Diffraction of light by a single slit and gratings

0. Introductory remarks

Diffraction of light at optical slits and gratings is an interference phenomenon characteristic for waves. The propagation of waves is dictated by so-called *wave equations*, which are differential equations in time and space. The mathematically correct description of wave propagation is rather complex but the *Huygens-Fresnel* principle introduced in the next chapter offers a rather instructive alternative approach to understand on a basic level interference phenomena like refraction and diffraction. If you encounter difficulties in understanding the next chapter, which is kept concise, please consult one of the many fundamental physics textbooks in which wave phenomena are explained. Some suggestions are given at the end of this handout.

It is very important to read the instructions in chapter 4 before you experiment with the laser.

1. Basic Theory

Light is an electromagnetic wave consisting of time-varying electric and magnetic fields, which oscillate in time and propagate in space similar to a mechanical wave or sound wave. Unlike mechanical waves electromagnetic waves require no medium and travel, e.g., in vacuum with a constant velocity ($c = 299792458$ m/s). The fields carry both energy and momentum. In the following it will suffice to consider the electric field $\vec{E}$ alone, which may be considered to play a role similar to the deflection from equilibrium position in a mechanical wave. A *plane wave* with frequency ν and wavelength λ travelling in the positive *x*-direction the electric field is mathematically described by a function

$$E(x,t) = E_0 \cdot \sin\left(2\pi\nu \cdot \left(t - \frac{x}{c}\right)\right) \tag{1}.$$

This equation tells us that the electric field $E(x,t)$ determined at a certain position x in space oscillates in time with a *frequency* ν and an *amplitude* E_0. The argument of the sin-function is often called *phase*. In three dimensions the points of a fixed phase in the wave of equation (1) form *yz*-planes, a fact which may be understood as motivation for the name. Note that with increasing time the spatial positions of fixed phase propagate into positive *x*-direction with the light velocity *c*. Because of the periodicity of the sin-function, points separated in *x*-direction by an integer multiple of c/ν have the same field value. Thus the wave field is spatially periodic with period $\lambda = c/\nu$, where λ is called the wavelength.

Plane waves, such as the one of equation (1), are mathematically very simple but by no means describe all possible wave fields. Much more complicated surfaces of constant phase are possible. In general, electromagnetic waves are generated by a number of oscillating point charges. A single point charge is the source of a *spherical wave*, which spreads with light velocity from the point of the charge into all spatial directions. The wave fronts then form sphere surfaces. The field value of such spherical waves must decrease with increasing distance from the source as indicated in Fig. 1.
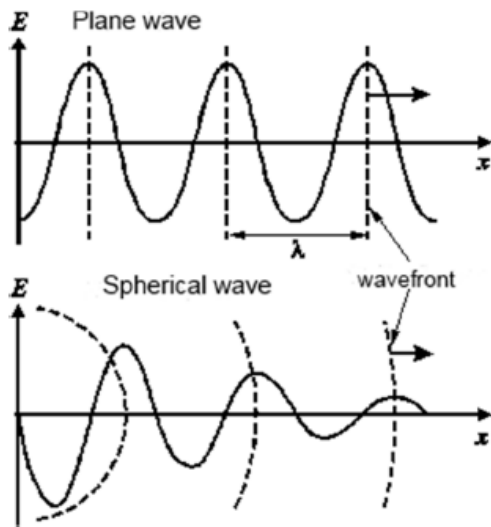
Fig. 1: Electric field and wave fronts of a plane wave (top) and a spherical wave (bottom).
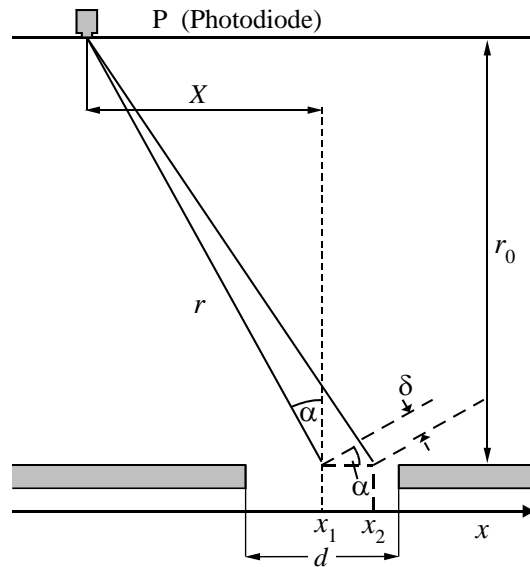
Fig. 2: Geometry of an optical slit. The positions x_1 and x_2 of two wavelet source points within the slit are marked. The lines connecting these points with the photo-diode denote the radii of wavelets inter-fering at the detector.

