1. Basic Theory

Light is an electromagnetic wave with time-varying electric field $\vec{E}$ and magnetic field $\vec{B}$ components, which carry both energy and momentum. The waves are transverse with both the $\vec{E}$ and $\vec{B}$ vectors perpendicular to each other and they are both perpendicular to the direction of propagation of the wave. Unlike mechanical waves electromagnetic waves require no medium and travel in vacuum with a constant velocity. The magnitudes of the $\vec{E}$-and $\vec{B}$-fields are related by $E=c B$, where $c$ is the velocity of light in vacuum ( $c=299792458 \mathrm{~m} / \mathrm{s}$ exactly). Hence we only need to consider the electric field component. For a monochromatic wave with a well-defined frequency $f$ and wavelength $\lambda$ travelling in the $+x$ direction we can describe the electromagnetic wave by means of a wave function

$$
\begin{equation*}
E_{y}(x, t)=E_{\max } \cdot \cos (k x-\omega t) \tag{1}
\end{equation*}
$$

This equation describes a sinusoidal plane wave. $E_{y}(x, t)$ represents the instantaneous value of the y-component of $\vec{E}, E_{\max }$ is the amplitude of the E-field, $\omega$ is the angular frequency ( $\omega=2 \pi f$ ) and $k$ is the wave number $(k=2 \pi / \lambda$ ). The wave fronts of this plane wave are infinite parallel $y z$-planes moving in the positive $x$ direction. The phase of the wave (the argument of the cos-function in equation (1)) remains constant on the wave front.

Both the $\vec{E}$-field and $\vec{B}$ - field transport energy and for a sinusoidal wave in vacuum the average value of the power per unit area is the intensity given by

$$
\begin{equation*}
I=\frac{1}{2} \varepsilon_{0} c E_{\max }^{2} \tag{2}
\end{equation*}
$$

So far we have only been discussing plane waves in which at any instant the fields are uniform over any plane perpendicular to the direction of propagation. Electromagnetic waves are produced by an oscillating point charge and such a point source will produce spherical waves and the amplitude of the wave diminishes with increasing distance from the source as shown in Fig. 1.


Fig. 1
We can use Huygens' principle to calculate from any known shape of a wave front at some instant, the shape of the wave front at some later time. Every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave. The superposition of the waves gives the fields at the point of observation.

Therefore the principle states : Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.
In this principle we consider not only the phenomenon of interference(the combination of two or more waves to form a composite wave based on the superposition principle) but a diffraction (the bending of waves as they pass by an object or through an aperture), observation of which is of the main interest in our experiment. Diffraction is representing the case when we cannot apply the laws of geometric optics while the waves are propagating, for example it is not possible to explain why we see the light in the area of the geometrical
shadow. This is one of the proofs of the wave nature of the light. Diffraction is always observed when in the path of the light beam there are obstacles, such as slits or apertures. In this case, the deviation from linear propagation of light is called diffraction. Depending on what the experiment displays we can consider two types of diffractions.

Fraunhofer diffraction is seen in the parallel beam of light with the plane wave front. In this case we are assuming that the light source is very far away from the obstacle (the reason of the diffraction). Moreover, the screen on which we see the diffraction picture is considered to be on the infinite distance from the obstacle.

In the case of the Fresnel diffraction both the source and the screen are located close to each other. And with the increase of these distances Frenel diffraction is starting to have the character of the Fraunhofer case. Due to the fact that computations of diffraction patterns are easier to make for the Fraunhofer diffraction, the experimental exercises suggested in this laboratory work are based on the assumption that the monitored diffraction is the diffraction on the parallel light beams. We will use laser as a light source, because it is characterize by low divergence, high coherence, high monochromity and intensity. For the case of a regular lamp we would need to use lenses.

In this experiment the wave properties of light will be demonstrated by observing diffraction and interference effects from a single slit, a double slit and a series of slits. We will learn about the properties of diffraction gratings, which are important components in spectroscopic instruments such as monochromators and spectrographs.

## 2. Diffraction from a single slit

Here we consider a monochromatic plane wave incident on a long narrow slit. According to geometric optics the transmitted beam should have the same cross section as the slit, but what we observe is a diffraction pattern. For the case of far-field, or Fraunhofer diffraction, where the distance to the point of observation, $r$, is very large compared to the slit width, $a$, so the rays can be considered to be parallel, we can calculate the resultant intensity using Huygens' principle. First, we consider the path length difference between a ray from one edge of the slit and a ray from the center of the slit. The difference in path length to a point $P$ is $(a \cdot \sin \theta) / 2$ where $a$ is the slit width and $\theta$ is the angle between the perpendicular to the slit and a line from the center of the slit to $P$.


