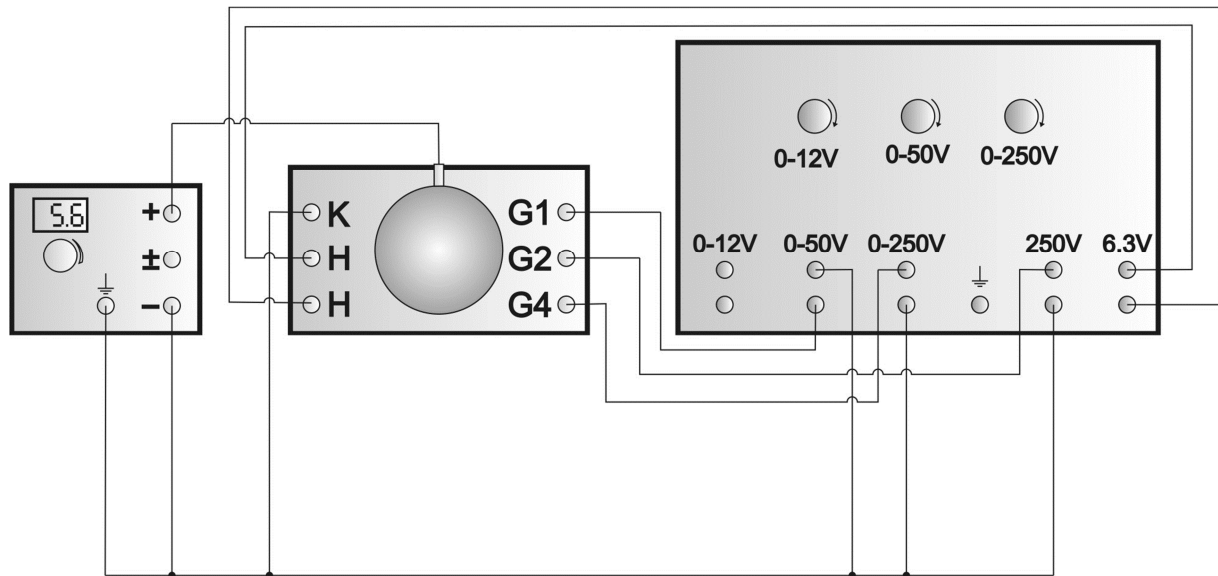


Figure 4: Wiring of the setup in the electron diffraction experiment.

Note: When performing the experiment it is advantageous to calculate from the constants h , m_e , e and R in formulae (6), (8) and (9) single numerical values to calculate λ , d and Δd . This saves time and decreases the likeliness of errors during calculation.

6. Tasks:

- For five different acceleration potentials ($3 \text{ kV} \leq U_a \leq 8 \text{ kV}$) measure two Debye-Scherrer rings, each four times to calculate a mean value and a standard deviation.
- From the data calculate the wavelengths of the electrons and the lattice constants d of graphite. For each potential U_a and each ring perform the error propagation using formula (9). As a second way to estimate the experimental precision, take the mean value and standard deviation of the different potentials U_a , according to equations (6) and (8) from the script on experimental error propagation.