If the sources or the wave fronts at some instant are given we may calculated the wave fronts at any later time by the so-called *Huygens' principle*:

- Every point on a wave front serves as the source of a spherical secondary wavelet that spreads out in all directions with a speed equal to the speed of wave propagation.

- The new wave front at a later instant is the surface tangent to all the secondary spherical waves.

Note that only the wave front in forward direction, i.e., in propagation direction, is considered. The wave front simultaneously formed on the back half is disregarded. It was about 150 years later when Fresnel pointed out that Huygens' principle can be extended in order to explain diffraction effects showing that optical apertures do not cast perfect shadows. This extended version of Huygens' Principle is therefore often named *Huygens-Fresnel principle*:

- Every unobstructed point of a wave front at a given instant serves as a source of a spherical secondary wavelet with the same frequency as that of the primary wave. The amplitude of the wave field at any point in forward direction results from the superposition of all these wavelets, considering their amplitudes and relative phases.

It is important to note that superposition here means addition of the electric field values in contrast to addition of intensities. However, for both the human eye and most detectors the light intensity is of relevance, which is proportional to the square of the electric field:

$$I_{\text{Ligt}} \propto |E|^2 \tag{2}.$$

The intensity of the light field at the point of observation arises from the square of the electric field that results from summing up the electric fields of all wavelets. A second crucial point is

that the extended version explicitly allows superposition of spherical wavelets with different radii. Considering the phase[1] of the wavelets suffices.

In this laboratory exercise interference phenomena will be studied in the limit of *Fraunhofer* diffraction (as opposed to *Fresnel* diffraction). This means that, firstly, the beam impinging on the diffracting obstacle can be assumed to be a plane wave and, secondly, the distance between the diffracting object and the point of observation is very large. In our experiment the first condition is met in good approximation since we use a laser as light source. The second condition is met if the distance between the obstacle and the point of observation is much larger than the extent of the diffracting object, i.e., the width of the slit or the illuminated grid.

In this experiment the wave properties of light will be demonstrated by observing diffraction and interference effects from a single slit, a double slit and a series of slits. We will learn about the properties of diffraction gratings, which are important components in spectroscopic instruments such as monochromators and spectrographs.

2.   Diffraction from a single slit

Here we consider a monochromatic plane wave incident on a long narrow slit. According to geometric ray optics the transmitted beam would have the same cross section as the slit, but what we indeed observe is a drastically different diffraction pattern. The main properties of the diffraction pattern can be understood with the aid of Fig. 2 and Fig. 3. For the case of Fraunhofer diffraction, where the distance r between the slit and the point of observation P is very large compared to the slit width d, all rays origination from the slit can be considered to be parallel in good approximation. To calculate the resultant intensity with Huygens' principle we consider the optical path length difference δ between wavelets originating at two different points x_1 and x_2 within the slit. The difference in path length to the point P is $\delta = (d \sin \alpha / 2)$ where d is the slit width and α the angle between the optical axis of the slit (line normal to the aperture) and a line from the slit to the point of observation P.
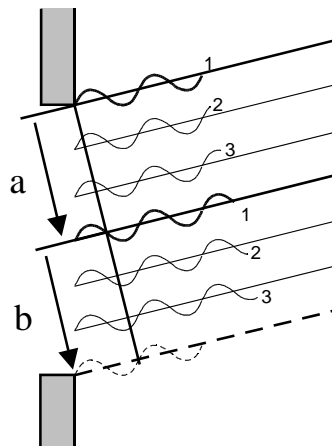

Fig. 3: Interference of wavelets originating from different positions within a single slit. The distance of the point of observation P is so far that the rays can be considered parallel. The numbers indicate wavelets that interfere destructively at P.