Figure 2: Single slit diffraction wave trains at the first diffraction minimum

If the path difference happens to be equal to $\lambda / 2$; as shown in Fig. 2 , then light from these two points arrives at point P with a half-cycle phase difference, and cancellation occurs. Similarly, light from pairs of points adjacent to the points we first chose will also cancel and light from all points on one half of the slit cancels out the light from corresponding points in the other half. The result is complete cancellation and a dark fringe in the diffraction pattern occurs whenever

$$
\begin{aligned}
& \qquad \frac{a}{2} \sin \theta=\frac{\lambda}{2} \\
& \text { or in general } \quad a \sin \theta=m \cdot \lambda \quad(m= \pm 1, \pm 2, \ldots)
\end{aligned}
$$

Frequently the values of $\theta$ are small, so the approximation $\sin \theta \simeq \theta$ (in radians) is valid.
The intensity distribution in the single-slit diffraction pattern can be derived by the addition of phasors. The resultant $\vec{E}$-field is

$$
E_{p}=E_{0} \cdot \frac{\sin \beta / 2}{\beta / 2}
$$

So the intensity distribution for a single slit is given by

$$
I_{s}=I_{0} \cdot\left(\frac{\sin \beta / 2}{\beta / 2}\right)^{2}
$$

The phase difference is $\frac{2 \pi}{\lambda}$ times the path difference so $\beta=\frac{2 \pi}{\lambda} \cdot a \sin \theta$

$$
\begin{equation*}
I_{\mathrm{S}}=I_{0}\left(\frac{\sin \left(\frac{\pi \cdot a}{\lambda} \sin \theta\right)}{\frac{\pi \cdot a}{\lambda} \sin \theta}\right)^{2} \tag{3}
\end{equation*}
$$

where $I_{0}$ is the intensity at $\theta=0$. The intensity distribution $I_{s}(\theta)$ has a central maximum at $\theta=0$ and minima for $\theta_{\text {min }}$ :

$$
\begin{equation*}
\sin \theta_{\min }=m \frac{\lambda}{a}, \quad m= \pm 1, \pm 2, \ldots \tag{4}
\end{equation*}
$$

where $m$ is the order of diffraction. The elementary waves from the two edges of the slits have a path difference $\delta=a \cdot \sin \theta=m \cdot \lambda$.

We might expect intensity maxima where the sine function has the value $\pm 1$, i.e.
$\beta= \pm \pi, \pm 3 \pi, \ldots$ or in general $\beta= \pm(2 m+1) \pi \quad(m=0,1,2,3, \ldots)$
There is no maximum at $\beta= \pm \pi$, so this is only an approximation, but we can use it to estimate the intensities of the $\mathrm{m}^{\text {th }}$ subsidiary maximum

$$
\begin{equation*}
I_{m} \approx \frac{I_{0}}{(|m|+1 / 2)^{2} \cdot \pi^{2}} \tag{6}
\end{equation*}
$$

The intensities of the side maxima decrease very rapidly - the actual intensities are $0.0472 \mathrm{I}_{0}$ $0.0165 \mathrm{I}_{0}, 0.0083 \mathrm{I}_{0}$


Figure 3: Diffraction pattern from a single slit

## The Diffraction Grating

A diffraction grating consists of a large number $N$ of slits each of width $a$ and separated from the next by a distance $d$, as shown in Figure 4.


Figure 4: Multiple slit interference

If we assume that the incident light is planar and diffraction spreads the light from each slit over a wide angle so that the light from all the slits will interfere with each other. The relative path difference between each pair of adjacent slits is $\boldsymbol{\delta}=\boldsymbol{d} \boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}$. If this path difference is equal to an integral multiple of wavelengths then all the slits will constructively interfere with each other and a bright spot will appear on the screen at an angle $\boldsymbol{\theta}$. Thus, the condition for the principal maxima is given by

$$
d \sin \theta=m \lambda, m=0, \pm 1, \pm 2, \pm 3
$$

If the wavelength of the light and the location of the m-order maximum are known, the distance $d$ between slits may be readily deduced. And vice versa, one can easily fine the wavelength knowing the distance between the slits and the order of the maximum.