A simple geometrical argument explains compellingly why we observe far behind the slit a number of parallel stripes with different intensities. As indicated in Fig. 3 we divide the wave front within the slit into two parts (a) and (b) of the same width. At a certain angle each

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[1] The phase relation of wavelets with different radii must be well defined, however. This so-called *coherence* condition is well obeyed in laser light.

wavelet of the second half will have a partner wavelet in the first half with precisely a half-wavelength optical path difference. This, however, means that at the point of observation all wavelets from the slit add with their partner wavelets to zero because of their 180° phase difference. As a result at this angle the intensity must be at a minimum. The same will occur at any angle, at which we may divide the slit into an even number of slices with each having half a wavelength path difference to the wavelets of the next neighbouring slice. This, finally, results into the condiction for angles at which stripes of with vanishing intensity are observed

$$\sin \alpha_{\min} = m \frac{\lambda}{d} \quad \text{with} \quad m = \pm 1, \pm 2, \cdots \tag{4}.$$

If, on the other hand, the number of slices with wavelets of 180° phase shift is odd, one slice remains unextinguished and an intensity maximum occurs:

$$\sin \alpha_{\max} = \left(m + \frac{1}{2}\right)\frac{\lambda}{d} \quad \text{with} \quad m = \pm 1, \pm 2, \cdots \tag{5}.$$

The integer number *m* is often called *order* of the maximum. Mathematically the electric field at the point of observation results from a sum of all wavelets originating from any location of a wave front within the slits. This sum turns into an integral since the sources of the wavelets are arbitrarily close to each other. Finally, the intensity at point P is obtained by taking the square of the sum:

$$I_S = I_0 \left(\frac{\sin\left(\frac{\pi \cdot d}{\lambda} \sin \alpha\right)}{\frac{\pi \cdot d}{\lambda} \sin \alpha} \right)^2 \tag{6}.$$

Here I_0 is the intensity of the maximum observed at angle $\alpha = 0$. Note that the intensity of equation (6) indeed vanishes whenever the angle α meets equation (4). Furthermore, according to equation (6), we expect an intensity maximum (in quite good approximation for $(d/\lambda)\sin \alpha > 1$) whenever the argument of the sin-function is an odd multiple of $\pi/2$, which
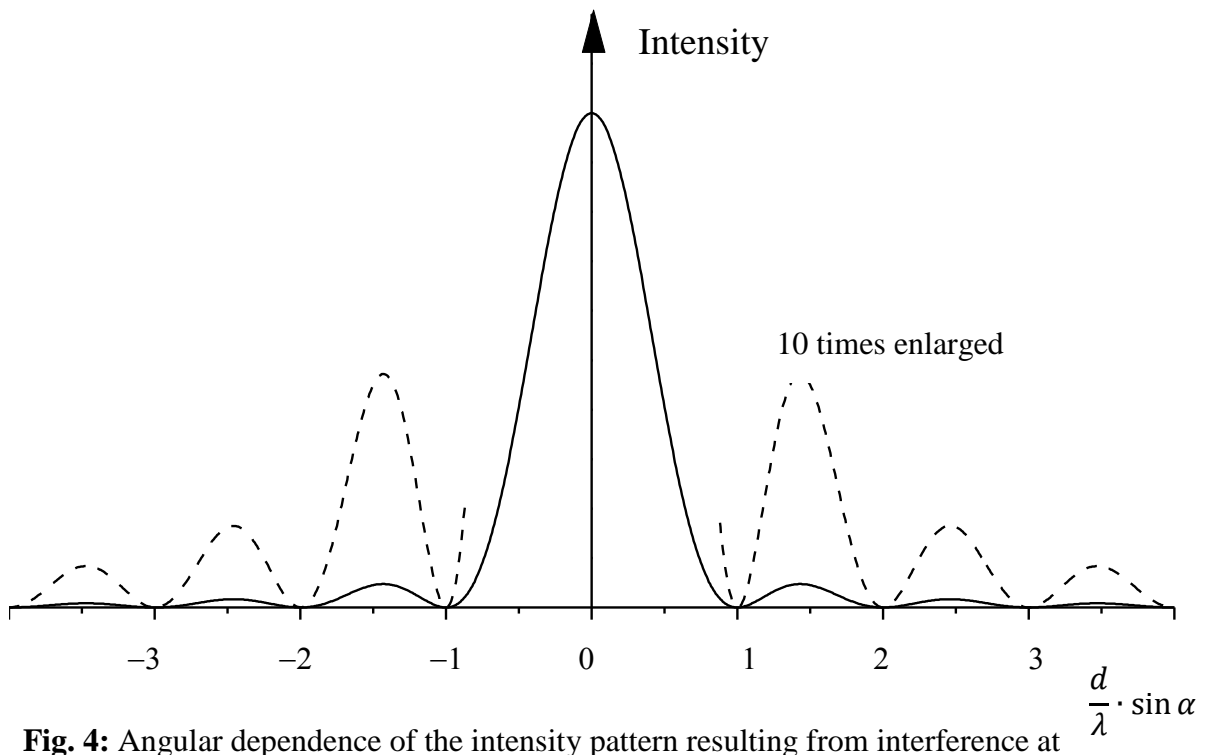

Fig. 4: Angular dependence of the intensity pattern resulting from interference at slit. The dashed line depicts the intensity ten times enlarged at angles beyond π.

is in correspondence to equation (5).

In Fig. 4 the intensity is depicted as function of the entity $(\pi d/\lambda) \cdot \sin \alpha$. A large main maximum is observed at zero deflection. The intensity of the much weaker side maxima decreases with increasing order m according to

$$I_{\max} = \frac{I_0}{\pi^2(|m| + 1/2)^2} \qquad (6).$$

3.   The Diffraction Grating

A diffraction grating consists of a large number N of identical slits each of width d and separated from the next by a distance D, as shown in Fig. 5. We will first consider the interference of wavelets originating from the slit centers. Of course, the result will then be only a good approximation if the slits widths are extremely small. But we will find an easy way to extend the result to the case of grids with slits of realistic widths. If we assume that the incident light is planar and perpendicular to the grid, the optical path difference of wavelets in neighboring slits is $\Delta = D \sin \alpha$, where the angle α is again taken between the normal to the grating and the line joining the grating and the point of observation P. If this path difference is equal to an integral multiple of wavelengths then all the slits will constructively interfere with each other and a bright spot will be measured at P:

$$\sin \alpha_{\max} = m\frac{\lambda}{D}, \qquad m = 0, \pm 1, \pm 2, \dots \ . \qquad (7).$$

These maxima are called principal maxima, because the superposition of the wavelets according to Huygens' principle will turn out to result in additional maxima, which are much weaker though. Mathematically the superposition results in a sum of the fields in the wavelets from all N slits in the grating. To obtain the intensity the result is squared again, which results in:
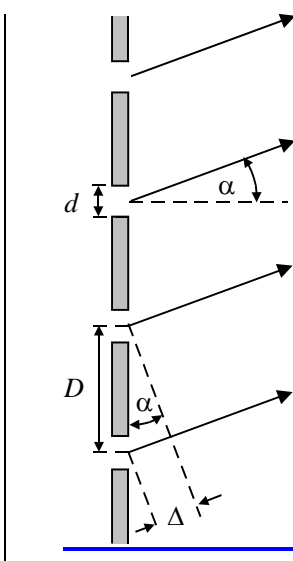

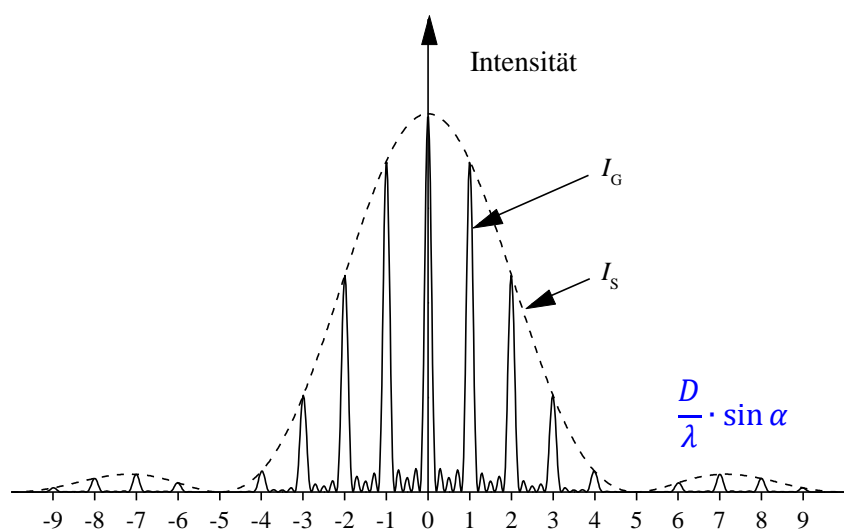**Fig. 5:** Geometry of an optical grating.