The location of the maxima does not depend on the number of slits, N. However, the maxima become sharper and more intense as $N$ is increased. For $N$ slits there are $N-1$ minima between each pair of principal maxima and the width of the maxima can be shown to be inversely proportional to $N$, while the height of each maxima is proportional to $N^{2}$. The chromatic resolving power of a grating spectrograph is given by

$$
R=\frac{\lambda}{\Delta \lambda}=m \cdot N
$$

Where $\Delta \lambda$ is the minimum wavelength difference that can be resolved, $N$ is the total number of slits, or lines, of the grating illuminated and $m$ is the order of diffraction.
The pattern produced by several very narrow slits consists of the principal maxima given by $d \cdot \sin \theta=m \cdot \lambda$ and between these maxima are small secondary maxima which become smaller with increasing N . This interference pattern is convolved with the diffraction pattern from the individual slits shown in Fig. 3 to give the overall intensity distribution shown in Fig. 5.


Figure 5: Intensity distribution from 5 equally spaced slits ( $N=5$ ).

Mathematically, the intensity distribution is given by the product of two functions - the slit diffraction function $\mathrm{I}_{\mathrm{S}}$ and the grating interference function:

$$
\begin{equation*}
I=I_{\mathrm{S}}\left(\frac{\sin \left(\frac{N \cdot \pi \cdot d}{\lambda} \sin \theta\right)}{\sin \left(\frac{\pi \cdot d}{\lambda} \sin \theta\right)}\right)^{2} \tag{7}
\end{equation*}
$$

## 5 The Experiments

## WARNING NEVER look directly into the laser beam:

Avoid reflecting the laser beam inadvertently.
Take off watches and jewelry.
Don't sit down, because the laser beam may be on the same level as your eyes.

### 5.3 Experimental Setup



Figure 6: Geometry of the diffraction experiment with a single slit


Figure 7: Mask with the single and multiple slits

The light source is a 2 mW helium-neon laser ( $\lambda=632.8 \mathrm{~nm}$ ). The beam should be incident normal to the mask with the apertures so the beam reflected from the mask should be adjusted to lie along the incident beam. The diffraction patterns are measured using a photodiode which can be translated perpendicular to the optical axis, defined by the incident laser beam. The intensity of the light $I_{L}$ is proportional to the current in the photodiode ( $I_{P}$ ), but of course the units are different. The current from the photodiode passes through a $200 \Omega$ resistor to produce a voltage $(\mathrm{V}=\mathrm{IR})$ which is measured using a digital voltmeter.

The laser needs about 20 minutes to warm up and stabilize.

### 5.2. Tasks to be performed

Measure the distance $r_{0}$ along the optical axis from the slit to the arrow marked on the photodiode. This value is needed for calculating the angle of deviation $\tan \theta=X / r_{0}$, where $X$ is the displacement of the photodiode from the optical axis (see Fig. 6 ).
a) First, for the 0.2 mm single slit, measure the positions of several maxima and minima on both sides of the central maximum. The displacement of the photodiode must be perpendicular to the optical axis and it is essential to move the diode in small steps to record all of the maxima and minima. You should then repeat this exercise using the 0.05 mm slit. For this measurement, since the peaks are not sharp enough don't make too many steps.
b) Observe the patterns from the double, triple and quadruple slits on a screen placed at a larger distance behind the photodiode. Make a drawing of the maxima and minima and describe the common features and the differences. Count the number of secondary maxima (for N slits you should observe N-2 secondary maxima).
c) Using the quadruple slit measure the positions and intensities of the principal and secondary maxima up to the second visible principal maximum, plus all of the principal maxima. This experiment is quite important for your future data analysis, so better to have about 200 points measured.

### 5.3. Data analysis

a) Make a table with the positions of the maxima and minima measured in 5.2 a and plot the cases for two slits together. Compare the effect when one slit being is thinner and explain what we see there. Compare the measured intensities of the maxima with the theoretical values from equation 6 for the appropriate values of $m$. $I_{0}$ corresponds to the intensity of the central maximum. Describe what you expect to observe for a wide $(a>1 m)$ and a very narrow $(a \sim \lambda)$ slits. Frequently circular pinholes are used to produce wider laser beams, e.g. for holography.
b) Make a table with the positions of the maxima and minima measured in 5.2 c , calculate the corresponding deviation angles $\theta_{\max }$ and $\theta_{\min }$ and use these values to calculate the wavelength of the laser. Calculate the average value of the wavelength and the standard deviation. Compare the result with the expected laser wavelength of 632.8 nm .
c) The intensity distribution from the 0.05 mm slit should be plotted together with the distribution from the quadruple slit with the same slit width in the same figure (similar to Fig. 6). Plot $I / I_{0}$ against $d \cdot \sin \theta / \lambda$.
d) Use the previous Figure to explain what happens when the grating spacing $d$ is twice the slit width
a. Is this consistent with the theory which predicts $\mathrm{N}-2$ secondary maxima? Indicate on the drawing, where you would expect secondary maxima if the slits were much narrower.