Fig. 6: Intensity pattern of an optical grid with $N = 5$ slits. The ratio between the width of the slits and the period of the grating is $\gamma = 1/5$. The dashed line marked I_S reflects the intensity pattern of a single slit in the grid.

$$I_G = I_S \left(\frac{\sin\left(\frac{N \cdot \pi \cdot D}{\lambda} \sin\alpha\right)}{\sin\left(\frac{\pi \cdot D}{\lambda} \sin\alpha\right)} \right)^2 \tag{8}.$$

Extinction occurs at angles α_{min} with

$$\sin\alpha_{min} = \frac{\ell}{N} \cdot \frac{\lambda}{D}, \qquad \ell = \pm 1, \pm 2, \dots \text{ und } \ell \neq N \tag{9}.$$

If the ℓ/N is an integer number, the denominator of I_G in equation (7) also vanishes. A mathematical consideration leads to the result that in this case the intensity must be $N^2 I_S$. These maxima occur at angles in accordance with equation (7), and they are much stronger as compared to the intensity observed at any other angle. According to equation (9), we expect between principal maxima $(N-1)$ intensity minima and $(N-2)$ weak secondary maxima, which strength relative to the principal maxima rapidly decreases with increasing N.

If in each slit we have only one source of a wavelet the factor I_S in equation (8) is the intensity of the wavelet. In a real grid we have to consider the finite width of the slit. If different wavelets originating in the same slit are interfering destructively, they will do so in a grating as well. The finite width of the slits is considered in equation (8) if the factor I_S contains the intensity pattern of a single slit of the grating. As a result we obtain the interference pattern depicted in Fig. 6. In the case depicted in this figure, the width of the slit amounts to $1/5^{th}$ of the grating period.

The location of the maxima does not depend on the number of slits. However, they become sharper and more intense as N is increased. Because of the wavelength dependence of the angles, at which principal maxima are observed, light consisting of different wavelengths can be spectrally resolved with an optical grating: After diffraction of the light beam at the grating the components with different wavelength are observed at different angles. Note that the resolving power of the grating, i.e., the resolution at which different wavelengths can be distinguished with the grating, increases with the number of illuminated slits because of the decreasing width of the principal maxima at gratings with increasing N.

4. The Experiments

> **WARNING: NEVER EVER look directly into the laser beam:**
>
> **Avoid reflecting the laser beam inadvertently.**
>
> **Take off watches and jewelry from hand and wrist.**
>
> **Don't sit down, because the laser beam may be on the same level as your eyes.**

4.1 Experimental set up

The light source is a 2 mW helium neon laser ($\lambda = 632.8$ nm). The mask containing the apertures is sketched in Fig. 7. The beam should be incident normal to the mask. This can be checked by observing the part of the beam that is reflected at the mask. The mask should be adjusted so that the reflected beam coincides with the incident beam. The diffraction patterns are measured using a photodiode which can be shifted perpendicular to the optical axis, defined by the incident laser beam. The intensity of the light I_L is proportional to the current of the photodiode I_P. The current of the photodiode is passed through an $R = 200\ \Omega$ resistor to produce a voltage $V = I_P R$, which is measured using a digital voltmeter. Note that I_L and I_P are different physical entities with different units.

The laser needs about 20 minutes to warm up and stabilize.

4.2. Tasks to be performed

Measure the distance r_0 along the optical axis between the slit and the arrow marked on the photodiode. This value is needed for calculating the angle of deflection $\tan \alpha = X/r_0$, where X is the displacement of the photodiode from the optical axis (see Fig. 2).

a) First, for the 0.2 mm single slit, measure the positions of several maxima and minima on both sides of the central maximum. Make sure that the displacement of the photodiode is perpendicular to the optical axis and it is essential to move the diode in small steps in order not to miss a maximum or minimum. You should then repeat this exercise using the 0.05 mm slit. For this measurement you may adjust the size of the steps because the peaks turn out to be not as sharp as in the case of the larger slit.

b) Observe the patterns from the double, triple and quadruple slits on a screen placed at a larger distance behind the photodiode. Make a drawing of the maxima and minima and describe the common features and the differences. Count the number of secondary maxima. For N slits one would expect $N - 2$ secondary maxima according to equations (7) and (9). Does your observation confirm this expectation?

c) Using the quadruple slit measure the positions and intensities of the principal and secondary maxima up to the second visible principal maximum, plus all of the principal maxima. This experiment is quite important for your future data analysis, so it is better to have about 200 points measured.

4.3. Data analysis

a) Make a table with the positions of the maxima and minima measured in 4.2a and prepare a plot showing the positions for both slits. Calculate the corresponding angles α_{min} and α_{max} and determine the wavelength from each of the angles. Determine an average wavelength together with a statistical error. Compare the thus determined wavelength with the one of the helium neon laser. Discuss possible error sources.

b) Compare the measured maxima intensities with the theoretical values according to equation 6 for the appropriate values of m. In equation (6) the factor I_0 corresponds to the intensity of the central maximum. How do you rate your experimental data?

c) Describe what you expect to observe for a wide ($d > 1$ m) and a very narrow ($a \sim \lambda$) slit. Frequently circular pinholes are used to produce wider laser beams for applications such as holography.

d) Make a table with the positions of the maxima and minima measured in 5.2c. Calculate the corresponding angles α_{min} and α_{max} and use these values to calculate the wavelength. Calculate the average wavelength and the standard deviation. Compare the result with the expected laser wavelength of 632.8 nm.

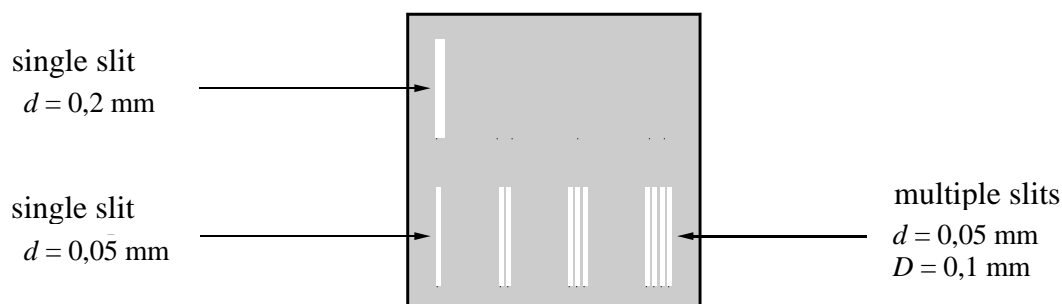e) The intensity distribution from the 0.05 mm slit should be plotted together with the distri-

single slit
$d = 0{,}2$ mm

single slit
$d = 0{,}05$ mm

multiple slits
$d = 0{,}05$ mm
$D = 0{,}1$ mm

Fig. 7: Sketch of the mask used in these experiments.

bution from the quadruple slit having the same slit width, similar to figure 6. In order to compare with figure 6, use for this plot the intensities I/I_0 normalized to the intensity of the zero order maximum and $(D/\lambda)\sin\alpha$ as the scale of the ordinate.

f)   Use the previous figure to explain what happens when the grating period D happens to be exactly twice the slit width d. How can this comply with the prediction of N - 2 secondary maxima? Indicate on the drawing, where you would expect secondary maxima if the slits were much narrower.

Hints for parts 4.3 b) and c)

The intensity distribution of N equally spaced slits can be plotted like in figure 6 substituting $x = (D/\lambda)\sin\alpha$ in equation (8) and normalizing to the intensity of the principal maxima:

$$\frac{I_G(N,\lambda,x)}{N^2 I_0} = \left(\frac{\sin(\gamma\cdot\pi\cdot x)}{\gamma\cdot\pi\cdot x}\right)^2 \cdot \left(\frac{\sin(N\cdot\pi\cdot x)}{\sin(\pi\cdot x)}\right)^2 \tag{10}$$

where N is the number of illuminated slits and $\gamma = d/D$ the ratio of the slit width to the grating period. The first factor on the right hand side in equation (10) reflects the contribution arising from the slit geometry. The second factor originates from the periodicity. Similarly the intensity of a single slit can be calculated from equation (3):

$$\frac{I_S(\gamma,x)}{I_0} = \left(\frac{\sin(\gamma\cdot\pi\cdot x)}{\pi\cdot\gamma\cdot x}\right)^2 \tag{11}$$

The values of the intensities can be calculated on a programmable calculator, but you have to leave out $x = 0$ and the values corresponding to the principal maxima (why?). More easily the intensity distribution can also be calculated using a plot program[2] running on a computer.

Hint for 4.3 c): It is instructive to plot the grating interference term, i.e., the second factor on the right hand side of equation (10) separately for comparison with the previous plot. Figures 8 and 9 show as an example the calculated interference pattern of a 5-fold slit grating with $\gamma = 0.25$. Every fourth principal maximum is suppressed by the diffraction minimum in the single-slit envelope term. This can be easily understood by comparison to the pattern in figure 9, in which the second factor of equation (10), which reflects the grating periodicity alone, is presented. Figures like the figures 8 and 9 can be easily calculated with the provided Mathematica notebook[3].

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[2] like, for instance, Maple, MathLab, MathCAD, Mathematica,
   GNU Octave (https://www.gnu.org/software/octave/),
   Maxima (http://maxima.sourceforge.net/) oder SciLab (http://www.scilab.org/)

[3] You can download a Mathematica notebook file (file extension .nb) from the TUHH-Praktikum homepage. Then you can calculate the intensity distributions for different numbers of slits and slit width to spacing ratios.

Note: First import the notebook in Mathematica and evaluate (command "evaluate Notebook"). Then you can enter the appropriate value of γ to create the required graph using the plot command. To evaluate cells in Mathematica use either Shift + <Enter> or numeric keypad <Enter>. A <Return> on the keyboard only produces a line break for multi-line input. Feel free to play around with the parameters in the plot command to get a feeling for the changes they produce. The size of the plots can be easily changed by dragging with the mouse. When you're happy with the figures you can export them with a right-click on the plot and save them by selecting in the context menu the command "save as …".
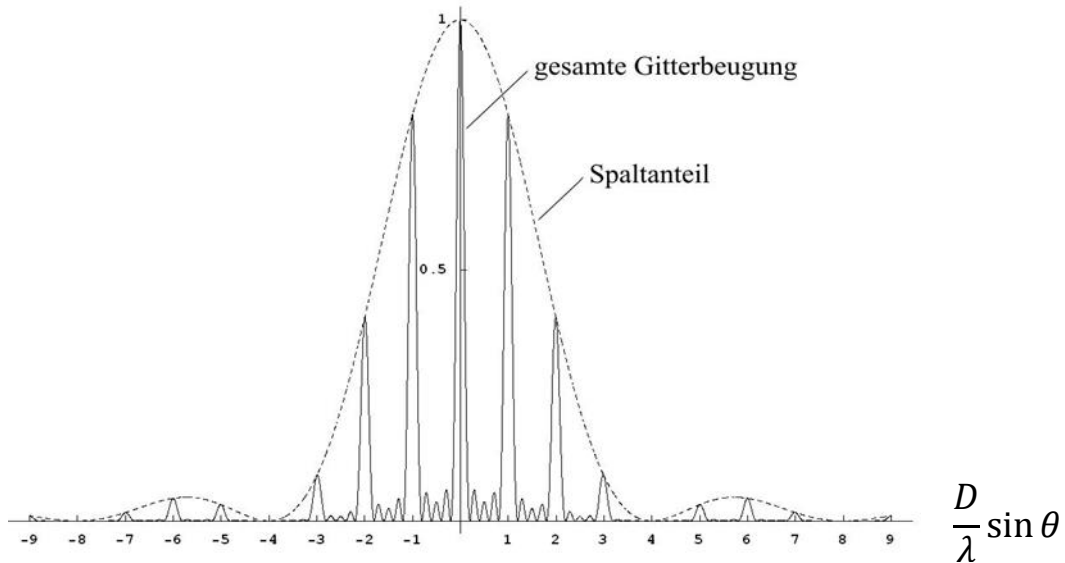
Figure 8: Calculated diffraction pattern of a grating with $N = 5$ slits and a ratio of slit-width to grating-period of ¼.
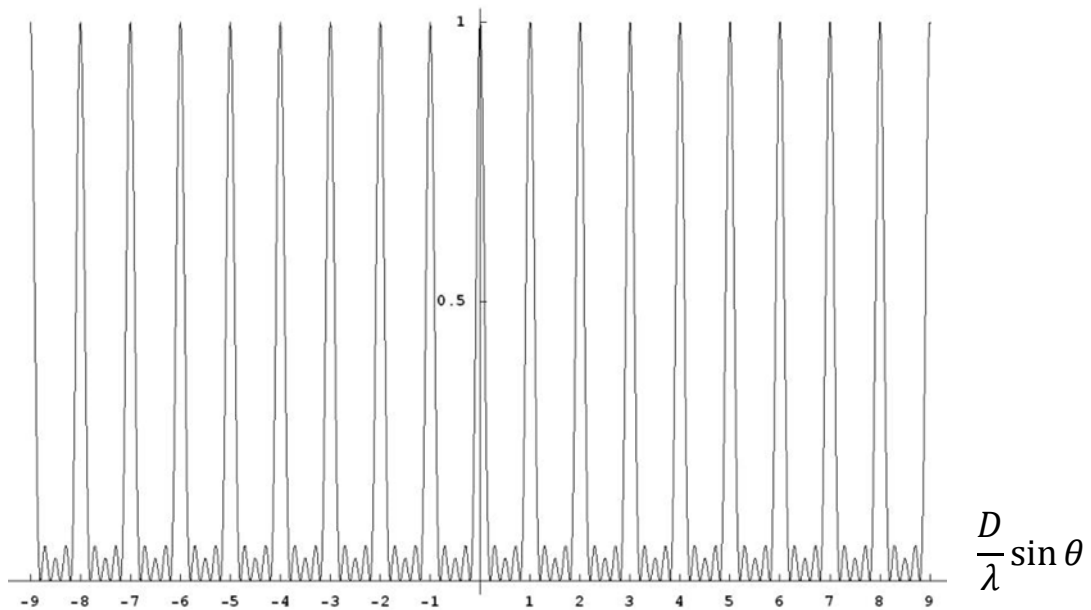

Figure 9: Diffraction pattern of a grating assuming a vanishing slit-width to period ratio.

5   Self-test questions

These questions are primarily intended to help you checking whether you understand the basic principles of interference. Similar questions might be asked by the experiment supervisor.

(1) How can light be described?

(2) Do our eyes detect light the amplitude or the intensity of light waves? What is the difference?

(3) Do light waves require a medium for propagation, as is the case for sound waves, or waves in water?

(4) How is a wave front defined? What is the difference between plane waves and those originating from a point source?

(5) Explain Huygens' principle. What is the optical path difference? How can you use Huygens' principle to understand the diffraction pattern from a narrow slit?

(6) What are the conditions are required for Fraunhofer diffraction? Sketch the diffraction pattern produced by a single slit under these conditions.

(7) How can you explain in a geometric consideration the result equation (3) for the positions of the diffraction maxima and minima of a single slit?

(8) Explain the diffraction pattern of an optical grating with same slit width d and grating spacing D when illuminated with coherent monochromatic light of a laser?

(9) Which effect does the width of the slits have on the intensity distribution?

(10) In the multiple slit diffraction pattern there are intense primary maxima and smaller secondary maxima. How does the number of secondary maxima between two adjacent primary maxima depend on the number of slits illuminated?

6 Preparation for the laboratory excercise on this experiment

Here are suggestions for preparations before your actually perform this laboratory excercise.

- Read these instructions carefully in order to know which measurement series have to be conducted.

- Plan your measurements and how you will tabulate the data and prepare the plots.

- Read the information on data analysis.

- The light intensity and thus the signal of the photodiode decrease with increasing distance r_0 between mask and detector. On the other hand, at too small distances the finite size of the photodiode limits spatial resolution. As a compromise, in the present set up the distance should be chosen in the interval $50\,\text{cm} \leq r_0 \leq 62\,\text{cm}$. Given the laser wavelength $\lambda = 632.8\,\text{nm}$ and a value for r_0 you can calculate and plot the expected positions X of the diffraction minima and maxima using equations (4) and (5).

7. Performing the Experiments

This experiment requires all the time available because the measurements have to be performed carefully.

- You can save time by assigning tasks among your peers. For example, one student can make the measurements, the second writes down the results, and the third student calculates the angles and compares sample data with theory. In this way errors can be spotted quickly, so you don't waste time.

- Prepare the tables for recording the data before the day of the experiment. Follow the hints given in Sect. 5.3.

- Follow the instructions given by the instructor. He may request you to do the measurements in a particular way. Make sure to obey the warning at the beginning of chapter 4: Never expose your eyes to the laser beam!

8 Literature

For further reading on the topic of wave phenomena, interference and diffraction you may consult any physics textbook like, for instance:

D. Halliday, R. Resnick: Physics, Wiley-VCH, 2005

P.M. Fishbane: Physics for scientists and engineers, Vol. 1, Prentice-Hall International, 2004

J.D. Cutnell: Physics, Student Solutions Manual, Wiley & Sons Inc, 2006

D.C. Giancoli: Physics for Scientists and Engineers, Prentice Hall, ISBN 0-13-021517-1.

D.C. Giancoli: Physik, Pearson Studium, 2006.

J. Orear: Physik, Hanser, 1989

E. Hering, R. Martin, M. Stohrer: Physik für Ingenieure, Springer 2007.

H. Vogel: „Gerthsen" Physik, Springer, 2004.

P.A. Tipler: Physik für Wissenschaftler und Ingenieure, Spektrum, 2004

Walcher: Praktikum der Physik, Springer 2013.